Co–moving frames for BSSNOK evolutions of dynamical spacetimes

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Introduction

Motivation: co–moving frame

- Significantly reduces metric dynamics relative to computational grid
- Numerical errors (spurious excitations) greatly reduced
- Physically: precessing binaries, better angular momentum conservation

Why desire the above properties?

- Improved phase accuracy for GWs (from both BH and NS binaries)
- Hydrodynamical spacetimes, e.g. eccentric NS binaries

Several previous efforts, notably before **moving puncture** technique, (Campanelli *et al*, 2006)

We build on and extend methods by (Bruegmann *et al*, 2004; Alcubierre *et al*, 2003; Diener *et al*, 2006).

Goal: Framework that can be easily integrated into mesh–refinement codes for

- relativistic vacuum (BH) spacetimes
- relativistic stellar (hydrodynamical) models

Moving Punctures

Puncture initial data

- Bowen–York solution to constraints
- singularities removed
- used to generate BBH data with momentum and spin

Pre–2005: punctures fixed using "static conformal factor" $g_{ab} = \psi^4 \tilde{g}_{ab}$

2005:

- Moving Punctures discovered, (Baker *et al*; Campanelli *et al*).
- removed static conformal factor
- allowed first stable BSSNOK evolutions

Hannam *et al*, 2007 showed instabilities at punctures due to enforcing the "static conformal factor" ψ

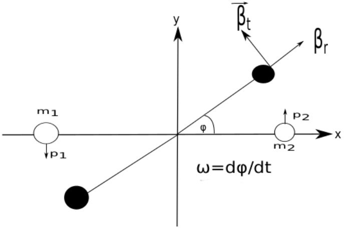
BSSNOK evolutions should be compatible with 'old-school' co-rotation.

Co-moving frames: basics

Consider orbiting binary (black holes or neutron stars), in $xy-$ plane

Transform to co–moving frame rotating with angular velocity $\vec{\omega}$,

$$t = \bar{t},$$
$$x = \bar{x}\cos(\omega t) - \bar{y}\sin(\omega t),$$
$$y = \bar{x}\sin(\omega t) + \bar{y}\cos(\omega t),$$
$$z = \bar{z}$$



Plugging transformation into line element $\rightarrow$ gauge transformation

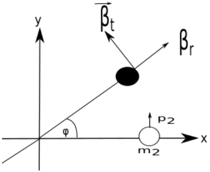
$$\bar{\alpha} = \alpha,$$
$$\bar{\beta}^i = \beta^i + (\vec{\omega} \times \vec{r})^i$$

Evolution equations invariant under gauge transformation

Co-moving frames: BSSNOK evolutions(2)

Implement co-moving frame by attaching the grid to one member of the binary.

- Removes the tangential shift components: $\vec{\beta}_\omega = \vec{\omega} \times \vec{r}$
- ω either specified or determined dynamically from BH motion



Possible adjustment strategies

1. $\vec{\beta} \to \vec{\beta} + \vec{\beta}_\omega$; as initial condition
2. $\vec{\beta} \to \vec{\beta} + \vec{\beta}_\omega$ plus update on $\vec{\omega}$, $\vec{\omega} = \left(\vec{r} \times \vec{\beta}\right)/r^2$.

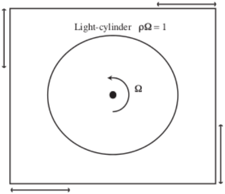
Moving puncture gauge, in particular, for the shift we adopt

$$\partial_t \beta^i - \beta^j \partial_j \beta^i = \frac{3}{4} B^i$$

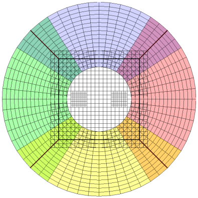
$$\partial_t B^i - \beta^j \partial_j B^i = \partial_t \tilde{\Gamma}^i - \beta^j \partial_j \tilde{\Gamma}^i - \eta B^i$$

Framework and tools

- Cactus (computational framework), Carpet (mesh refinement)
- Puncture initial data; BSSNOK evolution system
- Multi–patch system, (Pollney *et al*, 2011):
  - spherical outer boundary → no corners (problem for Cartesian codes)
  - angular grid spacing increases with r → avoids CFL instability

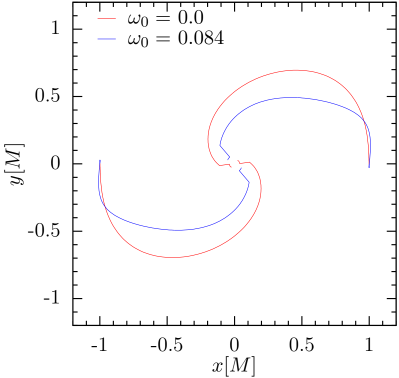


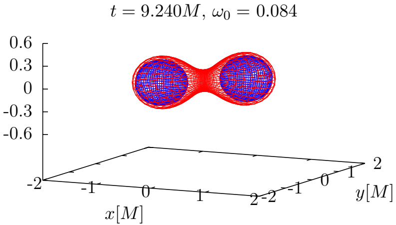
[Alcubierre, 2008]



Preliminary results: simple rigid rotation

Close–orbit test; equal mass binary black hole; about half an orbit



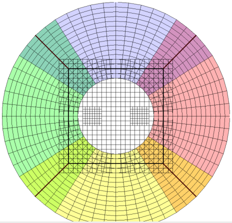


co-rotating shift, each BH moves slower than in non-rotating case.

Preliminary results: simple rigid rotation (2)

Comments:

- simple rigid rotation not adequate to remove grid dynamics;
- Investigating dynamical methods for updating shift
  - using motion of BH centroid
  - using value of shift at puncture location
- Superluminal shift near outer boundary
- Possibly need to consider specific boundary conditions for superluminal shift, Calabrese & Gundlach, 2006
  - At both outer and interpatch boundaries
  - Transition region between $|\beta| \leq 1$ and $|\beta| > 1$, blend the outer boundary conditions (?)



Status and outlook

Co-rotating frames represent a simple and effective way to improve phase accuracy for finite difference codes.

We have carried out initial experiments with "moving-puncture"-BSSNOK evolutions.

- No science (GW extraction) yet;
- Promising results from close–orbit tests.

In progress

- Investigating boundary effects
  - outer boundary
  - interpolation
- Tests with longer BBH models (multiple orbits)

If all goes well . . .

- Deploy on full realistic astrophysical models to reduce phase errors
- Apply to relativistic hydrodynamical models
  - core collapse
  - BNS models