Conserved charges of the extended Bondi-Metzner-Sachs algebra

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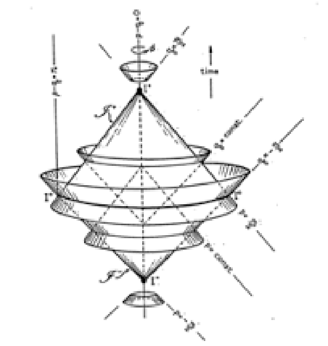
Brief review of asymptotic symmetries



R. Penrose, Les Houches, 1963

- Asymptotically flat spacetimes have BMS symmetry group
- BMS has semidirect form: Supertranslations (ST) $\rtimes$ Lorentz
- ST: infinite-dimensional, abelian, 4D translation subgroup; roughly "angle-dependent translations"
- Conjugate charges: 4-momentum, supermomentum, and angular momentum

An extended BMS algebra

Barnich & Troessaert: include singular solutions of conformal Killing equation

Nomencalture

- Lie algebra has form: ST $\rtimes$ Virasoro
- Virasoro: infinite-dimensional, called superrotations (SR), Lorentz subalgebra; roughly "angle-dependent boosts"
- SR charges: "super-center-of-mass" (super-CoM)

Physical relevance?

- Barnich & Troessaert show for a Kerr BH: super-CoM vanish in a Bondi frame
- SR symmetry gives rise to subleading soft theorem of gravitational scattering and also the spin-memory effect

Results and outline of this talk

Aim and Results

- Aim: compute super-CoM charges in stationary regions
- Results: show charges are finite and related to memory effect

Review of standard BMS

- Review Bondi coordinates and BMS asymptotic symmetries
- Summarize Wald-Zoupas approach to computing asymptotic charges and fluxes
- Explain relation between supermomenta and memory effect

Bondi-Sachs framework

Work in Bondi coordinates (u, r, θ^A):

$$
\begin{aligned}
ds^2 = & - du^2 - 2du dr + r^2 h_{AB} d\theta^A d\theta^B \\
& + \frac{2m}{r} du^2 + r C_{AB} d\theta^A d\theta^B + D^B C_{AB} d\theta^A du + \ldots
\end{aligned}
$$

Bondi-Sachs framework

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- θ^A: coordinates on S^2 with 2-metric h_{AB} and covariant derivative operator D_A
- $m = m(u, \theta^A)$: Bondi mass aspect
- $C_{AB} = C_{AB}(u, \theta^C)$: shear tensor (symmetric trace-free)

Bondi-Sachs framework

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- θ^A: coordinates on S^2 with 2-metric h_{AB} and covariant derivative operator D_A
- $m = m(u, \theta^A)$: Bondi mass aspect
- $C_{AB} = C_{AB}(u, \theta^C)$: shear tensor (symmetric trace-free)
- $N_{AB} = \partial_u C_{AB}$: news tensor (vanishes when stationary)
- $N_A = N_A(u, \theta^B)$: Bondi angular-momentum aspect

BMS symmetries in Bondi coordinates

Infinitesimal BMS symmetries on $\mathscr{I}^+$ generated by $\vec{\xi}$:

$$\vec{\xi} = f(\theta^A)\vec{\partial}_u + Y^A(\theta^B)\vec{\partial}_A$$

$$f(\theta^A) = \alpha(\theta^A) + \frac{1}{2} u D_B Y^B(\theta^A)\,, \quad 2D_{(A}Y_{B)} - D_C Y^C h_{AB} = 0$$

$\alpha \leftrightarrow$ ST; $Y^A \leftrightarrow$ Lorentz transformation

Can be extended into interior by requiring metric maintains Bondi form and scalings under pullback; e.g., C_{AB} transforms as

$$\delta C_{AB} = fN_{AB} - (2D_A D_B - h_{AB} D^2)f - \frac{1}{2} D_C Y^C C_{AB} + \mathcal{L}_{\vec{Y}} C_{AB}$$

Conjugate charges

- At $\mathscr{I}^+$, charges not conserved when fluxes nonvanishing
- Wald and Zoupas developed method to compute charges and fluxes associated with a symmetry $\vec{\xi}$ in this context.
- For $\vec{\xi}$ in BMS, charge and flux satisfy

$$Q[\mathcal{C},\vec{\xi}] = \int_{\mathcal{C}} \boldsymbol{\mathcal{Q}}\,, \quad Q[\mathcal{C}_2,\vec{\xi}] - Q[\mathcal{C}_1,\vec{\xi}] = \int_{\mathscr{I}^+} \boldsymbol{\mathcal{F}}$$

for 2-form $\boldsymbol{\mathcal{Q}}$ and (exact) 3-form $\boldsymbol{\mathcal{F}}$

- In stationary, vacuum, & Bondi coordinates $\boldsymbol{\mathcal{F}} = 0$ and

$$Q[\mathcal{C},\vec{\xi}] = \frac{1}{16\pi}\int_{\mathcal{C}} d^2\Omega \Big[4\alpha m - 2uY^A D_A m + 2Y^A N_A - \frac{1}{8}Y^A D_A(C_{BC}C^{BC}) - \frac{1}{2}Y^A C_{AB} D_C C^{BC} \Big]$$

