Bouncing cosmologies from condensates of quantum geometry

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arXiv:1602.05881 [gr-qc]
arXiv:1602.08271 [gr-qc]

GR21

Cosmology as a Condensate of Geometry

In any theory such as LQG which predicts that space-time is constituted of quanta of geometry, it seems reasonable to assume that in large cosmological space-times:

- there are many quanta of geometry,
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If all the quanta are indeed in the same state, this suggests using **condensate states** to extract cosmology from LQG.

This in turn directly leads to **group field theory**, a field theory for the quanta of geometry of LQG.

Group Field Theory with a Scalar Field

Group field theory (GFT) can be seen as a second-quantized language for loop quantum gravity, where the field operators

$$\hat{\varphi}^{j_1,j_2,j_3,j_4,\iota}_{m_1,m_2,m_3,m_4}(\phi), \qquad \hat{\varphi}^{\dagger\, j_1,j_2,j_3,j_4,\iota}_{m_1,m_2,m_3,m_4}(\phi),$$

create and annihilate quanta of geometry: spin network nodes [Oriti].

The j_i and m_i colour the links of the (four-valent) spin network node and the intertwiner ι and the scalar field ϕ both live on the spin network nodes. Connectivity is imposed via projectors on links.

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The classical GFT action $S(\varphi, \bar{\varphi})$ is typically chosen so that the perturbative expansion of the GFT partition function matches the sum over geometries of a spin foam model. In the simplest GFT actions for quantum gravity with $V(\phi) = 0$, the dominant terms are

$$S \sim \sum_{j_i,m_i,\iota_i} \int_{\phi_i} \left[\bar{\varphi}\, K_2^{(0)} \varphi + \bar{\varphi}\, K_2^{(2)} \partial_\phi^2 \varphi \right] + \sum_{j_i,m_i,\iota_i} \int_{\phi_i} \left[\bar{\varphi}^5\, \bar{\mathcal{V}}_5 + \varphi^5\, \mathcal{V}_5 \right].$$

Condensate States

A simple family of condensate states are the Gross-Pitaevskii condensate states, i.e., coherent states of the GFT field operator which are, up to a numerical prefactor, [Gielen, Oriti, Sindoni]

$$|\sigma\rangle \sim \exp\left(\sum_{j_i, m_i, \iota} \int \mathrm{d}\phi \, \sigma^{j_i,\iota}_{m_i}(\phi) \hat{\varphi}^{\dagger}{}^{j_i,\iota}_{m_i}(\phi)\right) |\mathbf{0}\rangle,$$

where $\sigma^{j_i,\iota}_{m_i}(\phi)$ is the condensate wave function. Note that $\sigma^{j_i,\iota}_{m_i}(\phi)$ is not normalized; rather, its norm gives the number of fundamental GFT quanta.

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where $\sigma^{j_i,\iota}_{m_i}(\phi)$ is the condensate wave function. Note that $\sigma^{j_i,\iota}_{m_i}(\phi)$ is not normalized; rather, its norm gives the number of fundamental GFT quanta.

Importantly, the massless scalar field can be used as a relational clock: $\sigma^{j_i,\iota}_{m_i}(\phi_o)$ can be understood as the condensate wave function evaluated at the 'time' ϕ_o. So, imposing the quantum equations of motion on $|\sigma\rangle$ gives relational dynamics of $\sigma^{j_i,\iota}_{m_i}(\phi)$ with respect to ϕ.

The Form of $\sigma_{m_i}^{j_i,\iota}(\phi)$ and its Dynamics

It is important to make choices for $\sigma_{m_i}^{j_i,\iota}(\phi)$ so that the condensate state represents a cosmological space-time.

