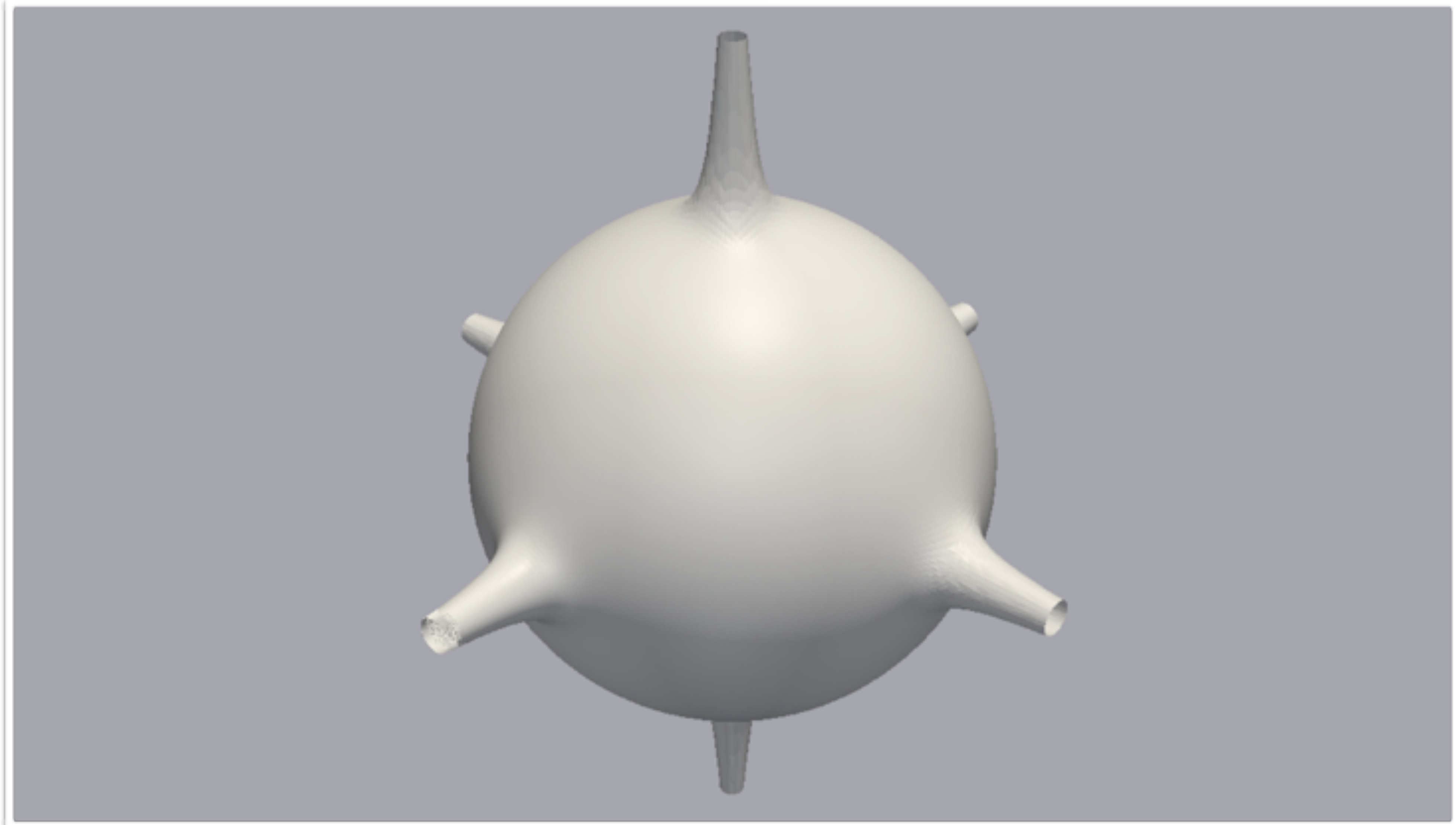
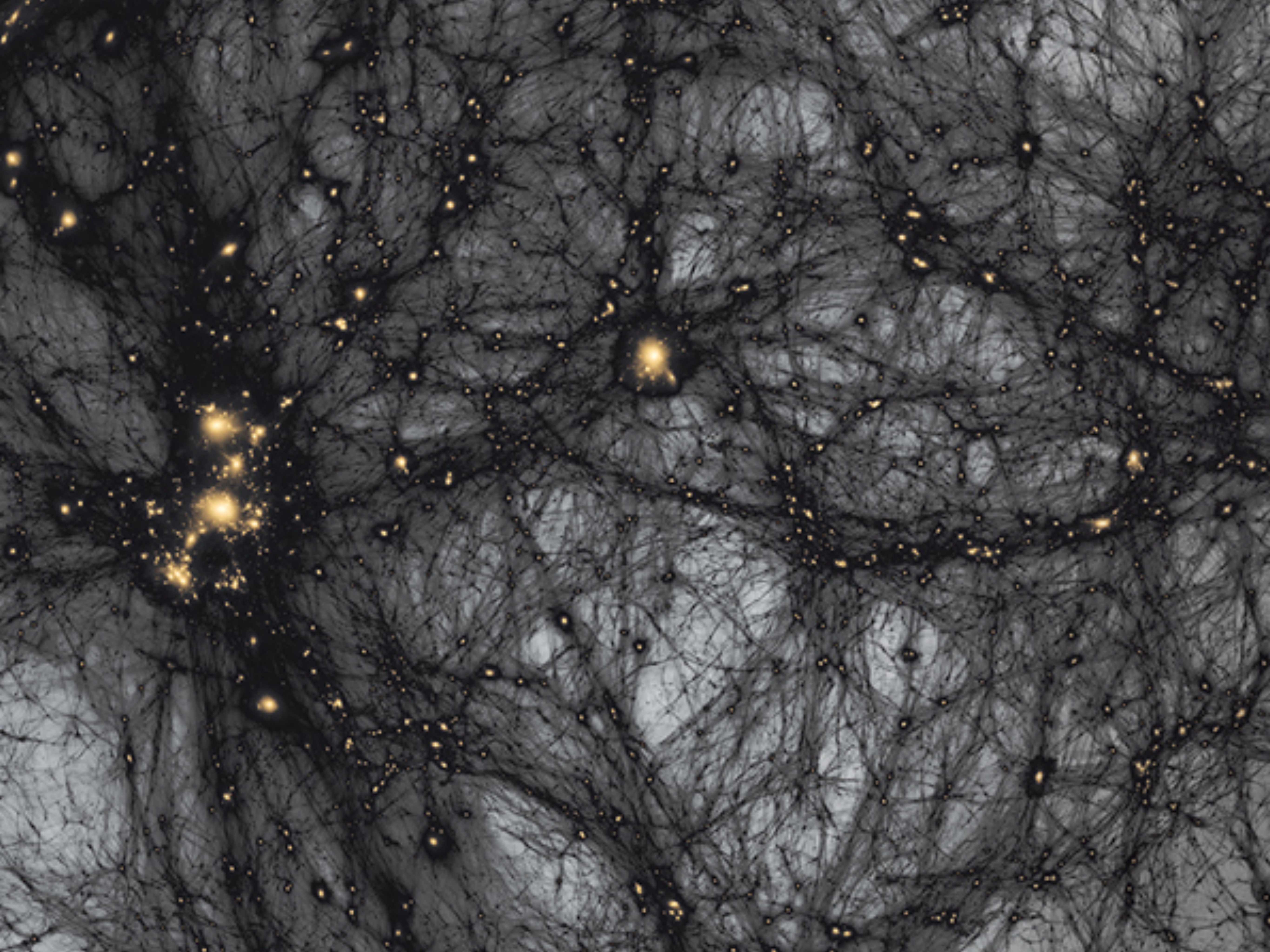
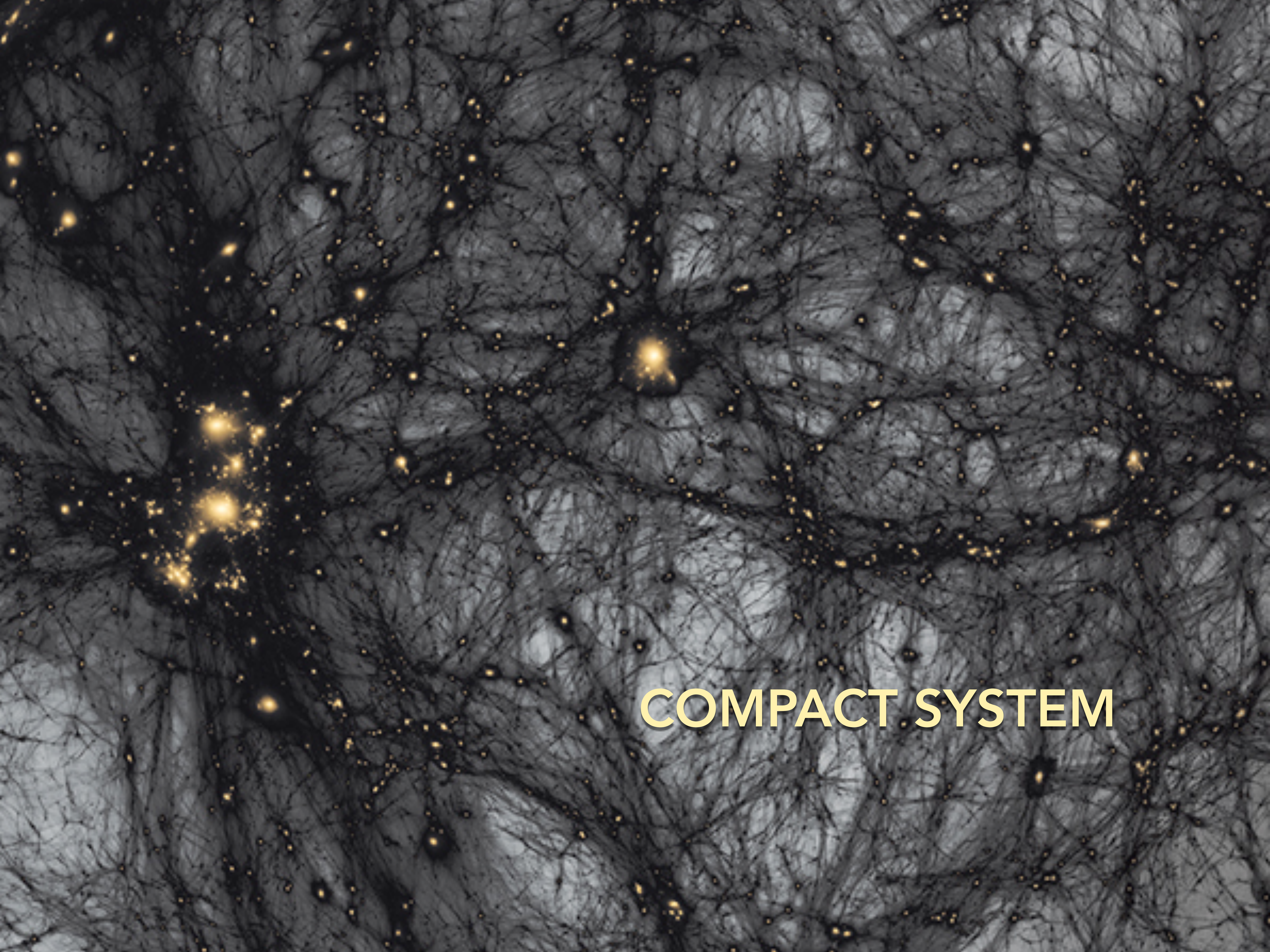


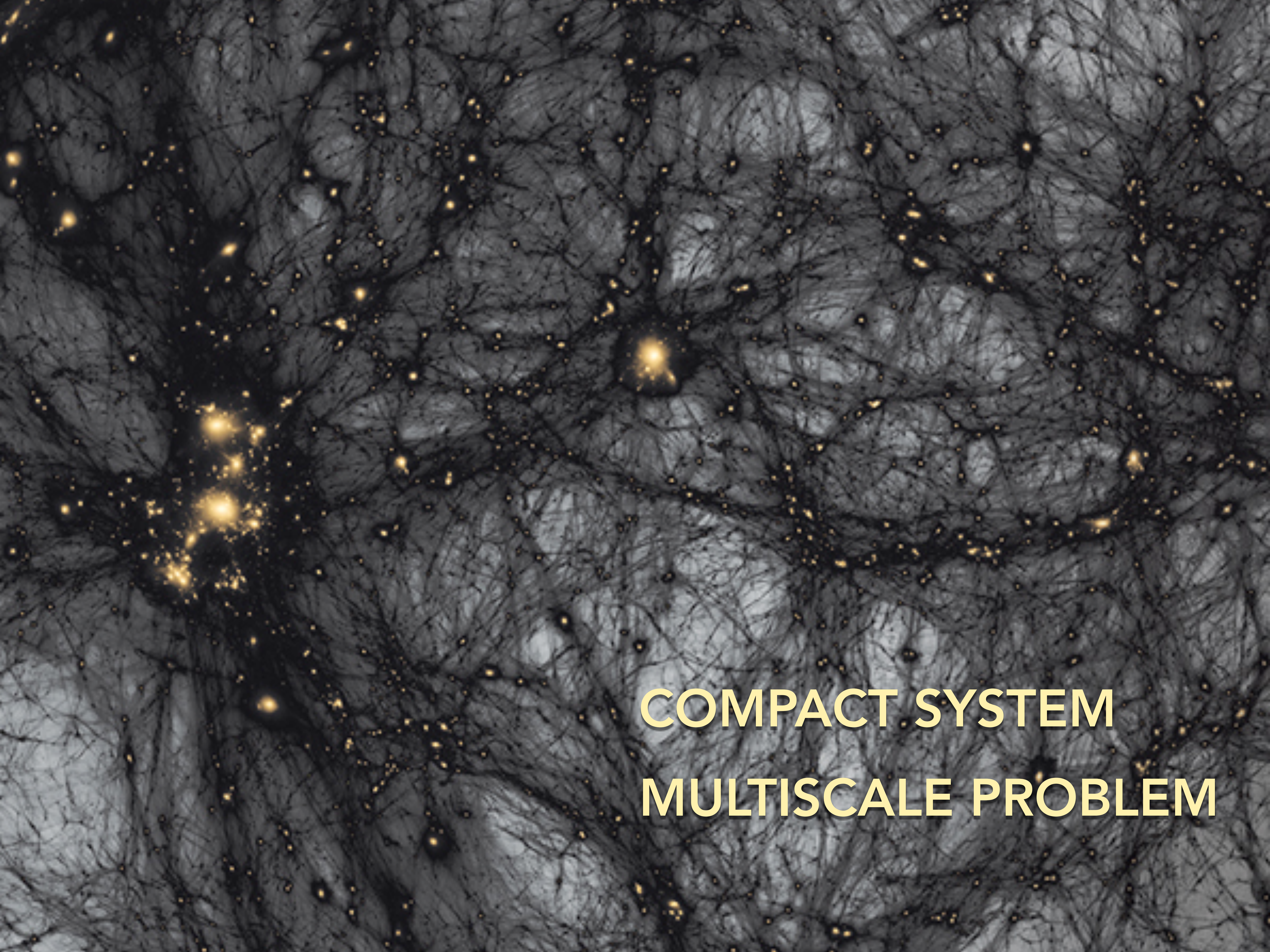
Eloisa Bentivegna, Università degli Studi di Catania

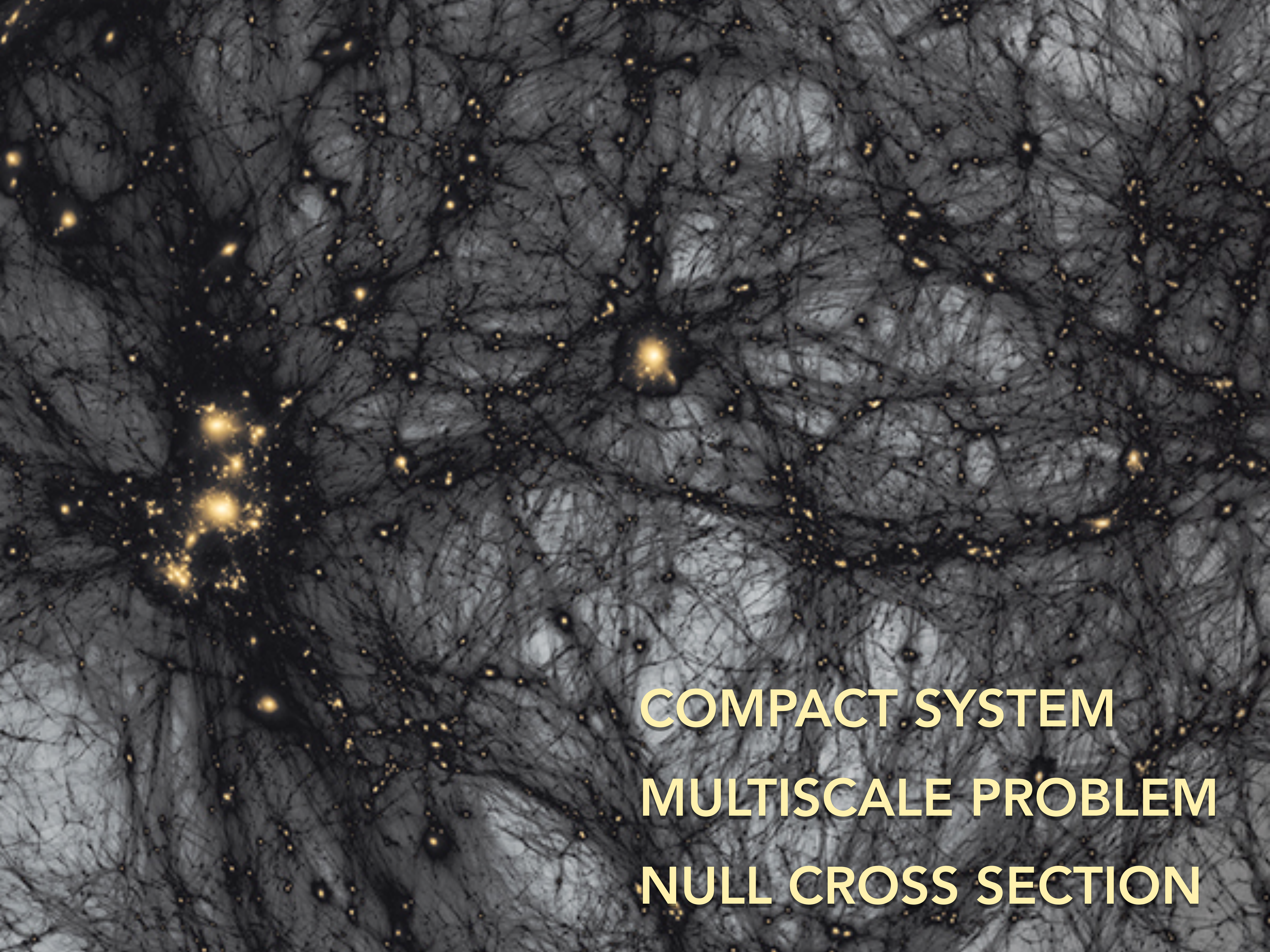
Cosmological Modelling with Numerical Relativity