Memory effect and supermomenta

In Bondi coords, memory effect is determined by

$$\Delta C_{AB}(\theta^C) = C_{AB}(u_2) - C_{AB}(u_1) = \frac{1}{2}(2D_A D_B - h_{AB} D^2)\Delta\Phi$$

Consider supertranslations with $\ell \geq 2$

$$\alpha(\theta^A) = \sum_{\ell \geq 2} \sum_{m=-\ell}^{\ell} \alpha_{\ell m} Y_{\ell m}(\theta^A)$$

Supermomenta on cut $\mathcal{C}$ (in stationary region) denoted $\mathcal{P}_{\ell m}$ with

$$Q[\mathcal{C}, \alpha\vec{\partial}_u] = \sum_{\ell,m} \alpha^*_{\ell m} \mathcal{P}_{\ell m} = \frac{1}{4\pi} \sum_{\ell,m} \alpha^*_{\ell m} m_{\ell m}$$

$m_{\ell m}$: $Y_{\ell m}$ moments of $m(u, \theta^A)$

Memory effect and supermomenta cont...

Flux formula $Q[\mathcal{C}_2, \vec{\xi}] - Q[\mathcal{C}_1, \vec{\xi}] = \int_{\mathscr{I}^+} \boldsymbol{\mathcal{F}}$ gives

$$\sum_{\ell,m} \alpha^*_{\ell m} \Delta \mathcal{P}_{\ell m} = \sum_{\ell,m} \alpha^*_{\ell m} (-\Delta \mathcal{E}_{\ell m} + \mathcal{D} \Delta \Phi_{\ell m})$$

with $\mathcal{D} = D^2(D^2 + 2)/(32\pi)$ and

$$\Delta \mathcal{E}_{\ell m} = \left[\int_{u_1}^{u_2} du \left(r^2 T_{uu} + \frac{1}{32\pi} N_{AB} N^{AB} \right) \right]_{\ell m}$$

- $\Delta \Phi_{\ell m}$: $Y_{\ell m}$ moments of total memory
- $\Delta \mathcal{P}_{\ell m}$: $Y_{\ell m}$ moments of "ordinary" memory
- $\Delta \mathcal{E}_{\ell m}$: $Y_{\ell m}$ moments of "null" memory

Supermomenta only part of memory effect; is there a charge encompassing the full effect?

Superrotations

Superrotation vector fields

- Besides 6 smooth $Y^A(\theta^B)$, and infinite number of Y^A satisfying $2D_{(A}Y_{B)} - D_C Y^C h_{AB} = 0$
- Common basis is $l_m = -z^{m+1}\partial_z$ and $\bar{l}_m = -\bar{z}^{m+1}\partial_{\bar{z}}$ with $z = e^{i\phi}\cot\theta/2$ and $m \in \mathbb{Z}$
- Must take real part of complex l_m and $\bar{l}_m$

Super-center-of-mass charges

- Compute charges using Wald-Zoupas prescription
- Formally valid, but might need modification for singular Y^A?
- Computed on a single cut via $\mathcal{Q}$
- Also compute change in charges via $\mathcal{F}$

A canonical Bondi frame

Consider 2 frames, in stationary, vacuum regions in which metric is u-independent

1. Canonical frame:

$$m(\theta^A) = m_0 = \text{const.}, \quad C_{AB}(\theta^C) = 0\,,$$

$$N_A(\theta^B) = \text{magnetic parity}, \ \ell = 1$$

2. One supertranslated by $\Delta\Phi(\theta^A)$, and linearized (for simplicity) from canonical frame

$$m(\theta^A) = m_0 = \text{const.}, \quad N_A(\theta^B) = N_A^{\ell=1} - \frac{3}{2} m_0 D_A \Delta\Phi\,,$$

$$\Delta C_{AB}(\theta^C) = \frac{1}{2}(2D_A D_B - h_{AB} D^2)\Delta\Phi$$

Super-center-of-mass charges

Charges in frames 1 and 2

- In frame 1, super-CoM vanish; in frame 2 given by

$$Q[\mathcal{C}, \vec{Y}] = -\frac{3m_0}{16\pi} \int_{\mathcal{C}} d^2\Omega Y^A D_A \Delta\Phi$$

- Super-CoM charges are finite & contain (incomplete) information about ΔC_{AB}

Change in charges between two general stationary frames

- Integrate change in flux

$$\int_{\mathscr{I}^+} \mathcal{F} = Q[\mathcal{C}_2, \vec{Y}] - Q[\mathcal{C}_1, \vec{Y}] - \frac{1}{32\pi} \int_{\mathscr{I}^+} du\, d^2\Omega Y^A \epsilon_{AB} D^B (\epsilon^{CD} D_D D^E C_{CE})$$

- Flux is difference between change in super-CoM charges and spin-memory

Conclusions and discussion of results

- On a single cut, super-CoM charges of stationary spacetimes are trivial (vanish in canonical frame)
- Flux of super-CoM contains information about total memory and spin memory
- Both points hold for spacetimes with black holes
- Supermomentum and super-CoM are *not* BH hair
- Changes in supermomentum and super-CoM may have some relevance for dynamical black holes & information paradox