- We are interested in the spatially flat FLRW space-time.
  So **we neglect connectivity**: the main observable is the total volume where connectivity is unimportant, and the space-time is spatially flat so we do not need to worry about encoding the spatial curvature in the connectivity of the graph [Gielen, Oriti, Sindoni].
- We are only interested in isotropic observables.
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The dynamics of the condensate state are determined by the first Schwinger-Dyson equation, $\langle\sigma| \frac{\delta S}{\delta\bar{\varphi}} |\sigma\rangle = 0$. The other Schwinger-Dyson equations should be approximately satisfied in the regime where $|\sigma\rangle$ is an approximate solution (i.e., where interactions are small).

The Small Interactions Approximation

The Gross-Pitaevskii condensate approximation assumes that interactions are small. Thus, the regime of validity of this approximation is where the interaction term is negligible. To consider cases when the interaction term becomes important, it will be necessary to go beyond the Gross-Pitaevskii approximation and include interactions (i.e., connectivity information).

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As can easily be checked in the equation of motion for σ_j, the interaction term will become large when $|\sigma_j|$ becomes sufficiently large. This is the large volume limit: **the Gross-Pitaevskii condensate approximation breaks down at large volumes**.

Interactions becoming important at large volumes may be related to the fact that the connectivity information has been ignored: all GFT quanta are interacting with all other quanta, not only their neighbours. Restoring connectivity information may well fix this.

The Mesoscopic Regime

In the remainder, I will consider the mesoscopic regime where the Gross-Pitaevskii approximation can be trusted ($|\sigma_j|$ sufficiently small) and where there are enough quanta for a continuum space-time interpretation to be viable ($|\sigma_j|$ sufficiently large.)

Such a regime will exist for some GFT actions (but not all), depending on the parameters in the action. Here, I will take such a GFT action and only work in this mesoscopic regime.

Coarse-Grained Equations of Motion

From the equations of motion for the condensate wave function, we can extract relational dynamics for coarse-grained observables like the total spatial volume and the momentum of the massless scalar field:

$$\pi_\phi' = 0,$$

$$\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_j V_j\,|\sigma_j|\,\mathrm{sgn}(|\sigma_j|')\,\sqrt{E_j - \frac{Q_j^2}{|\sigma_j|^2} + m_j^2|\sigma_j|^2}}{3\sum_j V_j|\sigma_j|^2}\right)^2,$$

where E_j, Q_j are constants of the motion, m_j depends on the coupling constants in the GFT action, $V_j \sim j^{3/2}\ell_{\rm Pl}^3$, and $f' := \partial_\phi f$.

The first equation is exactly the continuity equation for a massless scalar field.

The Condensate Friedmann Equation

At large volumes, $|\sigma_j|$ is large and in this limit, choosing the coupling constants in the GFT action such that $m_j^2 = 4\pi G$,

$$\left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3},$$

which is exactly the Friedmann equation for a spatially flat FLRW space-time in terms of the relational time ϕ.

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$\Rightarrow$ The correct Friedmann dynamics are recovered in the semi-classical large volume limit, for an appropriate choice of coupling constants in the GFT action.

Furthermore, a closer analysis of the more general Friedmann equation shows that there is always a bounce in the volume: $V = 0$ is never reached and so the big-bang and big-crunch singularities are generically resolved.

Recap

- Motivated by simple arguments combined with insights from LQC, we made a specific ansatz on the type of state in (the GFT reformulation of) LQG that corresponds to cosmological space-times: GFT condensate states.
- The equations of motion for the condensate states are determined by the GFT action, and from these equations of motion we can extract the continuity and Friedmann equations.
- The classical Friedmann equations are recovered in an appropriate semi-classical limit for some choices of parameters in the GFT action.
- The classical singularity is resolved and is generically replaced by a bounce.
- The LQC effective Friedmann equations are (almost) recovered for a natural choice of the condensate wave function.

Outlook

There are many open questions, including:

- Study other condensate wave functions and GFT actions [Gielen],
- Allow for scalar fields with non-trivial potentials,
- Calculate error in higher order Schwinger-Dyson equations,
- Include spatial curvature and anisotropies,
- Understand how to handle large interactions [de Cesare, Pithis, Sakellariadou],
- Include connectivity information in the analysis,
- And many more. . .

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Thank you for your attention!