21st General Relativity Conference
Columbia University
July 14th, 2016




COMPACT SYSTEM


COMPACT SYSTEM
MULTISCALE PROBLEM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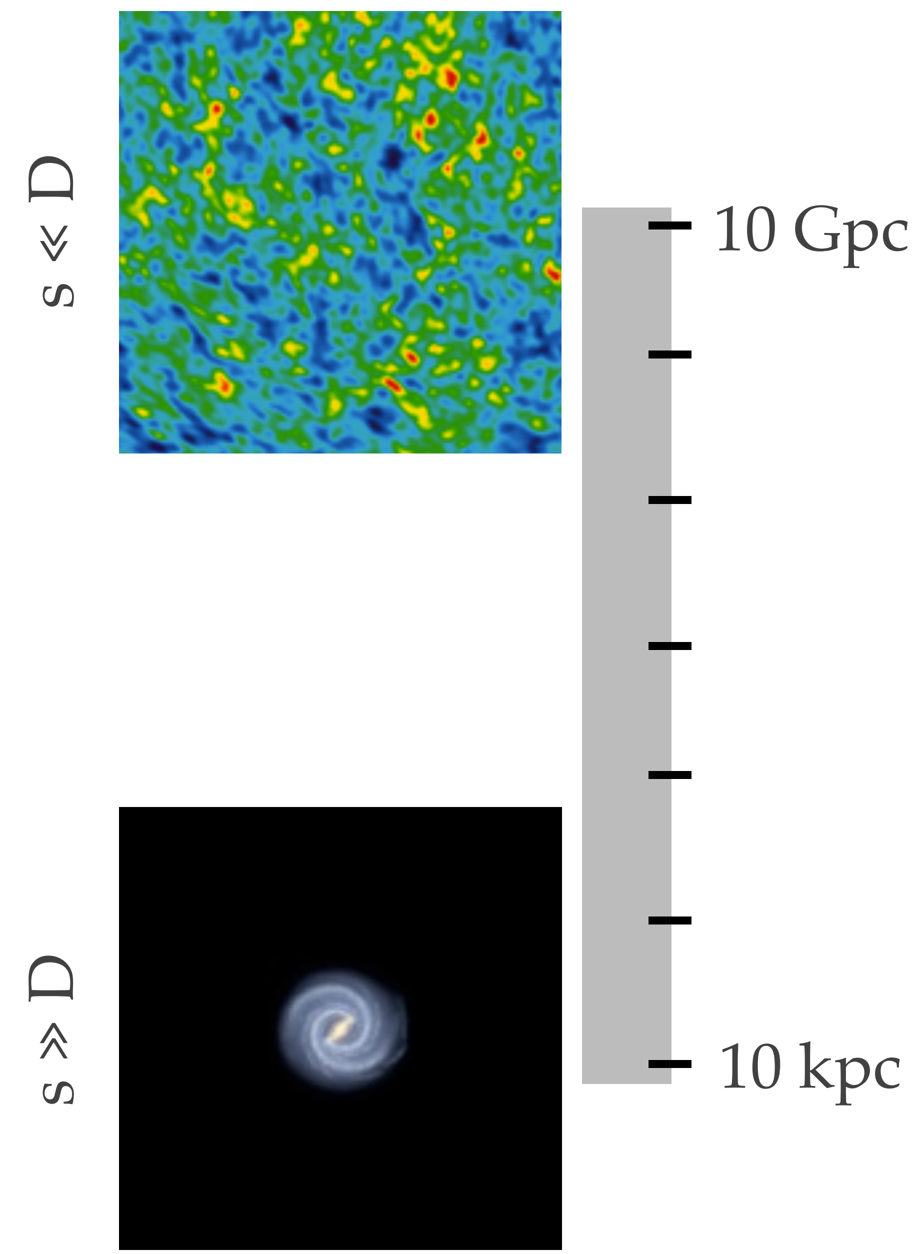

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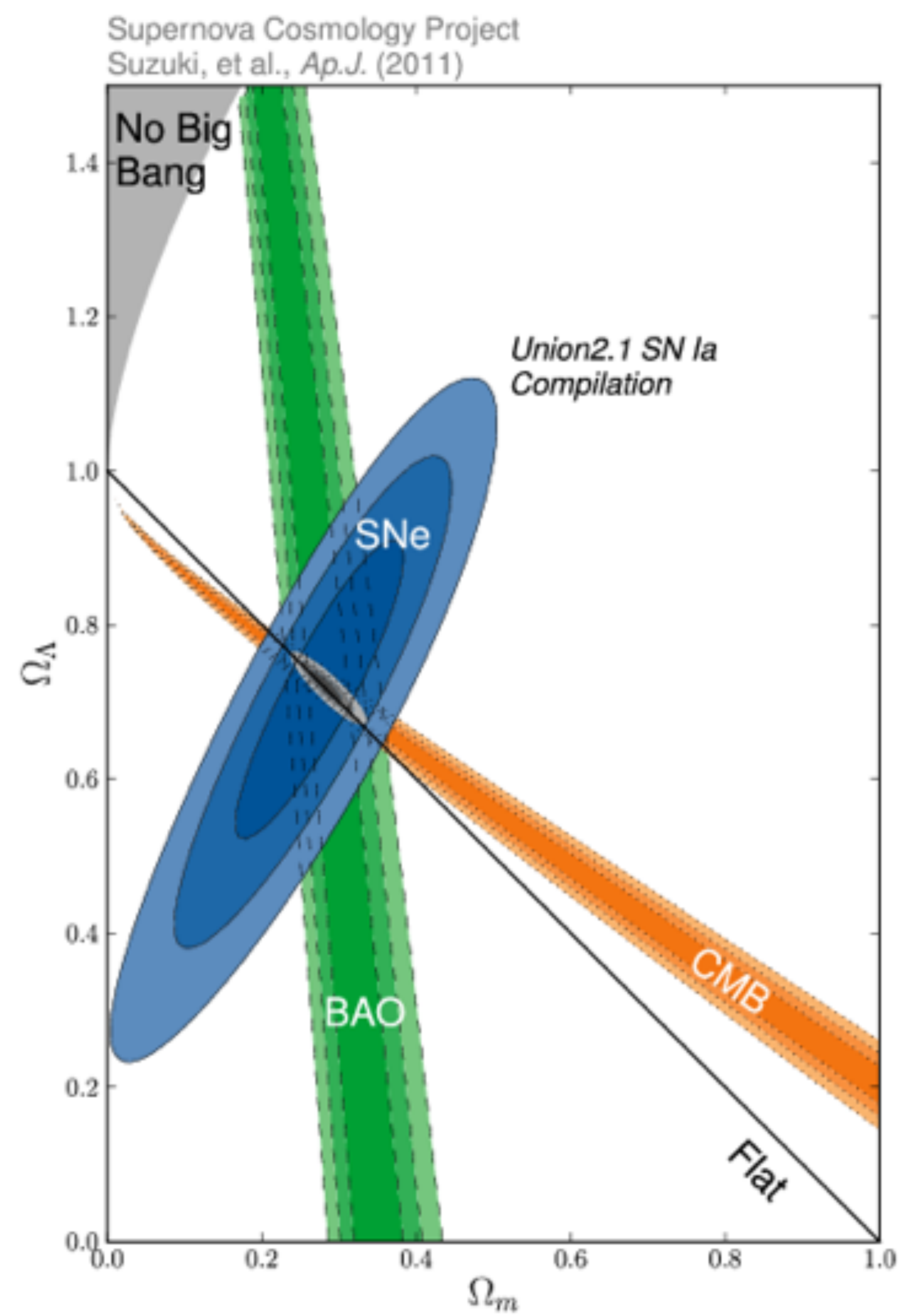

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Parameter	Prior range	Baseline	Definition
$\omega_b \equiv \Omega_b h^2$	[0.005, 0.1]	...	Baryon density today
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τ	[0.01, 0.8]	...	Thomson scattering optical depth due to reionization
Ω_K	[−0.3, 0.3]	0	Curvature parameter today with $\Omega_{tot} = 1 - \Omega_K$
$\sum m_\nu$	[0, 5]	0.06	The sum of neutrino masses in eV
$m^{\rm eff}_{\nu,\,\rm sterile}$	[0, 3]	0	Effective mass of sterile neutrino in eV
w_0	[−3.0, −0.3]	−1	Dark energy equation of state[a], $w(a) = w_0 + (1-a)w_a$
w_a	[−2, 2]	0	As above (perturbations modelled using PPF)
$N_{\rm eff}$	[0.05, 10.0]	3.046	Effective number of neutrino-like relativistic degrees of freedom (see text)
Y_P	[0.1, 0.5]	BBN	Fraction of baryonic mass in helium
A_L	[0, 10]	1	Amplitude of the lensing power relative to the physical value
n_s	[0.9, 1.1]	...	Scalar spectrum power-law index ($k_0 = 0.05{\rm Mpc}^{-1}$)
n_t	$n_t = -r_{0.05}/8$	Inflation	Tensor spectrum power-law index ($k_0 = 0.05{\rm Mpc}^{-1}$)
$dn_s/d\ln k$	[−1, 1]	0	Running of the spectral index
$\ln(10^{10}A_s)$	[2.7, 4.0]	...	Log power of the primordial curvature perturbations ($k_0 = 0.05\,{\rm Mpc}^{-1}$)
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σ_8		...	RMS matter fluctuations today in linear theory
z_{re}		...	Redshift at which Universe is half reionized
H_0	[20,100]	...	Current expansion rate in ${\rm km\,s^{-1}Mpc^{-1}}$
$r_{0.002}$		0	Ratio of tensor primordial power to curvature power at $k_0 = 0.002\,{\rm Mpc}^{-1}$
$10^9 A_s$		...	$10^9\times$ dimensionless curvature power spectrum at $k_0 = 0.05\,{\rm Mpc}^{-1}$
$\omega_m \equiv \Omega_m h^2$		...	Total matter density today (inc. massive neutrinos)
z_*		...	Redshift for which the optical depth equals unity (see text)
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k_D		...	Characteristic damping comoving wavenumber (${\rm Mpc}^{-1}$)
$100\theta_D$		...	$100\times$ angular extent of photon diffusion at last scattering (see text)
z_{eq}		...	Redshift of matter-radiation equality (massless neutrinos)
$100\theta_{eq}$		...	$100\times$ angular size of the comoving horizon at matter-radiation equality
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[Planck 2013 results XVI]

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A^{PS}_{100}	[0, 360]	Contribution of Poisson point-source power to $\mathcal{D}^{100\times100}_{3000}$ for *Planck* (in $\mu\mathrm{K}^2$)
A^{PS}_{143}	[0, 270]	As for A^{PS}_{100}, but at 143 GHz
A^{PS}_{217}	[0, 450]	As for A^{PS}_{100}, but at 217 GHz
$r^{\mathrm{PS}}_{143\times217}$	[0, 1]	Point-source correlation coefficient for *Planck* between 143 and 217 GHz
A^{CIB}_{143}	[0, 20]	Contribution of CIB power to $\mathcal{D}^{143\times143}_{3000}$ at the *Planck* CMB frequency for 143 GHz (in $\mu\mathrm{K}^2$)
A^{CIB}_{217}	[0, 80]	As for A^{CIB}_{143}, but for 217 GHz
$r^{\mathrm{CIB}}_{143\times217}$	[0, 1]	CIB correlation coefficient between 143 and 217 GHz
γ^{CIB}	[−2, 2] (0.7 ± 0.2)	Spectral index of the CIB angular power ($\mathcal{D}_\ell \propto \ell^{\gamma^{\mathrm{CIB}}}$)
A^{tSZ}	[0, 10]	Contribution of tSZ to $\mathcal{D}^{143\times143}_{3000}$ at 143 GHz (in $\mu\mathrm{K}^2$)
A^{kSZ}	[0, 10]	Contribution of kSZ to $\mathcal{D}_{3000}$ (in $\mu\mathrm{K}^2$)
$\xi^{\mathrm{tSZ}\times\mathrm{CIB}}$	[0, 1]	Correlation coefficient between the CIB and tSZ (see text)
c_{100}	[0.98, 1.02] (1.0006 ± 0.0004)	Relative power spectrum calibration for *Planck* between 100 GHz and 143 GHz
c_{217}	[0.95, 1.05] (0.9966 ± 0.0015)	Relative power spectrum calibration for *Planck* between 217 GHz and 143 GHz
β^i_j	(0 ± 1)	Amplitude of the jth beam eigenmode ($j = 1$–5) for the ith cross-spectrum ($i = 1$–4)
$A^{\mathrm{PS,ACT}}_{148}$	[0, 30]	Contribution of Poisson point-source power to $\mathcal{D}^{148\times148}_{3000}$ for ACT (in $\mu\mathrm{K}^2$)
$A^{\mathrm{PS,ACT}}_{218}$	[0, 200]	As for $A^{\mathrm{PS,ACT}}_{148}$, but at 218 GHz
$r^{\mathrm{PS}}_{150\times220}$	[0, 1]	Point-source correlation coefficient between 150 and 220 GHz (for ACT and SPT)
$A^{\mathrm{ACTe}}_{\mathrm{dust}}$	[0, 5] (0.8 ± 0.2)	Contribution from Galactic cirrus to $\mathcal{D}_{3000}$ at 150 GHz for ACTe (in $\mu\mathrm{K}^2$)
$A^{\mathrm{ACTs}}_{\mathrm{dust}}$	[0, 5] (0.4 ± 0.2)	As $A^{\mathrm{ACTe}}_{\mathrm{dust}}$, but for ACTs
y^{ACTe}_{148}	[0.8, 1.3]	Map-level calibration of ACTe at 148 GHz relative to *Planck* 143 GHz
y^{ACTe}_{217}	[0.8, 1.3]	As y^{ACTe}_{148}, but at 217 GHz
y^{ACTs}_{148}	[0.8, 1.3]	Map-level calibration of ACTs at 148 GHz relative to *Planck* 143 GHz
y^{ACTs}_{217}	[0.8, 1.3]	As y^{ACTs}_{148}, but at 217 GHz
$A^{\mathrm{PS,SPT}}_{95}$	[0, 30]	Contribution of Poisson point-source power to $\mathcal{D}^{95\times95}_{3000}$ for SPT (in $\mu\mathrm{K}^2$)
$A^{\mathrm{PS,SPT}}_{150}$	[0, 30]	As for $A^{\mathrm{PS,SPT}}_{95}$, but at 150 GHz
$A^{\mathrm{PS,SPT}}_{220}$	[0, 200]	As for $A^{\mathrm{PS,SPT}}_{95}$, but at 220 GHz
$r^{\mathrm{PS}}_{95\times150}$	[0, 1]	Point-source correlation coefficient between 95 and 150 GHz for SPT
$r^{\mathrm{PS}}_{95\times220}$	[0, 1]	As $r^{\mathrm{PS}}_{95\times150}$, but between 95 and 220 GHz
y^{SPT}_{95}	[0.8, 1.3]	Map-level calibration of SPT at 95 GHz relative to *Planck* 143 GHz
y^{SPT}_{150}	[0.8, 1.3]	As for y^{SPT}_{95}, but at 150 GHz
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H_0 (km s^{-1} Mpc^{-1})	
Local Universe **[Riess et al. 2016]**	73.24 ± 1.74
Planck+WMAP+ACT+SPT+BAO	69.3 ± 0.7

