# BLACK HOLE LATTICES & COSMOLOGY

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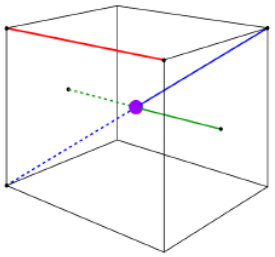
## 1. Lattice construction

| Lattice Type | Number of Faces | Number of Edges | Number of Vertices | Vertex Figure | Schläfli Symbols |
|---|---|---|---|---|---|
| 5−cell (Tetrahedra) | 10 (Triangles) | 10 | 5 | Tetrahedron | {333} |
| 8−cell (Cubes) | 24 (Squares) | 32 | 16 | Tetrahedron | {433} |
| 16−cell (Tetrahedra) | 32 (Triangles) | 24 | 8 | Octahedron | {334} |
| 24−cell (Octahedra) | 96 (Triangles) | 96 | 24 | Cube | {343} |
| 120−cell (Dodecahedra) | 720 (Pentagons) | 1200 | 600 | Tetrahedron | {533} |
| 600−cell (Tetrahedra) | 1200 (Triangles) | 720 | 120 | Icosahedron | {335} |

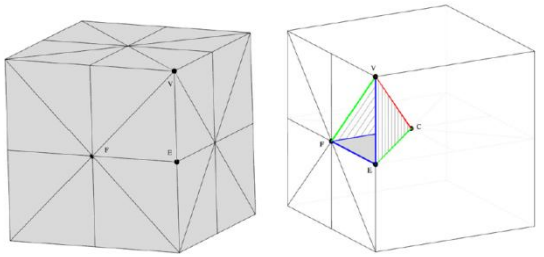
Table 1: *All possible lattices that can be constructed on a 3-sphere using the regular convex polychora. The Schläfli symbols {pqr} denote the number of edges to a face, p, the number of faces that meet at the vertex of a cell, q, and the number of cells that meet at an edge, r.*

We wish to regularly arrange a finite number of masses on a 3-sphere: consider the six possible convex regular polychora. Then we put a mass at the centre of each cell.

2. Identification of the curves with LRS

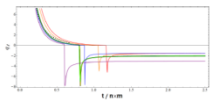


3. Identification of the reflection symmetric surfaces



4. Time evolution of the edge of a cell

### DECELERATION PARAMETER IN A LATTICE BLACK HOLE UNIVERSE



$$\ell(t) = \int_{\chi_1}^{\chi_2} \sqrt{(g_{\chi\chi})}\, d\chi = \int_{\chi_1}^{\chi_2} a_{||}\sqrt{(g_{\chi\chi})_0}\, d\chi$$

$$q_\ell \equiv -\frac{\ddot{\ell}\ell}{\dot{\ell}^2}$$

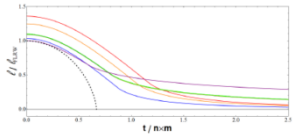
From highest to lowest at early times, the six solid lines correspond to the 5-cell (red), the 8-cell (orange) , the 16 and 24-cells (yellow and green), the 120-cell (blue), and the 600-cell (purple). The dotted line is the curve associated with a spatially closed FLRW solution with the same total proper mass, and time is displayed in units of total proper mass.

Note the sign of the deceleration parameter in vacuum! In a spatially flat FLRW solution such behaviour would be taken to correspond to a "phantom fluid", with $p < -\rho$.

5a. Effective deceleration parameter

### EVOLUTION OF THE EDGE LENGTH

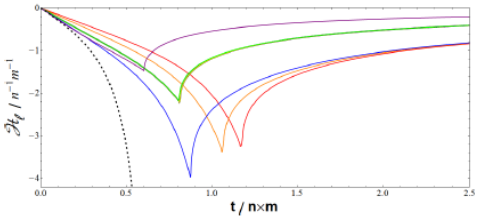
$$\ell(t) = \int_{\chi_1}^{\chi_2} \sqrt{(g_{\chi\chi})}\, d\chi = \int_{\chi_1}^{\chi_2} a_{||}\sqrt{(g_{\chi\chi})_0}\, d\chi$$



$l_{\rm FLRW}$: $dl^2 = \frac{16M^2}{9\pi^2}(d\chi^2 + \sin^2\chi d\theta^2 + \sin^2\chi\sin^2\theta d\phi^2)$

Legend: 5-cell (red), 8-cell (orange), 24-cell (green), 16-cell (yellow), 120-cell (blue), 600-cell (purple), closed Friedmann model with the same total proper mass (dotted). n = number of sources

5b. Effective Hubble function



6. Reduction of the equations

- Edges of the cells
- Curves that connect cell centres through cell faces
- Curves that connect cell centres with vertices

Evolution equations (ignoring curl H):

$$\begin{aligned} \mathbf{e}_0(\mathcal{H}_\perp) + \mathcal{H}_\perp^2 &= -\frac{1}{3}E_+ \\ \mathbf{e}_0(E_+) + 3\mathcal{H}_\perp E_+ &= 0 \\ \mathbf{e}_0(\mathcal{H}_{||}) + \mathcal{H}_{||}^2 &= \frac{2}{3}E_+ , \end{aligned}$$

where we have defined the Hubble expansion rates in the directions parallel and perpendicular to the LRS curve:

$$\begin{aligned} \mathcal{H}_{||} &\equiv \frac{1}{3}(\Theta - 2\sigma_+) \\ \mathcal{H}_\perp &\equiv \frac{1}{3}(\Theta + \sigma_+) \end{aligned}$$

7. On the reflection symmetric surfaces

Note that on the reflection surface $E \cdot H = 0$
Introduce the super-Poynting vector $P_a = \epsilon_{abc}E^b{}_d H^{dc}$
and note that $P_\parallel = 0$ on the reflection symmetric surface

↓

There is no gravitational radiation crossing these surfaces according to Bel's first criterion

↓

Piecewise silence in discrete cosmological models

8. Interpretation of the results

### PROPERTIES OF THE FULL SPACETIME

- The big bang/crunch manifests itself at the marginally trapped surfaces of the black hole long before it is ever arrived at in the corresponding Friedmann solution
- The edges of the cells never become singular at any time during their evolution
- The corners of the lattice cells are strongly isometric to Minkowski space
- The relation $E \cdot H = 0$ holds on the cell faces due to reflection symmetry
- Gravitational waves are trapped within small chambers
- In these cosmological models large-scale acceleration is possible without any violation of the energy conditions
- The discrete symmetries impose these models to be piecewise silent (there is no gravitational radiation crossing the boundaries of the cells)

**1.** "Exact Evolution of Discrete Relativistic Cosmological Models", T. Clifton, D. Gregoris, K. Rosquist, R. Tavakol, JCAP vol. 11, Article 010, arXiv: 1309.2876

**2.** "Piecewise silence in discrete cosmological models", T. Clifton, D. Gregoris, K. Rosquist, Class. Quantum Grav. 31 (2014) 105012 (17pp), arXiv:1402.3201

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