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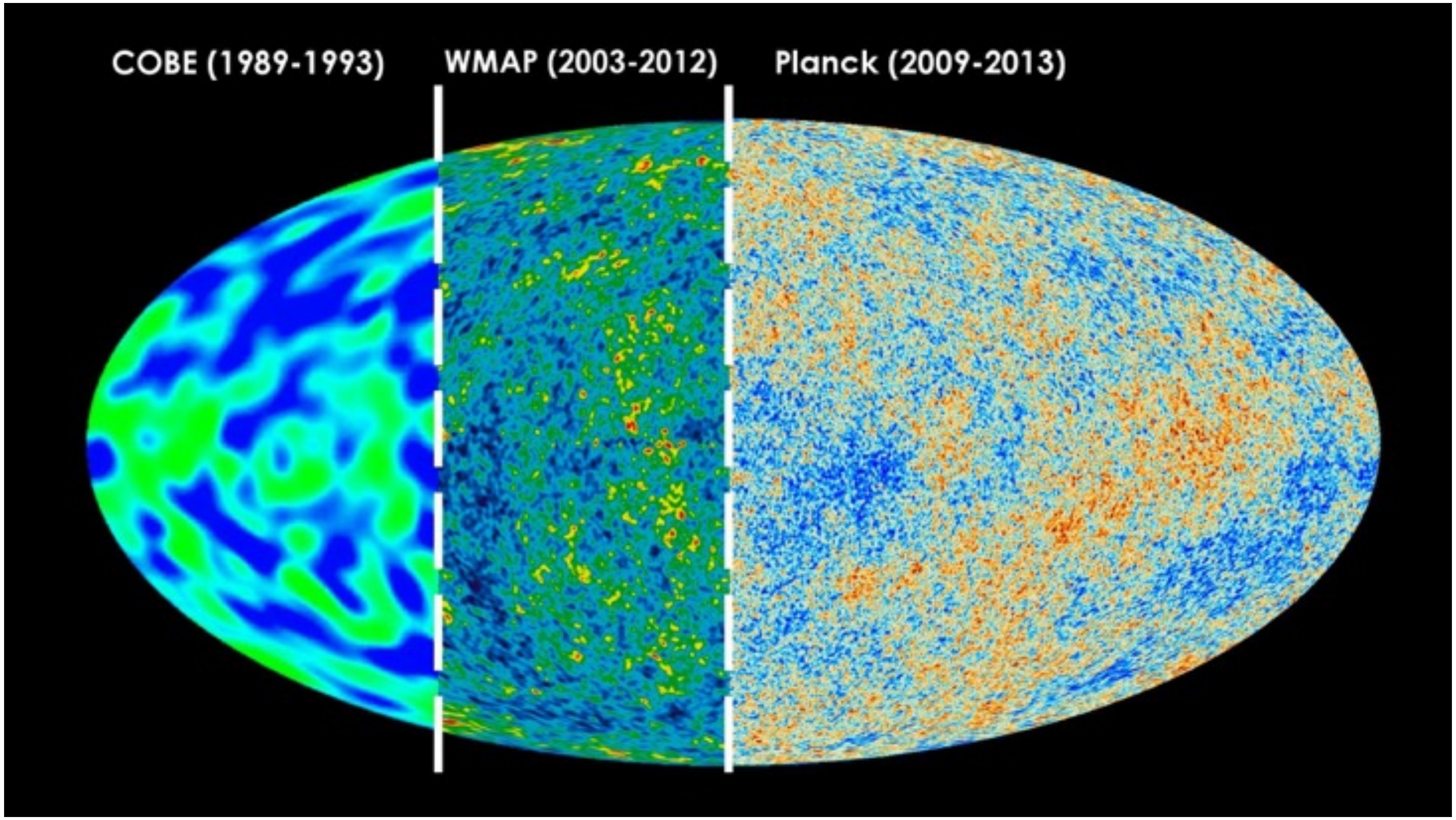

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Beyond Concordance

- Hybrid approach: Newtonian two-body interaction plus relativistic "touches" **[Thomas 2014, Adamek 2014-2016]**.
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In spite of its great importance in modern cosmology, general relativity suffers from two major misconceptions about its relevance and usefulness. The first of these points is:

Misconception 1: General relativity is not important in astrophysics.

This situation is all for the better since the second point states:

Misconception 2: General relativity is intractable.

We claim that these statements are not true, and will now support this claim with several examples.

Numerical Relativity for Cosmology

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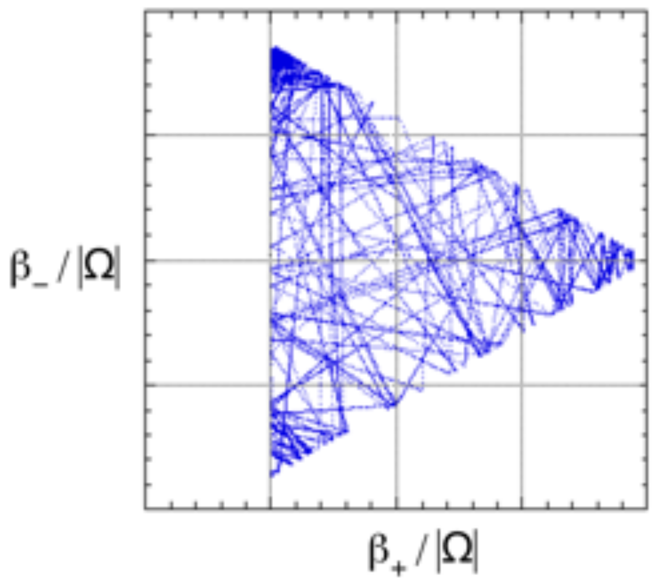
Singularities: **Matzner, Weaver (1970), Berger, Garfinkle (1991, 1997, 2004)**



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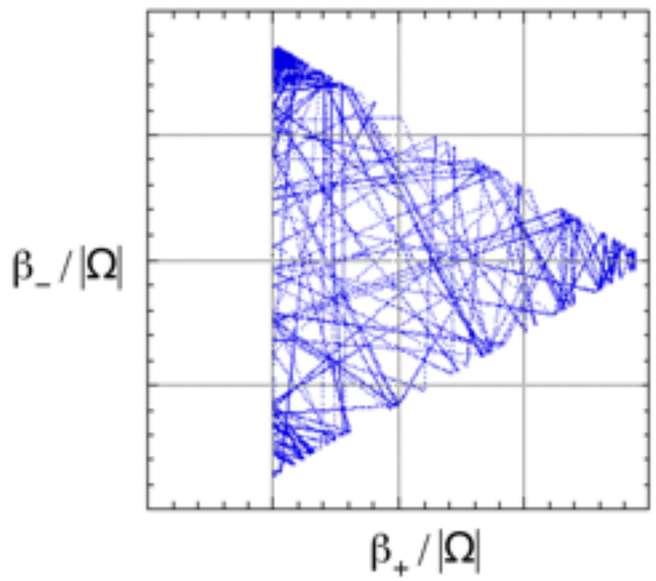

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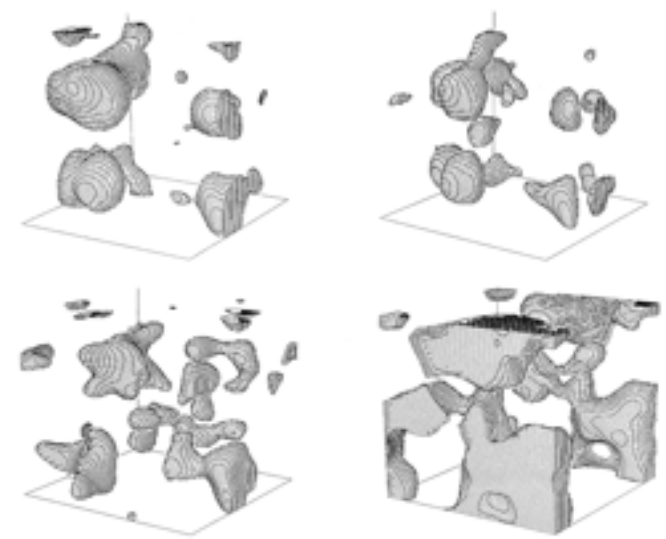

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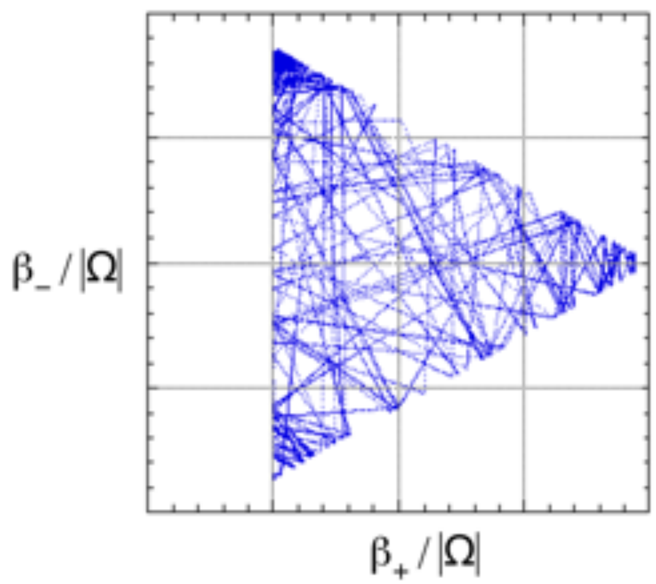

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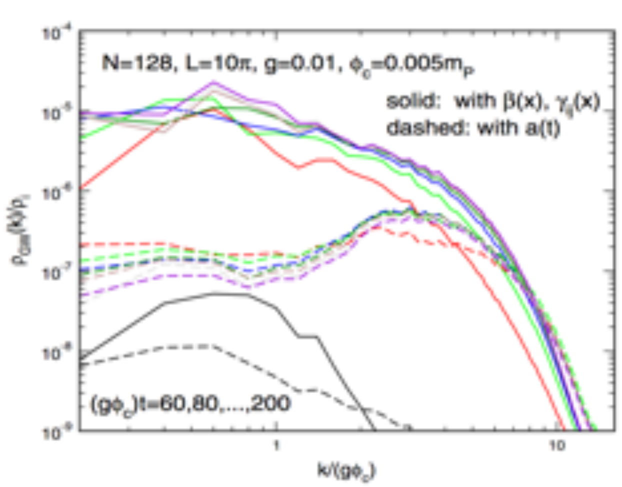

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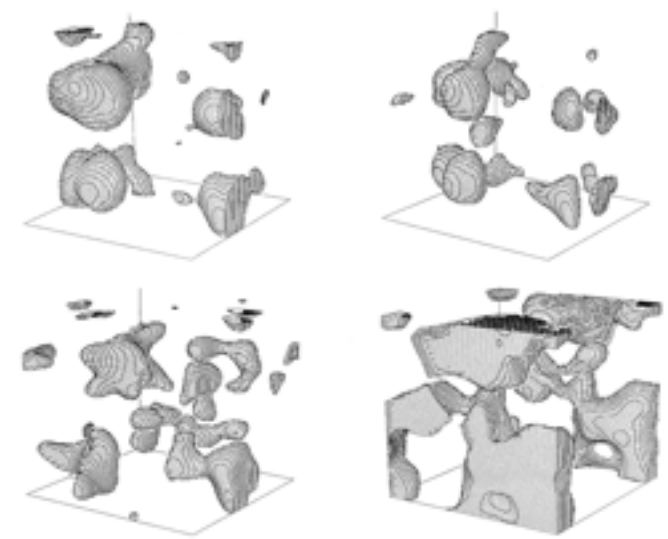

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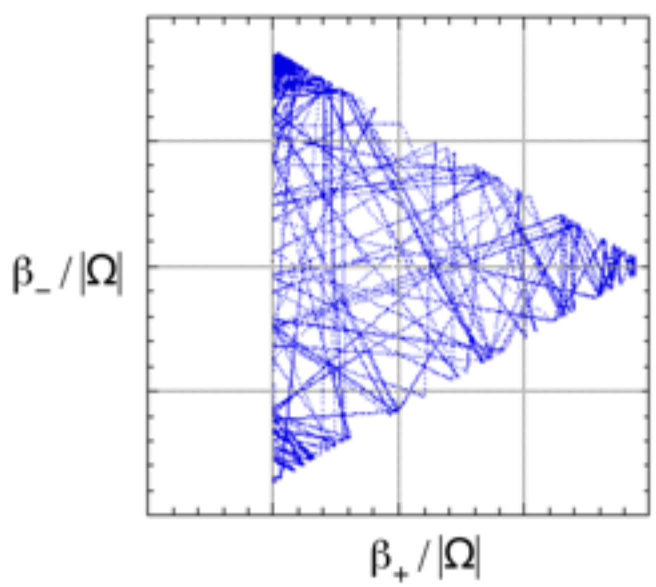

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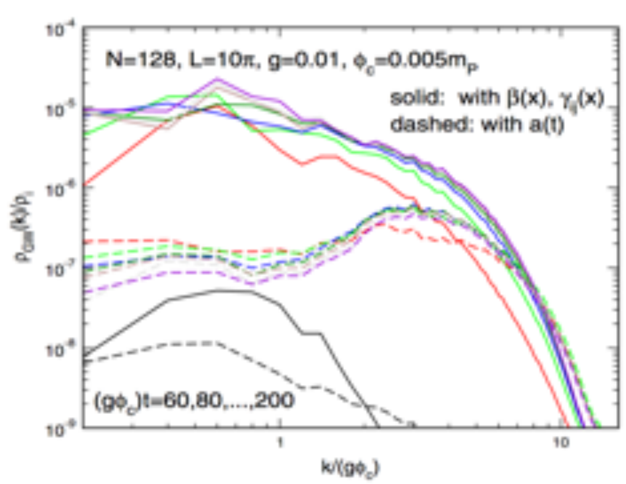

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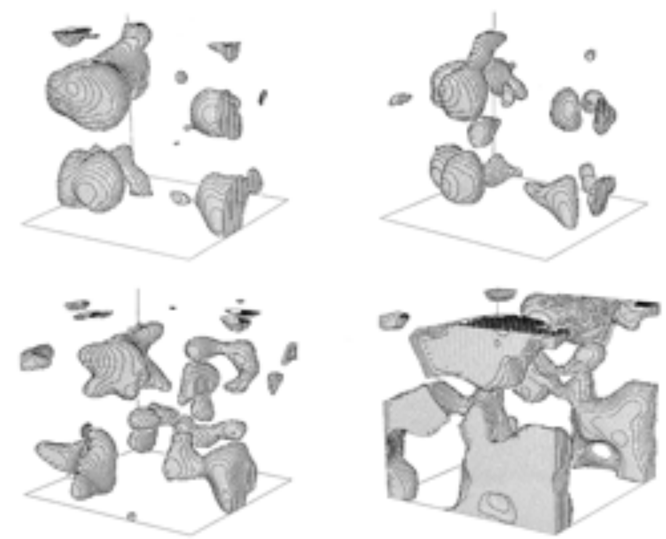
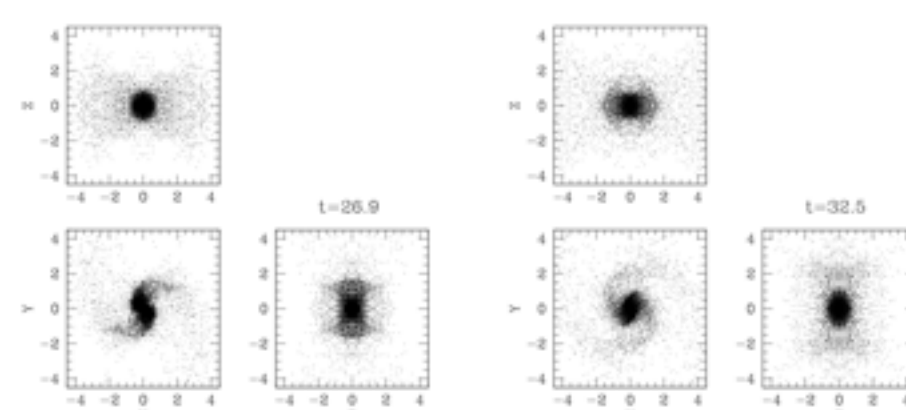


Large-scale structure, black-hole formation: **Anninos, Centrella, McKinney, Wilson (1984, 1985, 1999), Shibata (1999), Bentivegna, Korzyński, Hinder, Bruni (2012-2015), Yoo, Okawa, Nakao (2012-2014), Torres, Alcubierre, Diez-Tejedor, Nunez, de la Macorra (2014-2015), Rekier, Cordero-Carrion, Fuzfa (2015)**



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einsteintoolkit.org

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- **[Support from project CosmoToolkit (PIRG05-GA-2009-249290)]**

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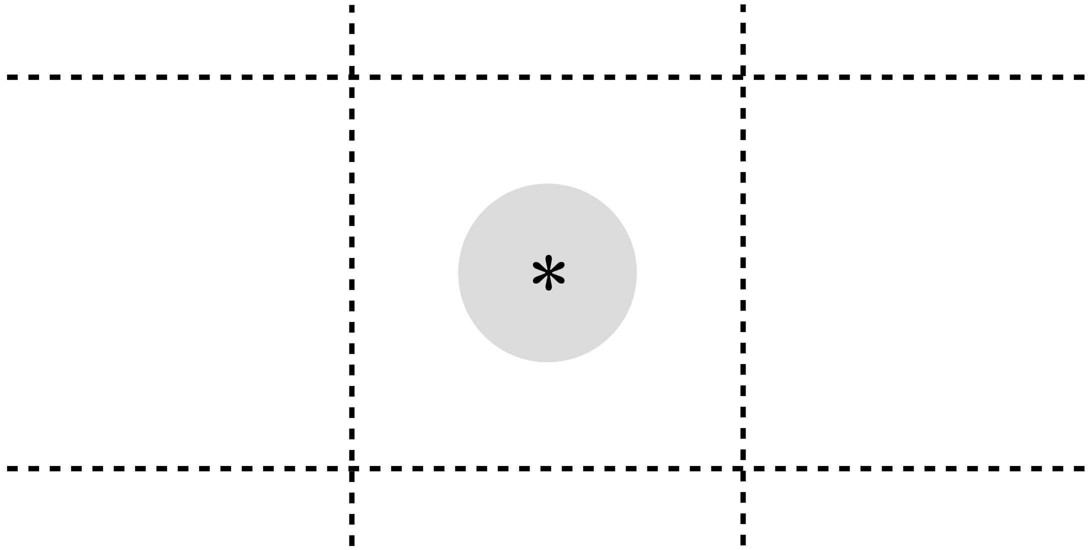

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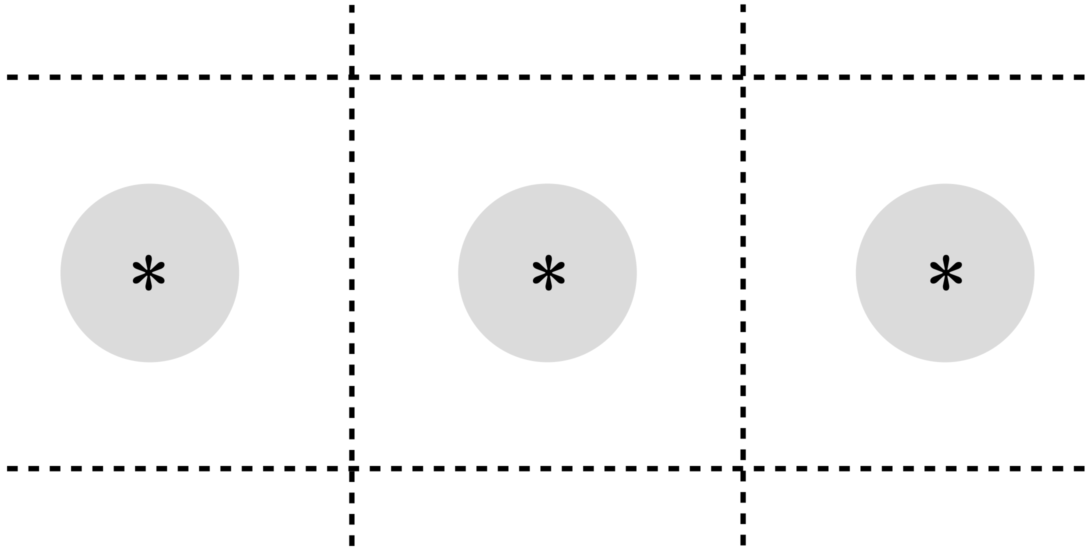

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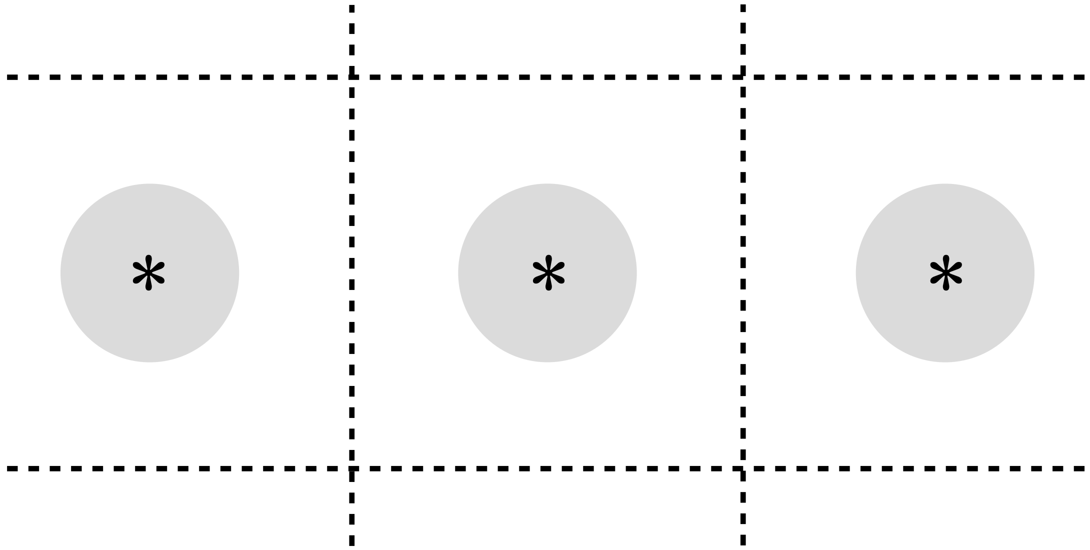

[Lindquist&Wheeler 1957]

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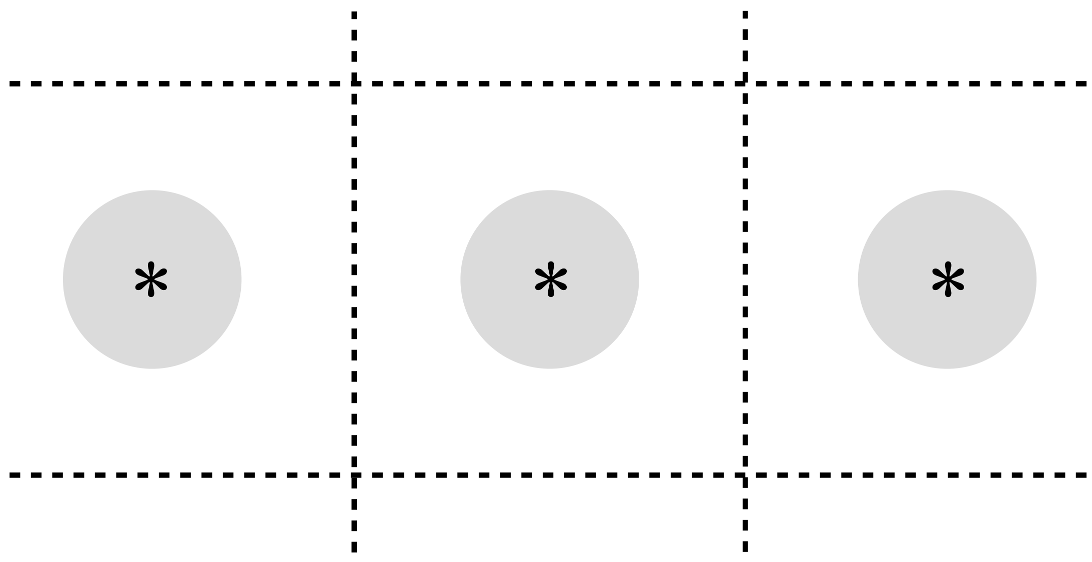

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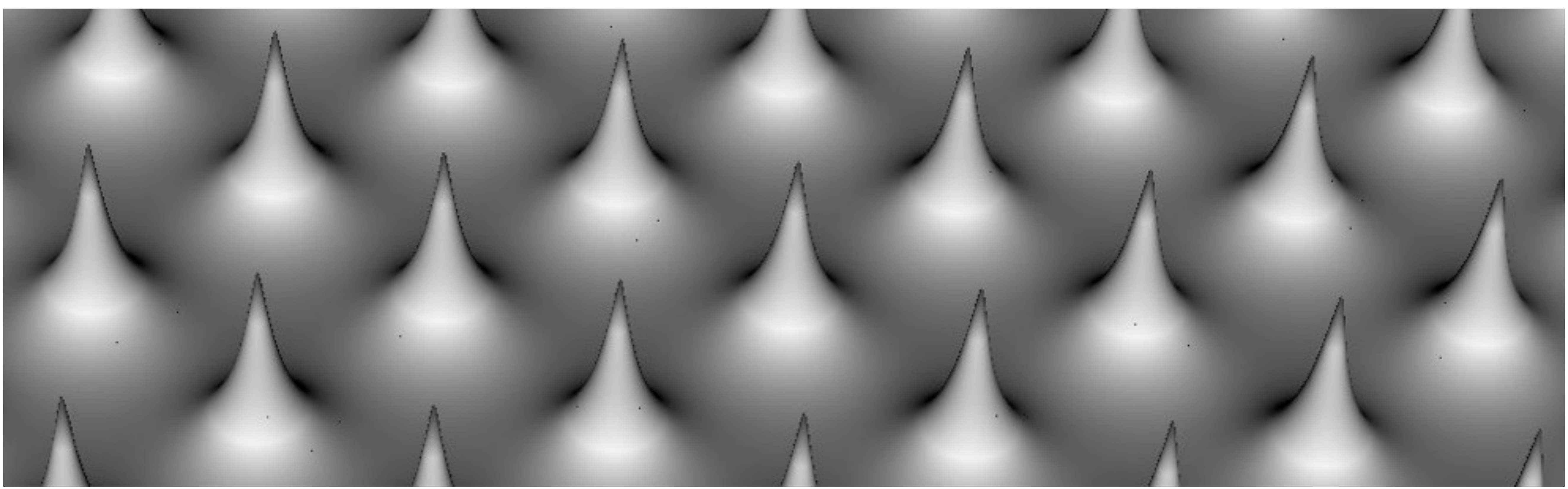

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$$r_{ijk} = \sqrt{(x - iL)^2 + (y - jL)^2 + (z - kL)^2}$$

Can this be generalized to an infinite number of black holes?

$$\psi = 1 + \sum_{ijk} \frac{m_{ijk}}{2r_{ijk}} \geq \sum_{ijk} \frac{m_{ijk}}{|a+i| + |b+j| + |c+k|}$$

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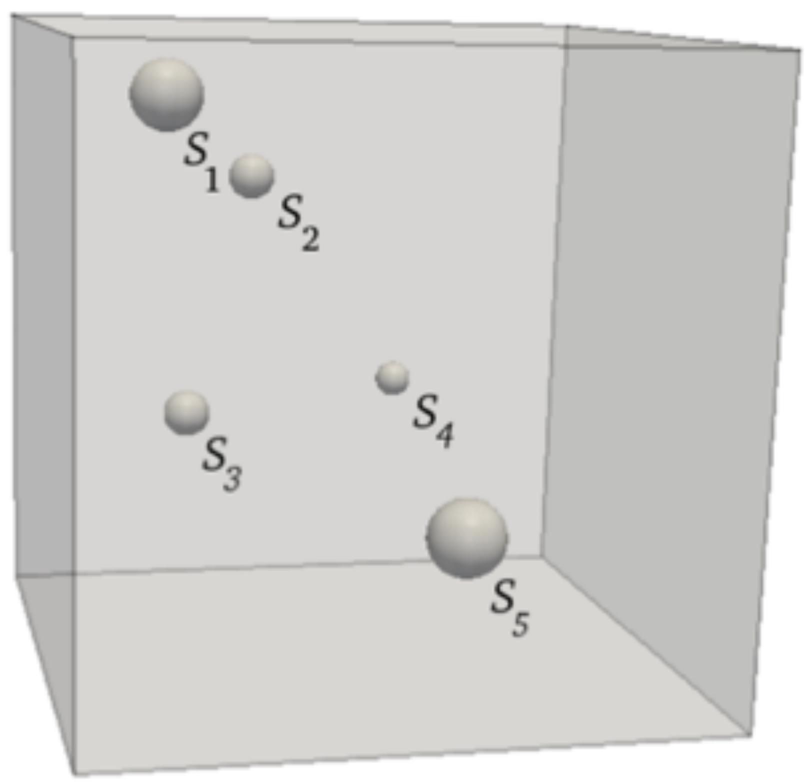

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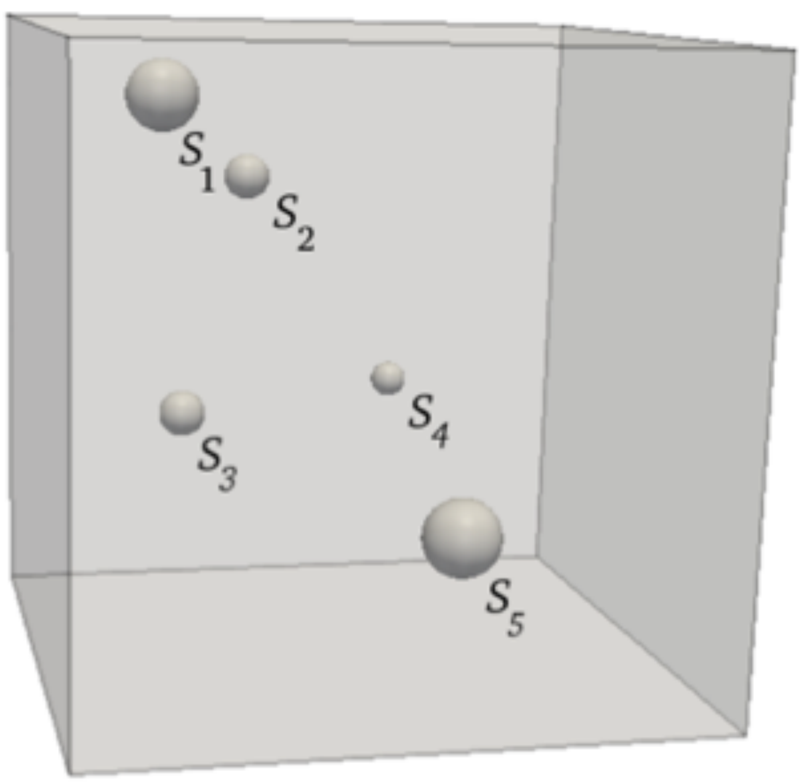

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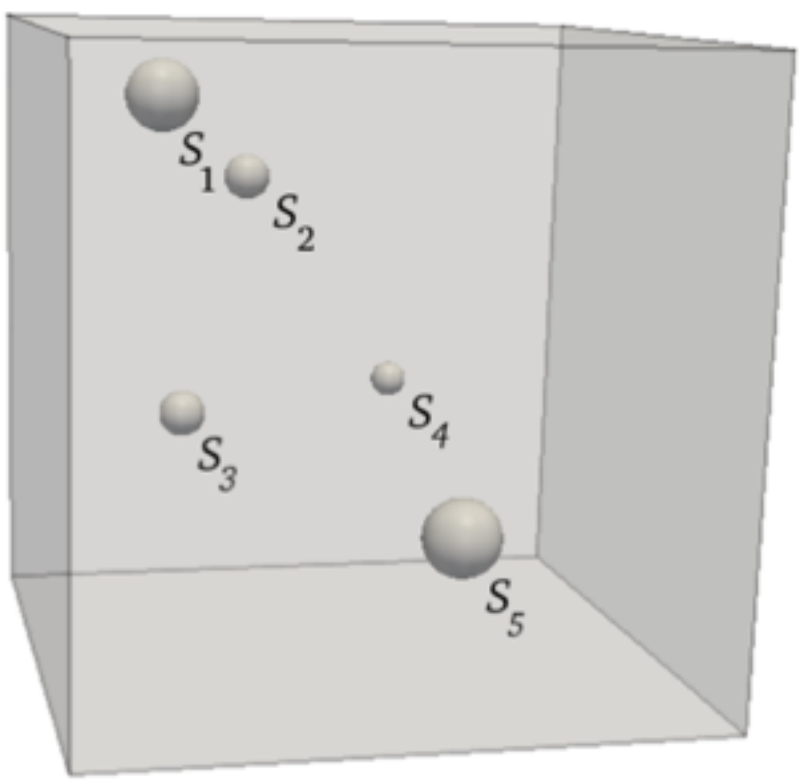

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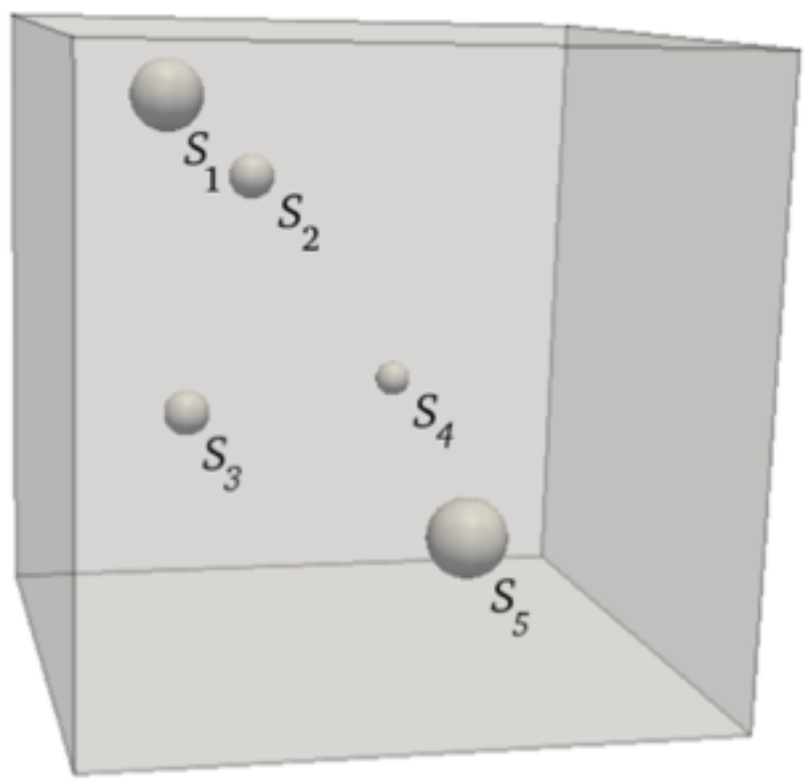

However, there is no surface term on the periodic boundaries. In a periodic space, the extrinsic curvature and the scalar curvature cannot both be zero! No time symmetric, spatially-flat solution (**homogeneous dust models have the same properties**).

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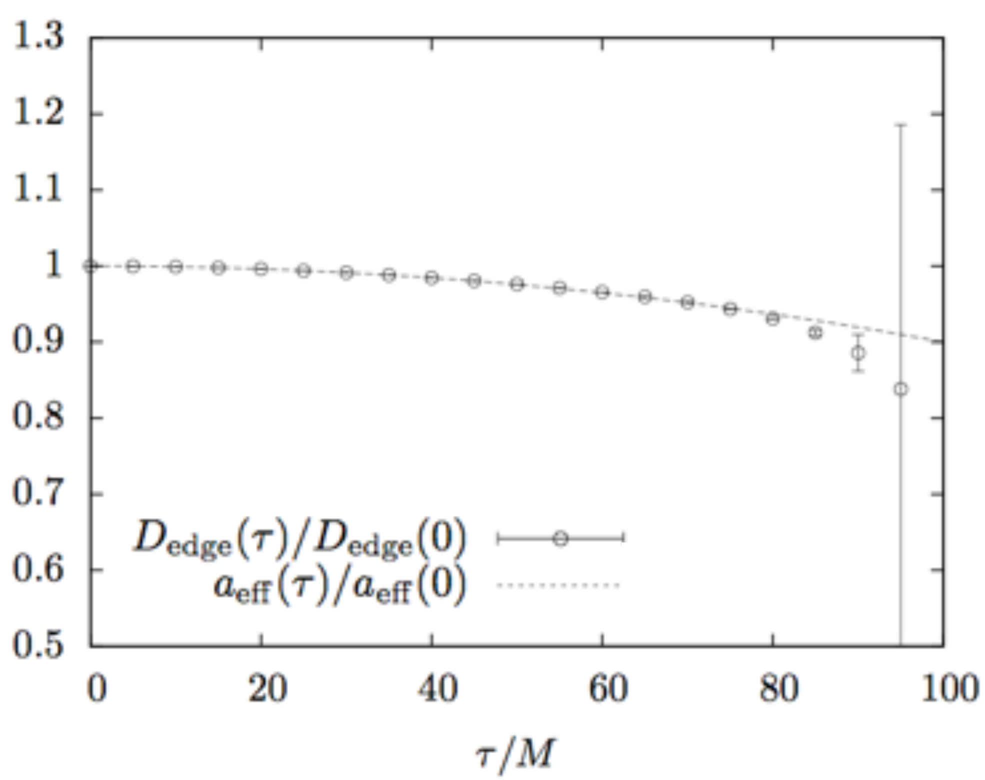

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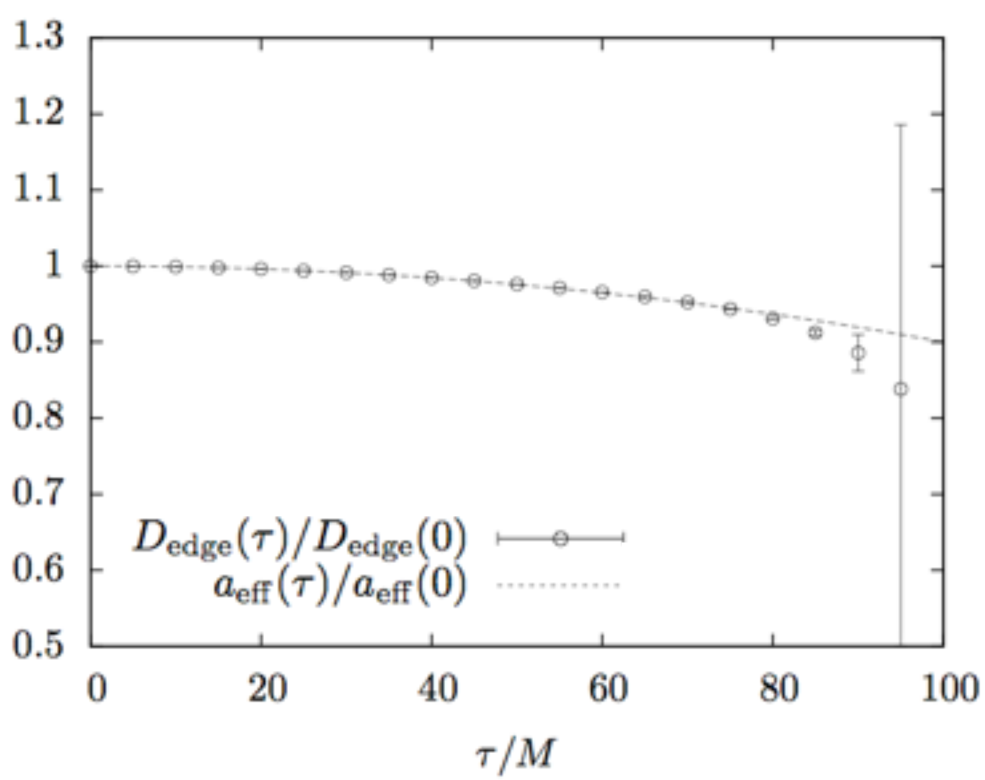

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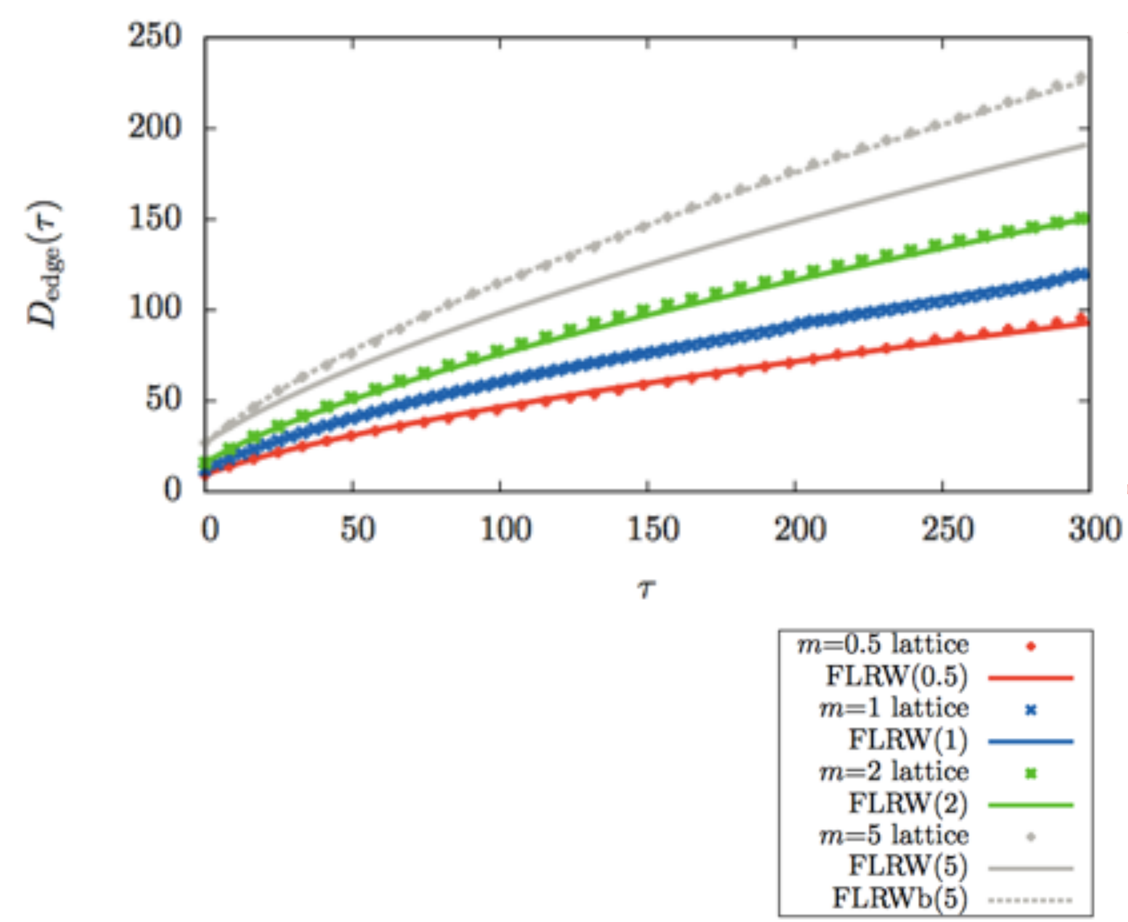

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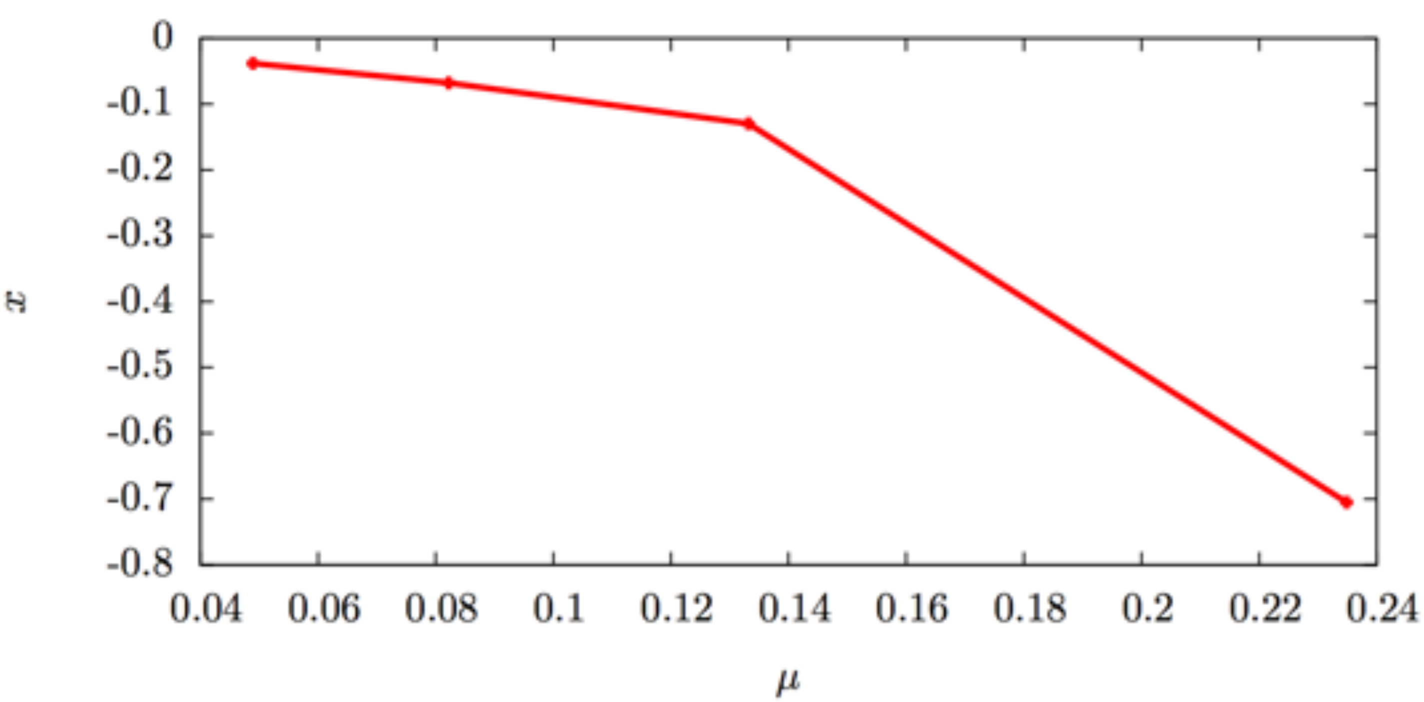

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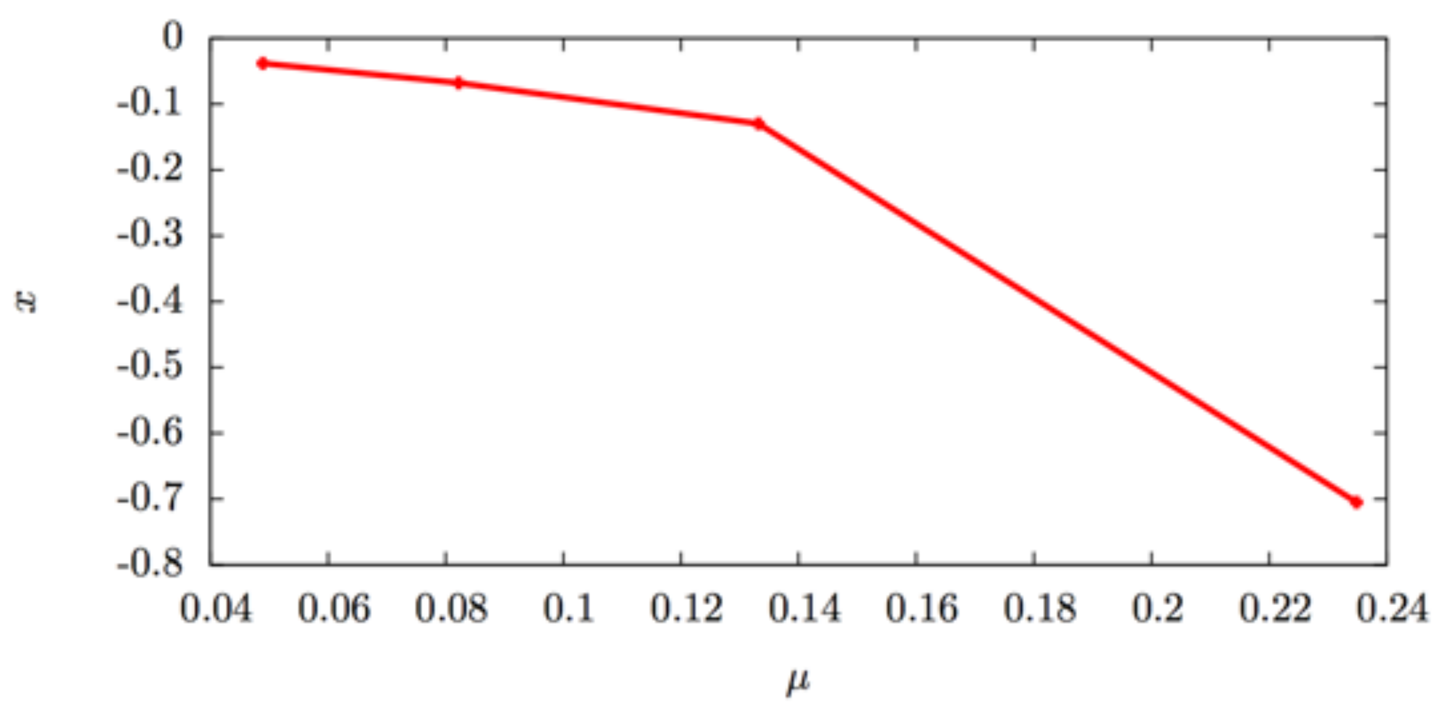

One must choose which mapping to use (fitting problem **non trivial**). Fitting one observable leads to a degradation in the quality of fit to the others.

Dust cosmologies

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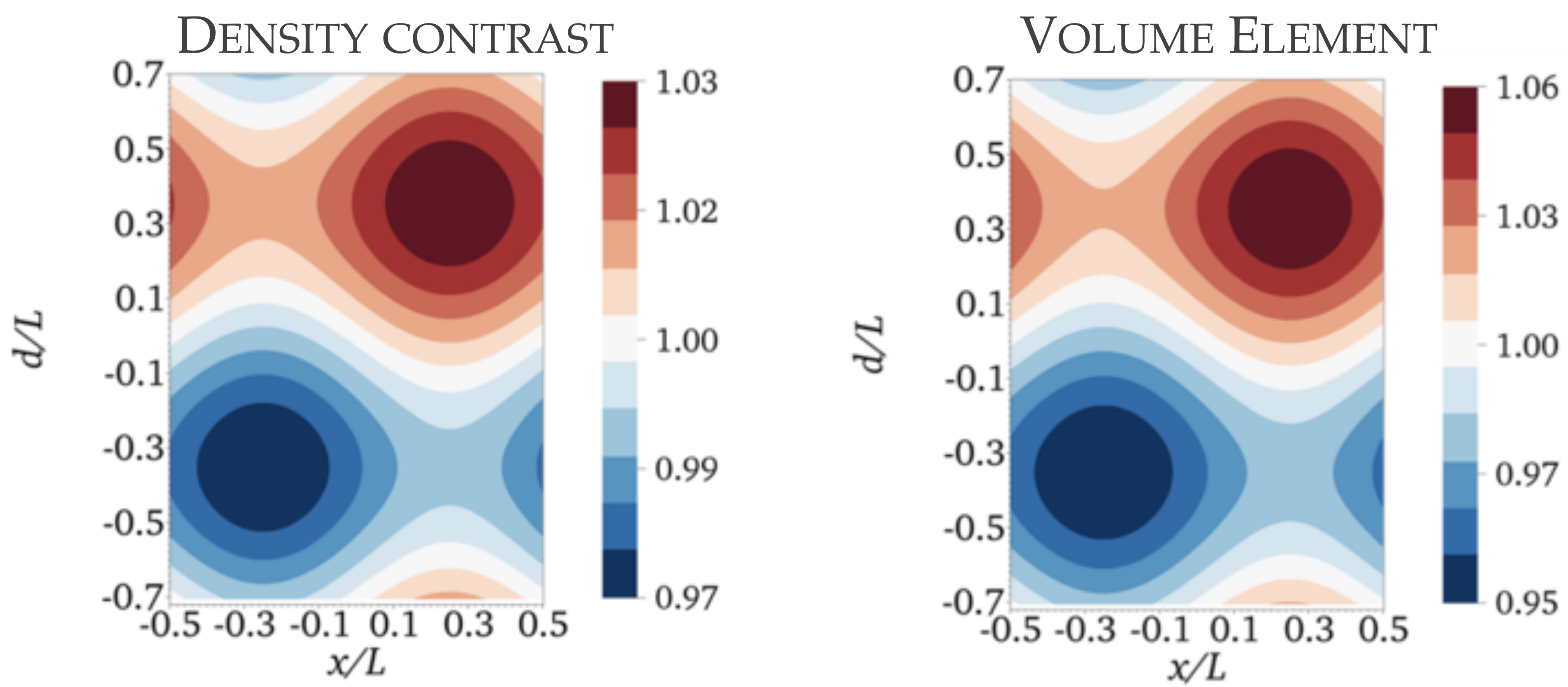

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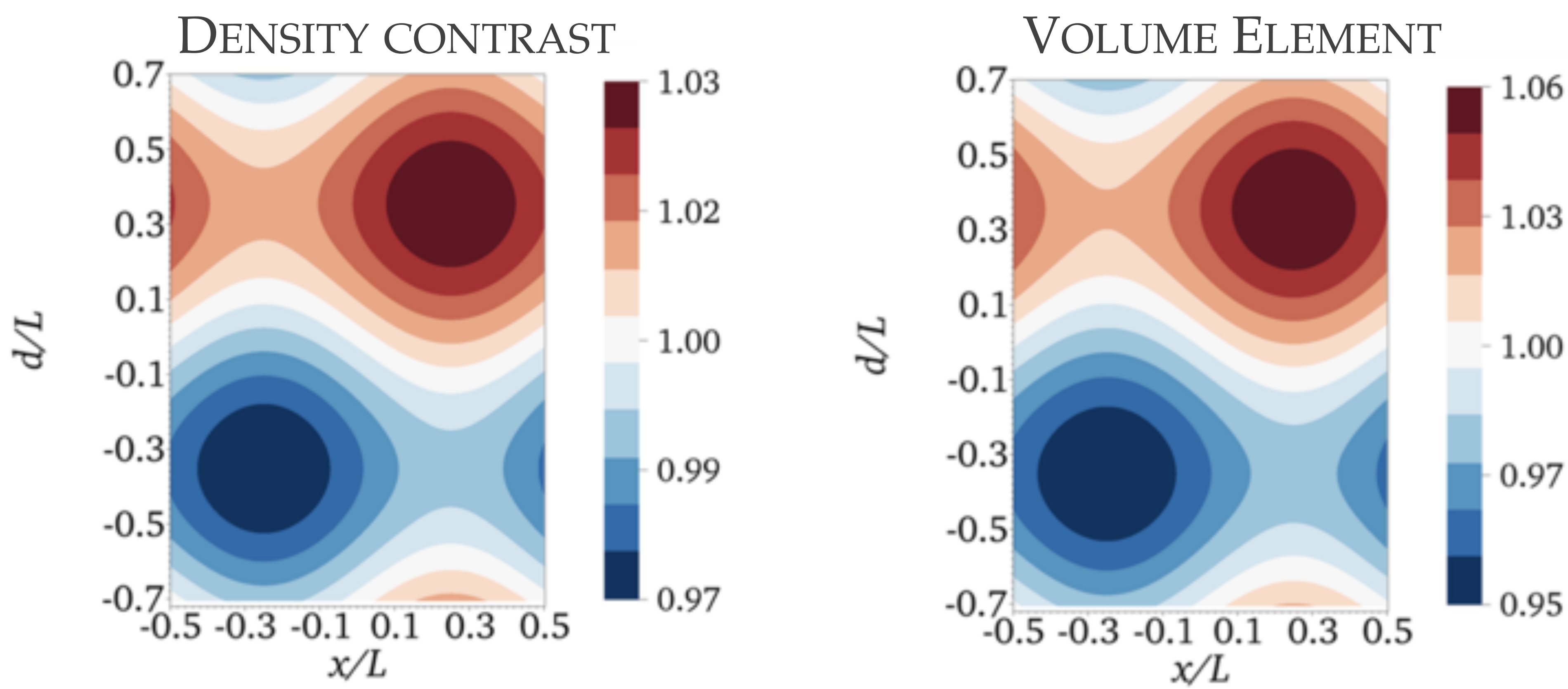

Approach much more similar to standard cosmological treatments of perturbed fluids. Many analytical approximations available in various regimes.

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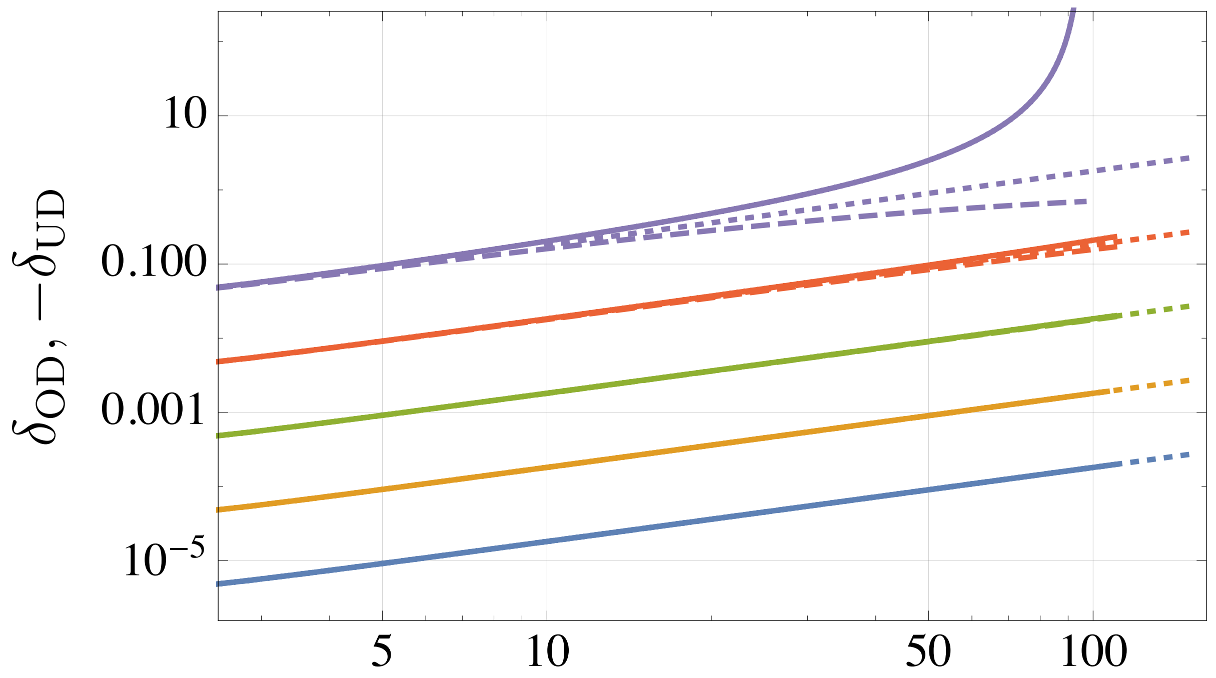

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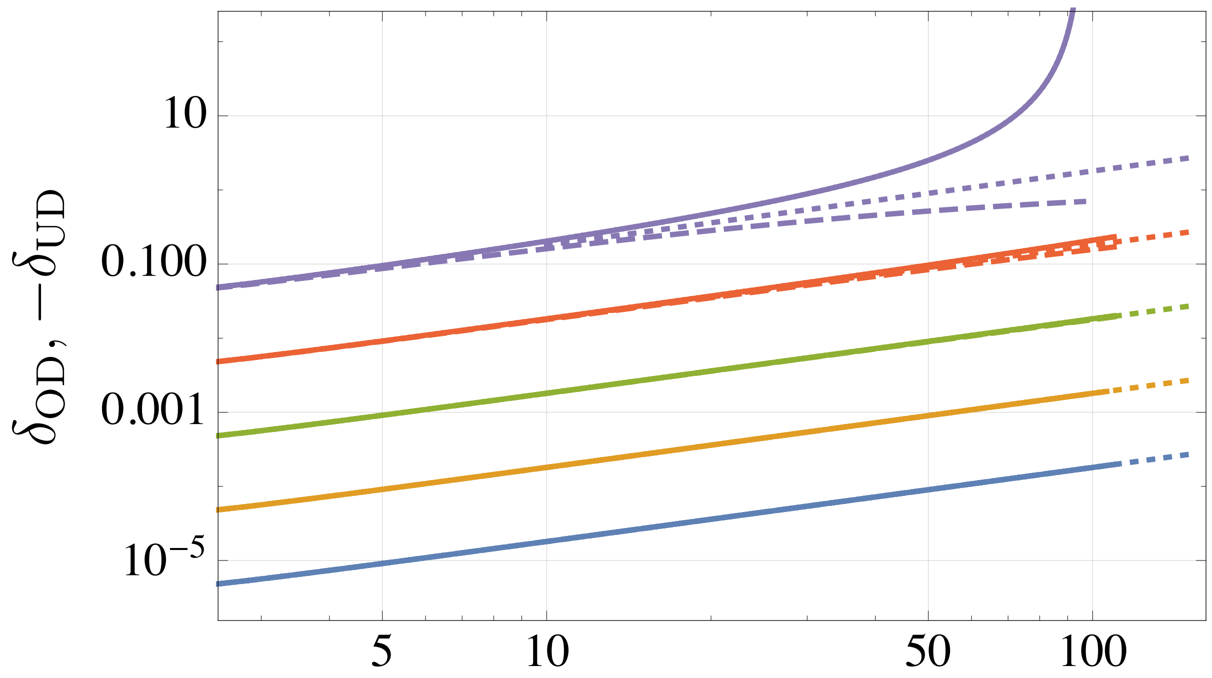

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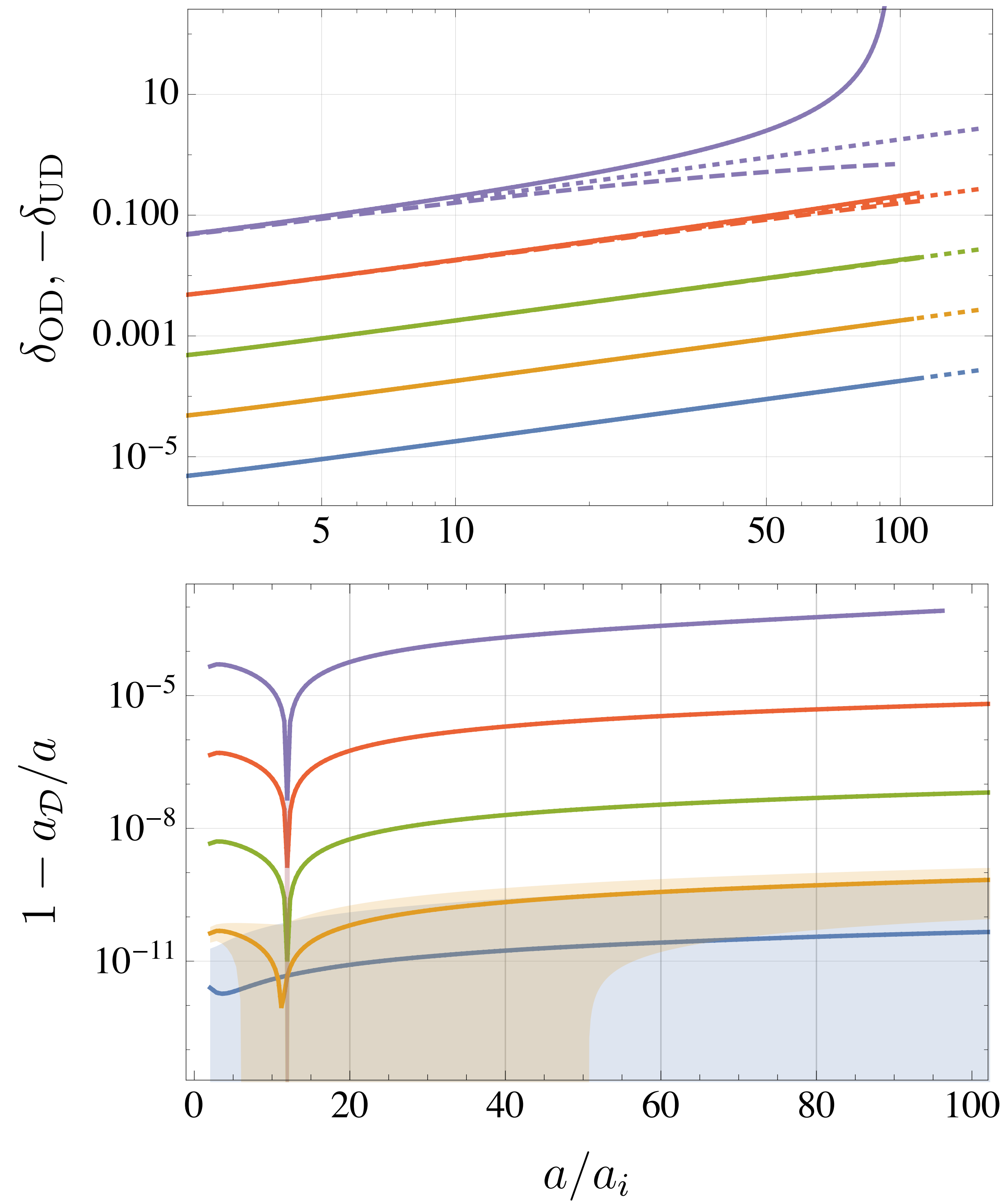

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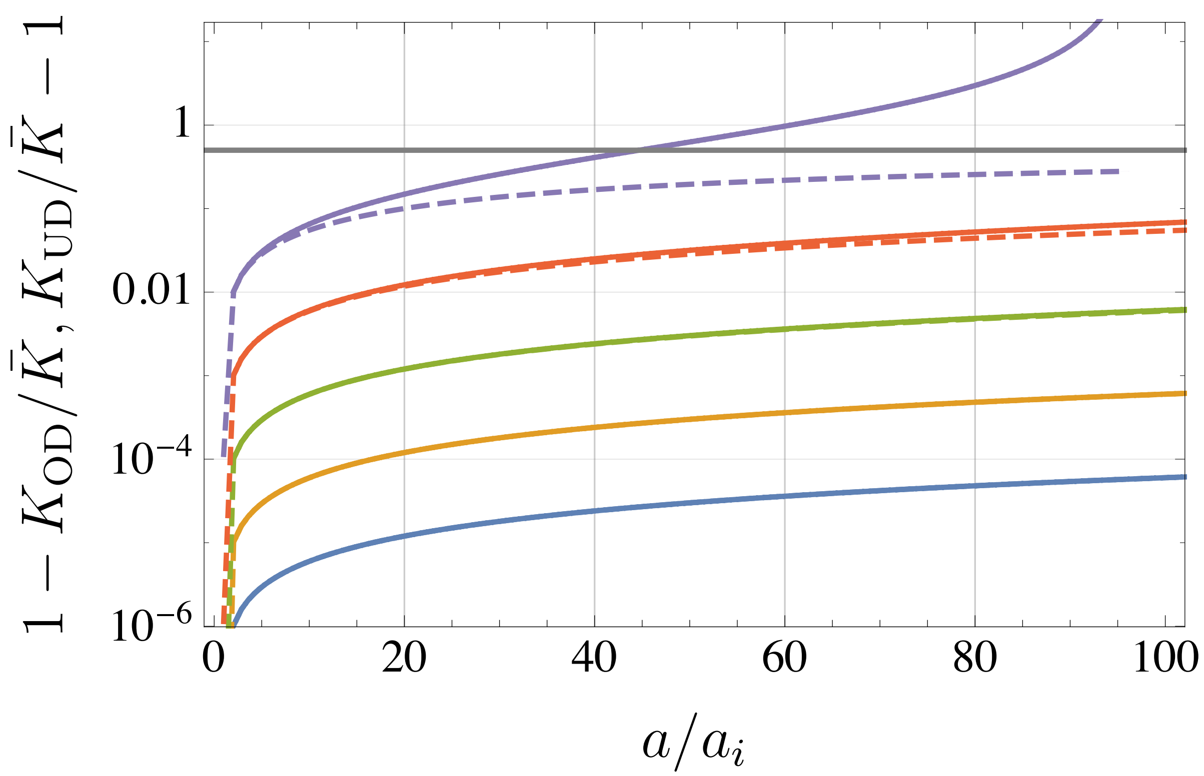

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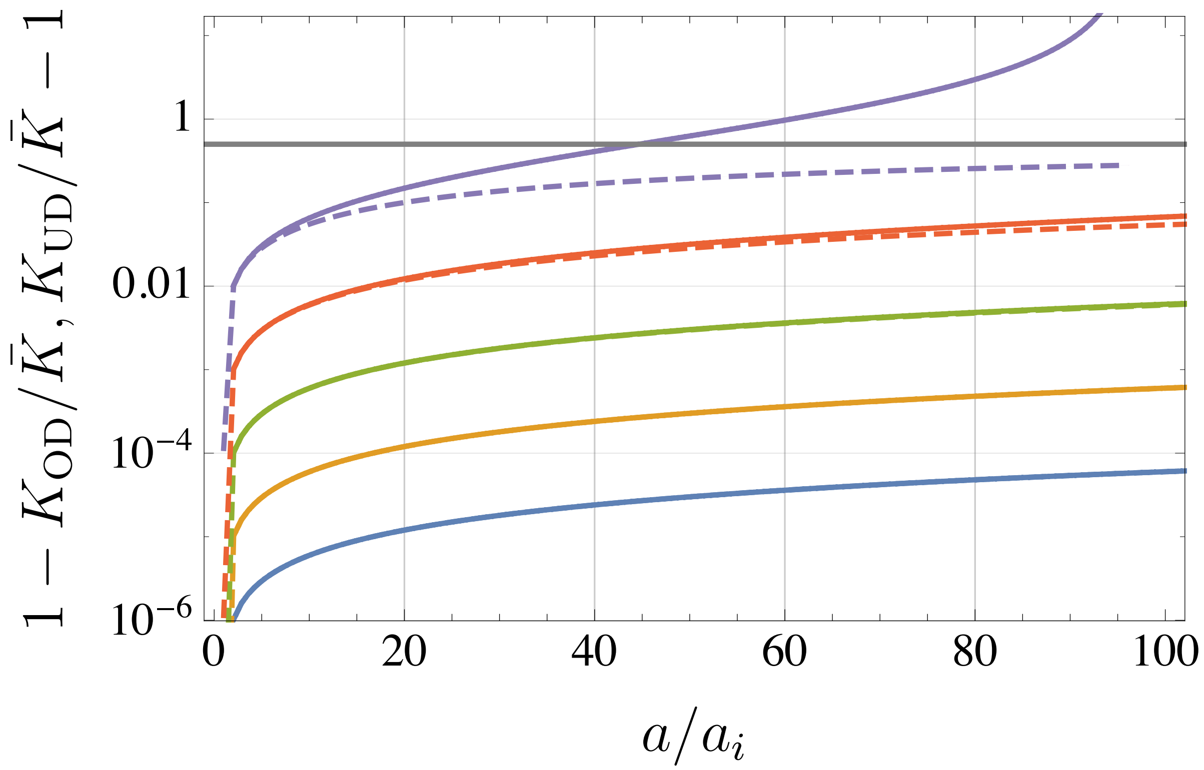

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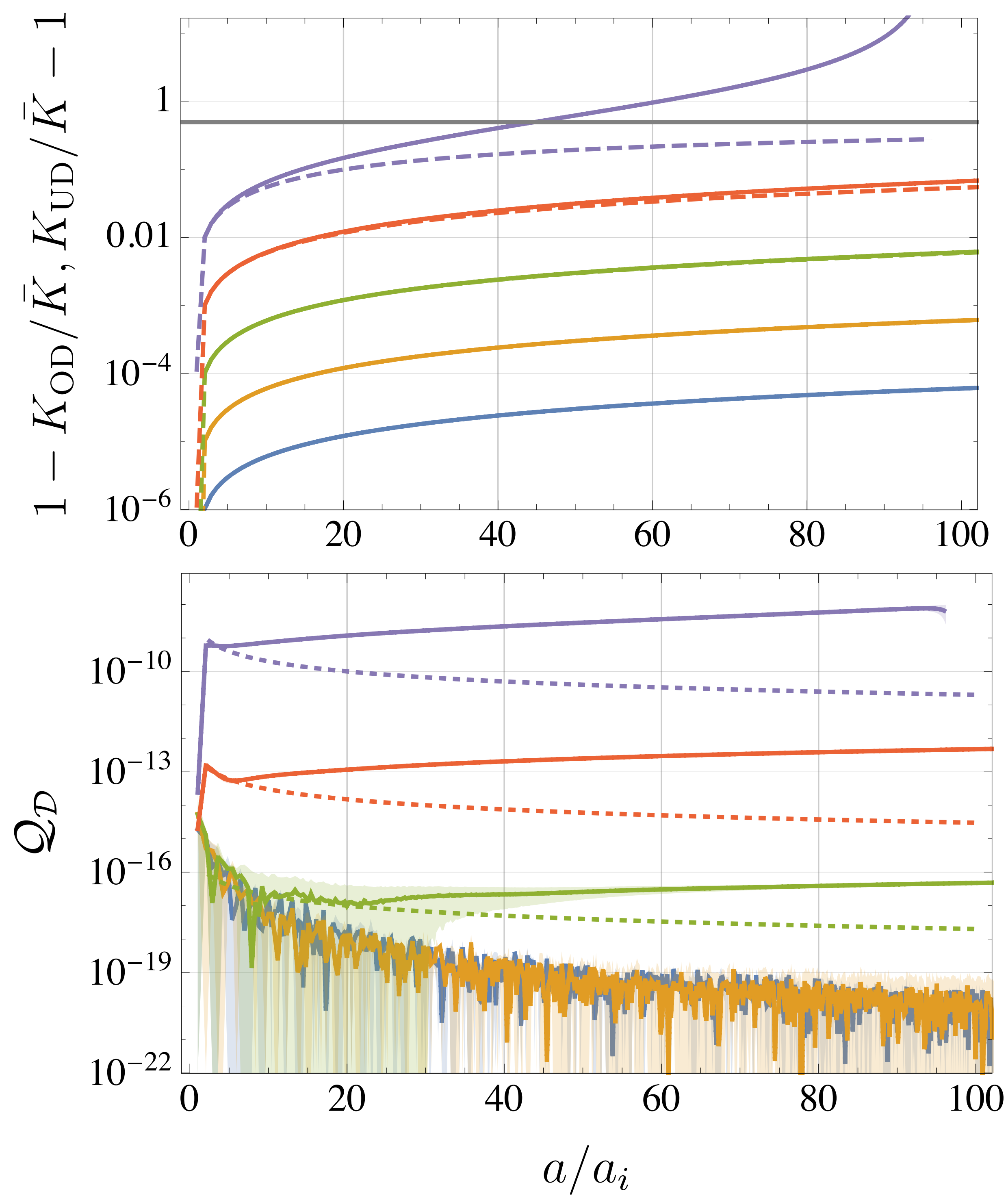

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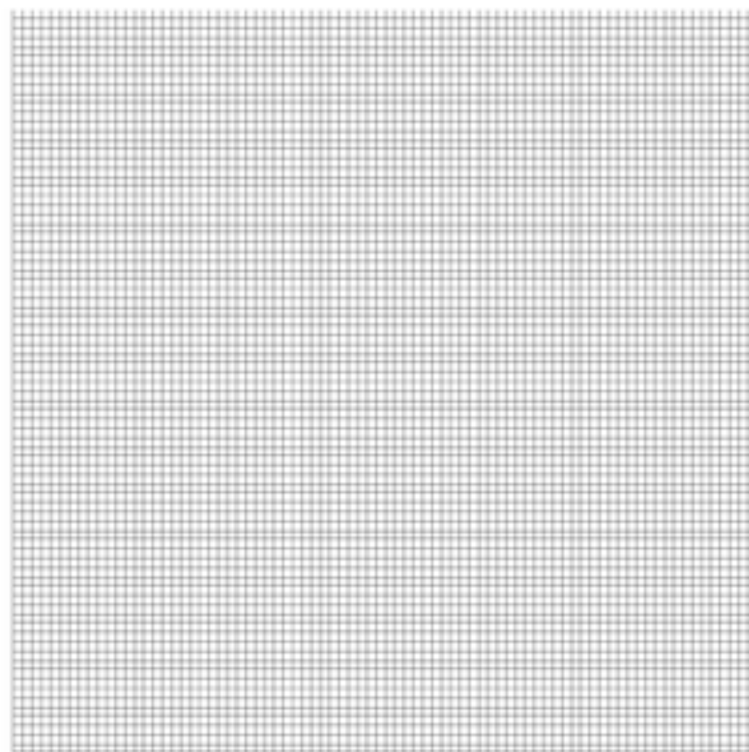

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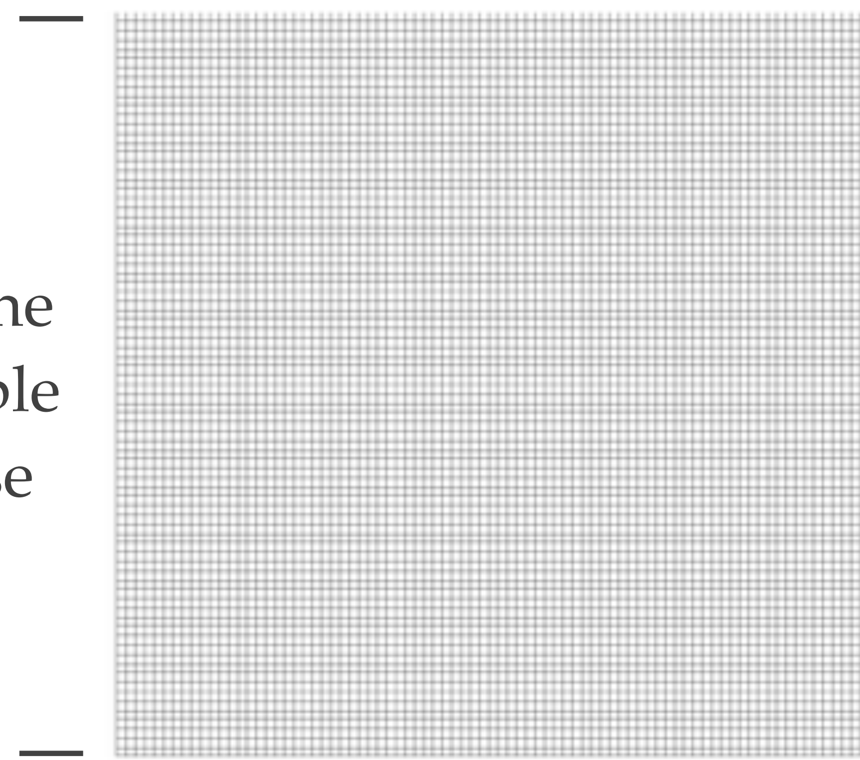

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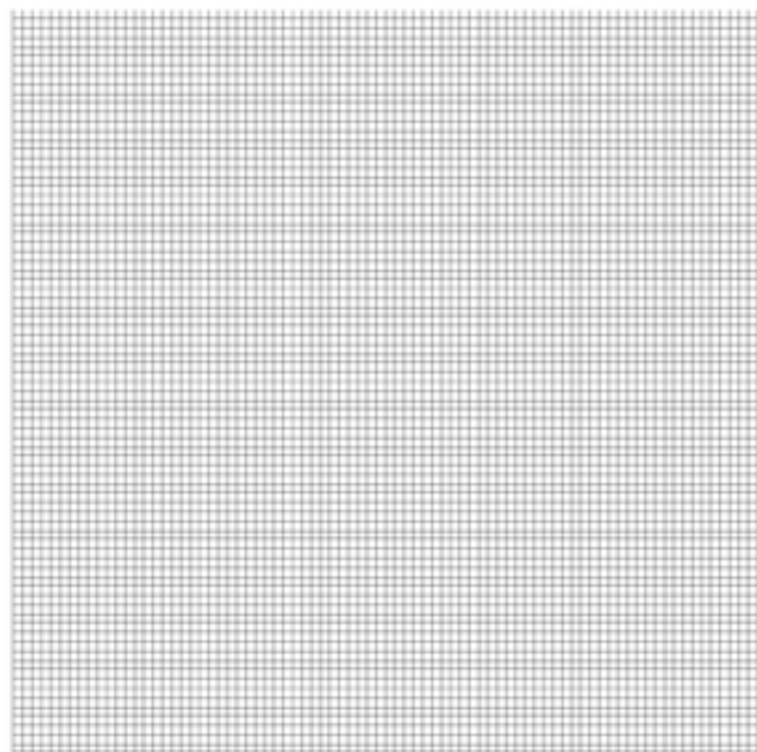

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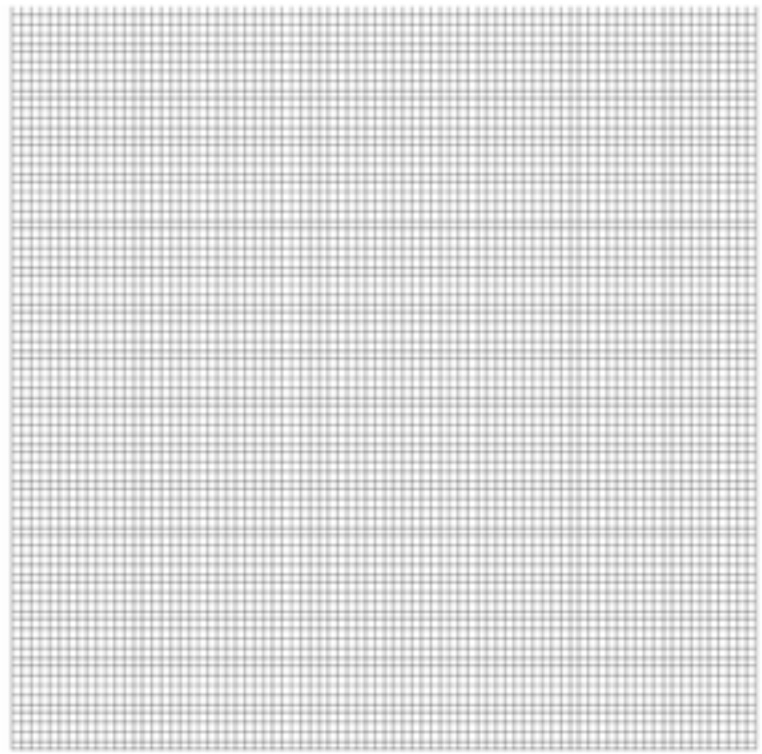

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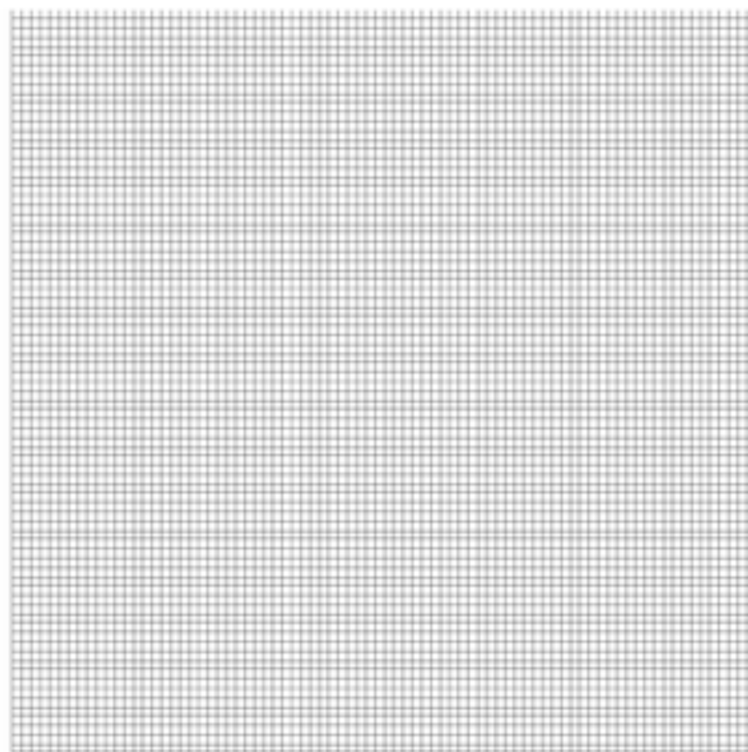

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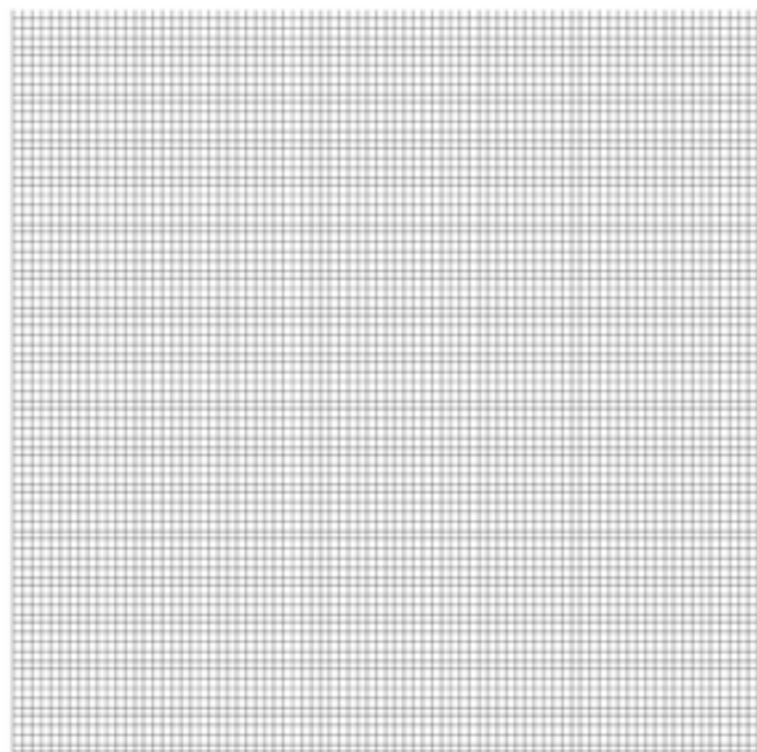

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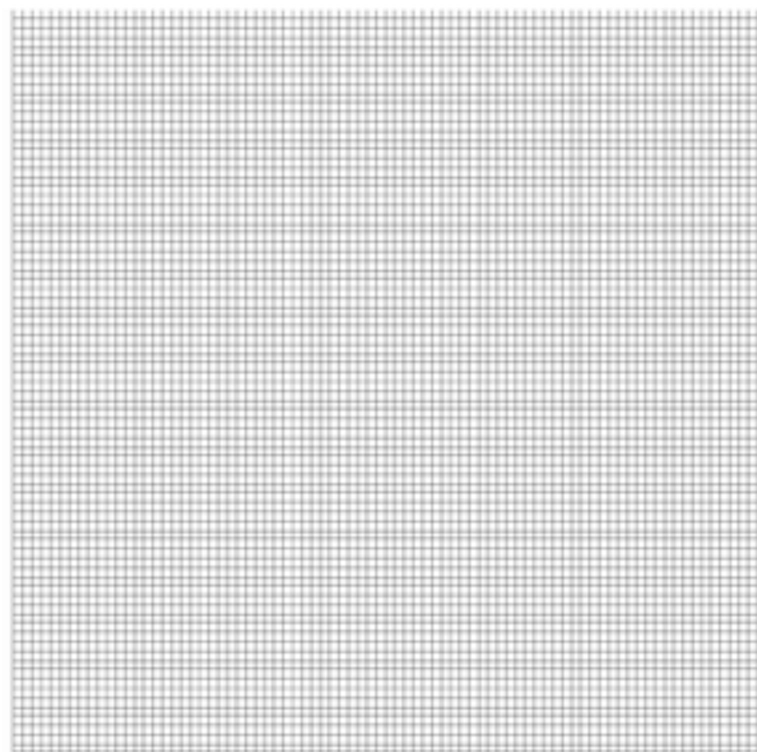

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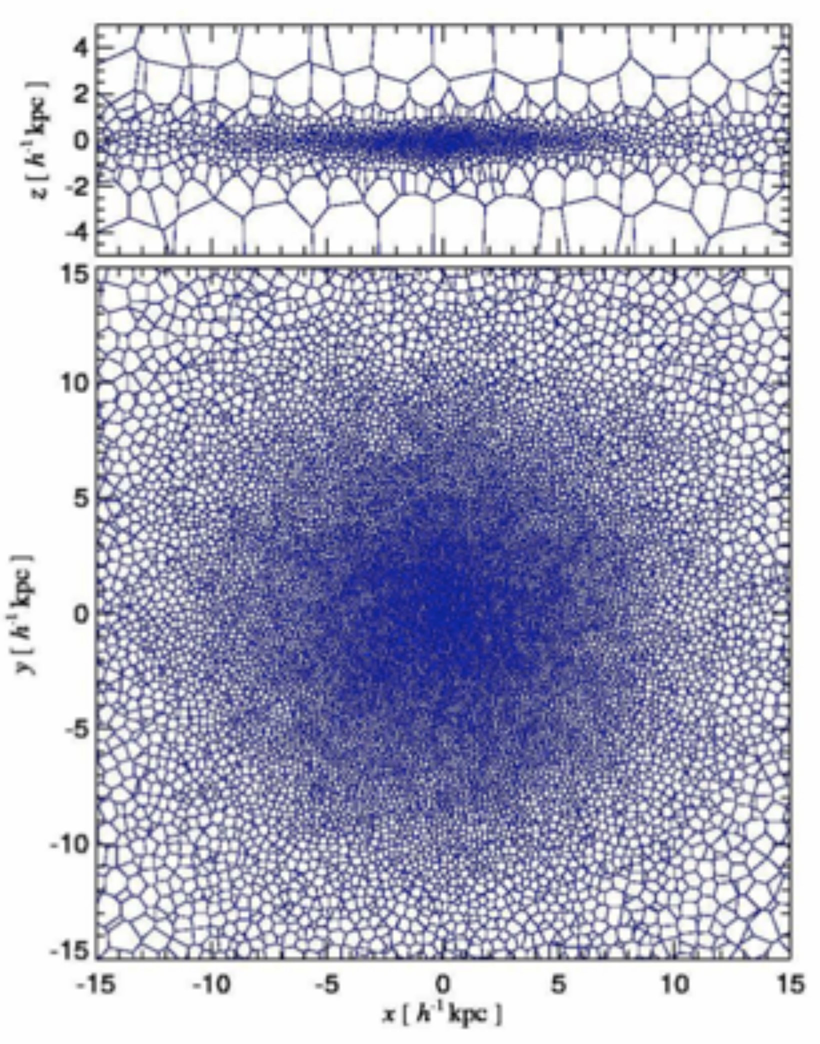

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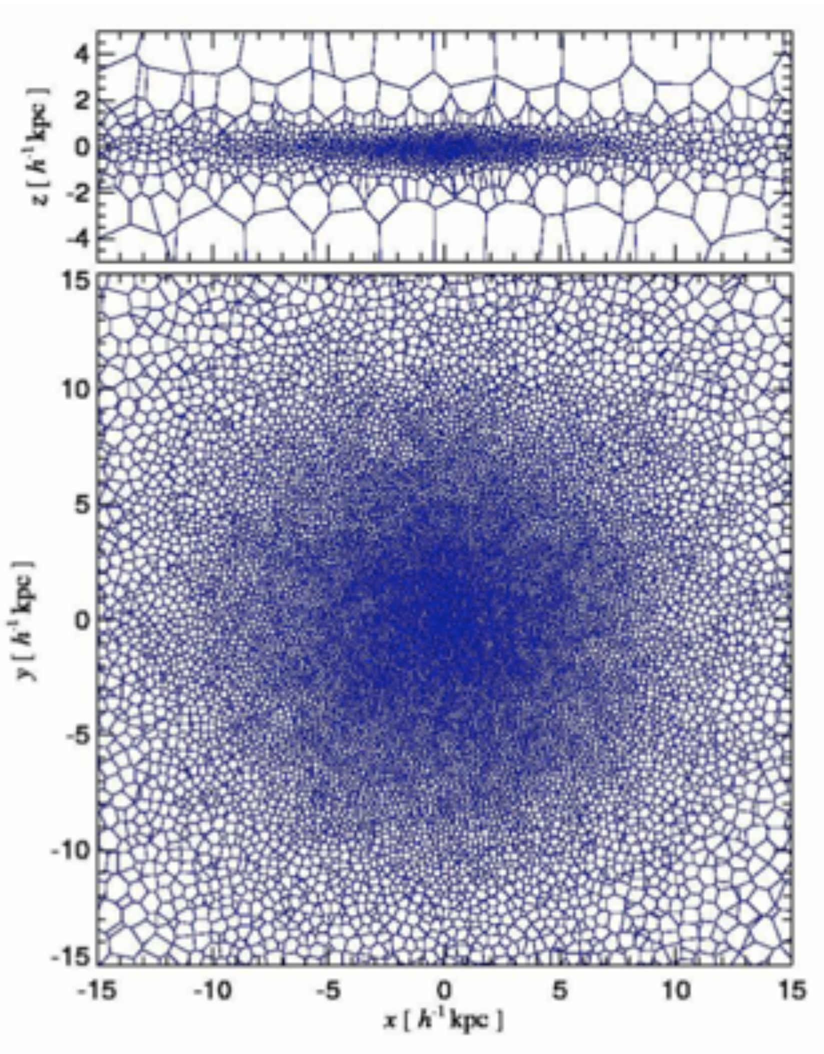

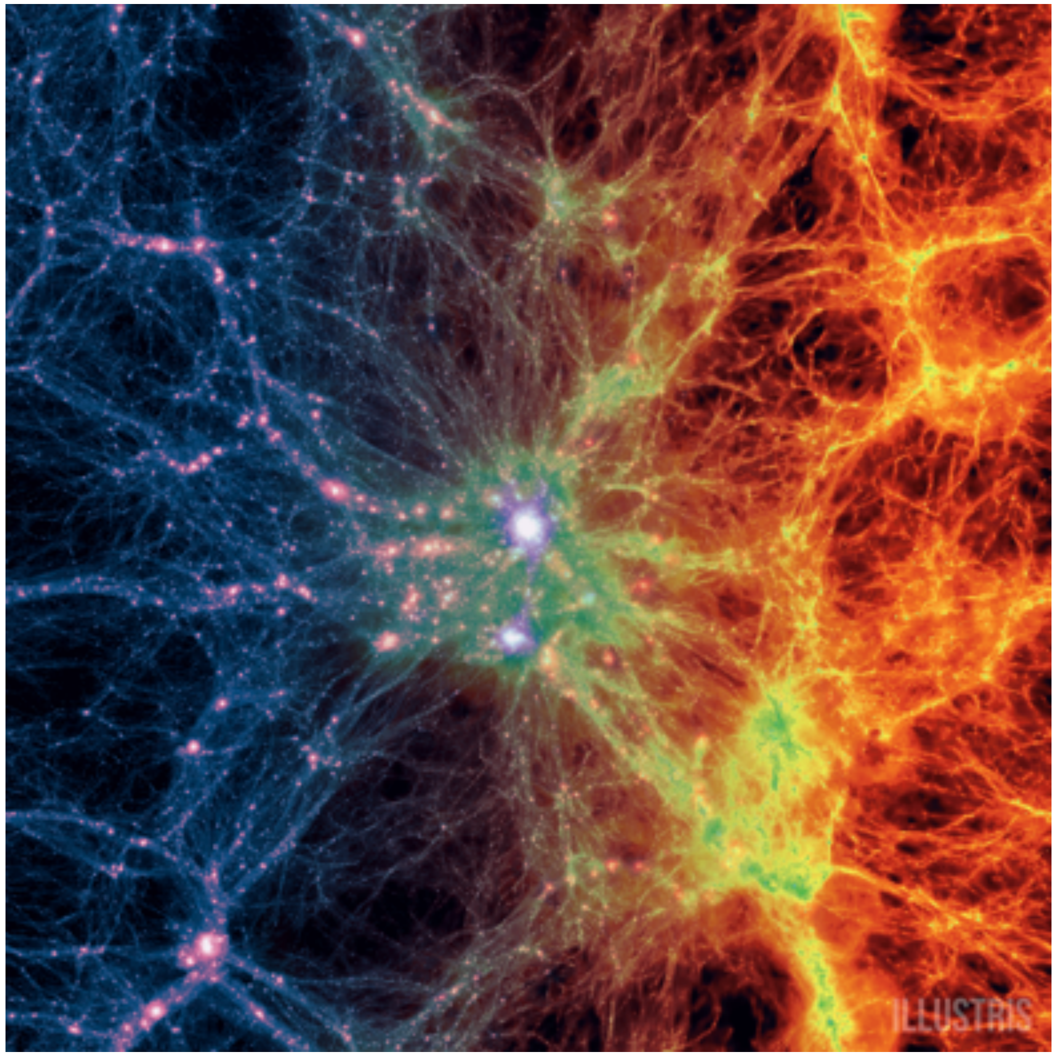

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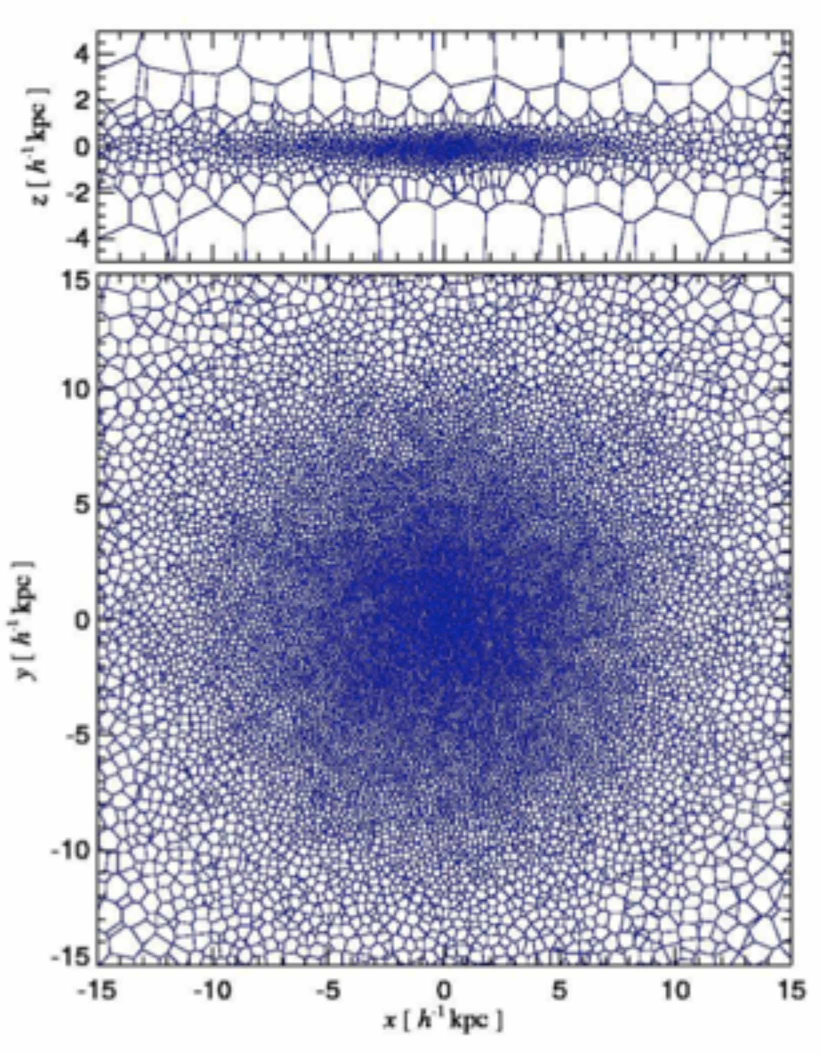

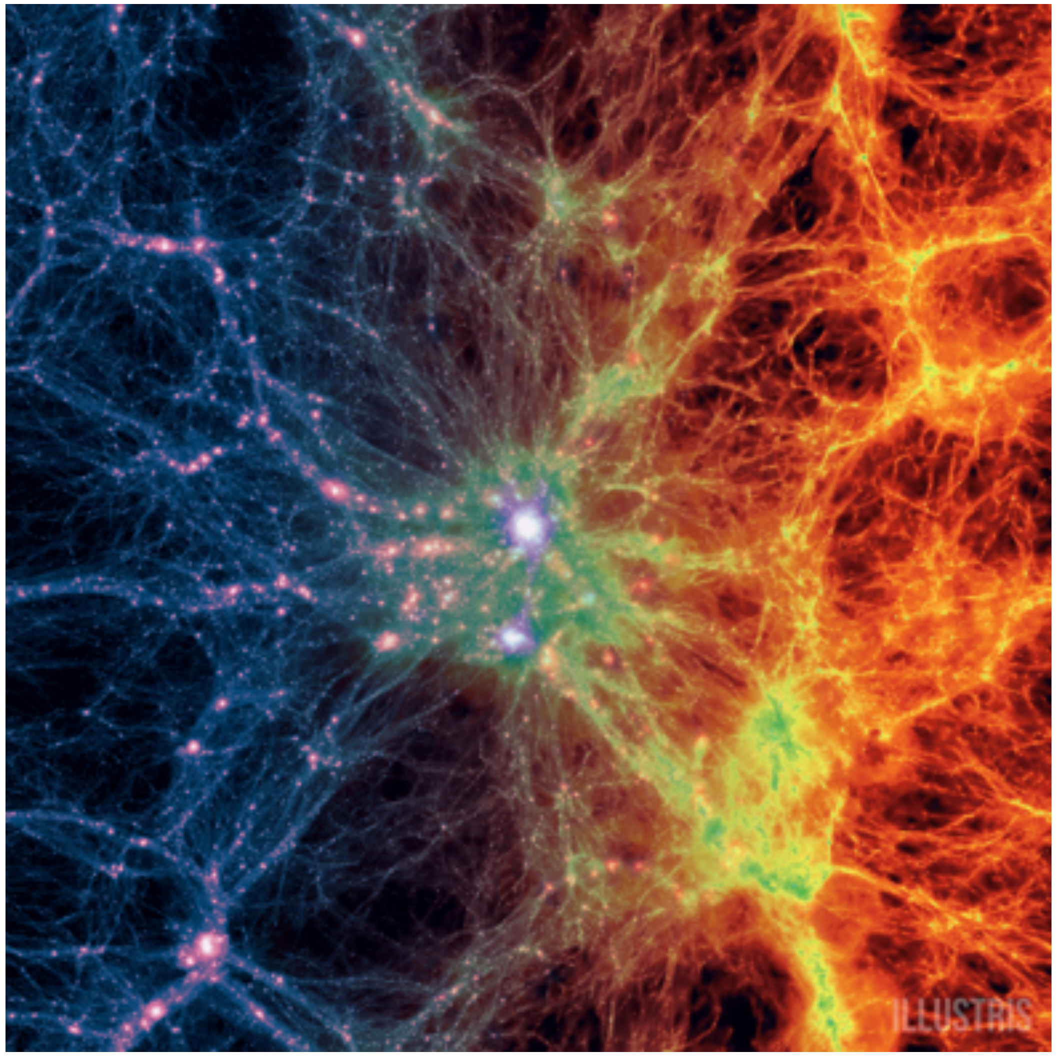

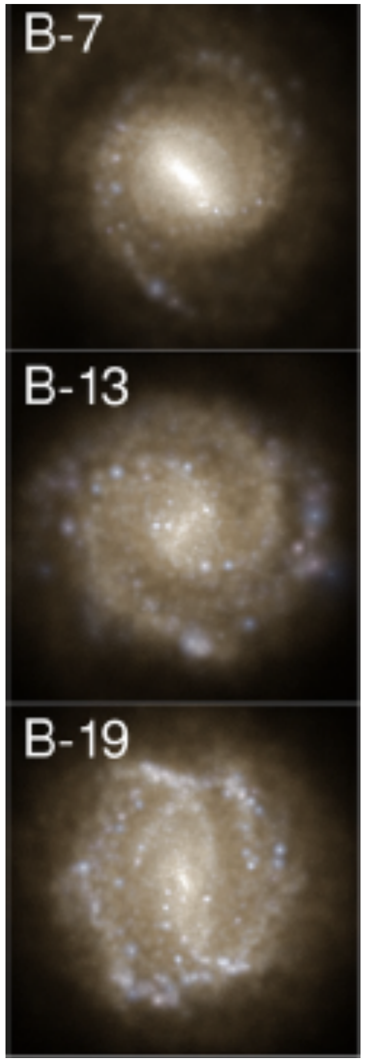

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