Searching for a Continuum Limit in CDT Quantum Gravity

Daniel Coumbe*
with J. Ambjorn, J. Gizbert-Studnicki, J. Jurkiewicz

Niels Bohr Institutet

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*daniel.coumbe@nbi.ku.dk

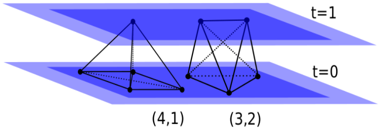
JAGIELLONIAN UNIVERSITY IN KRAKÓW

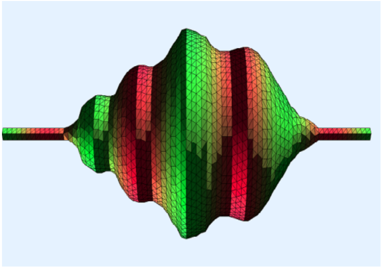
Orientation

1. Introduction to CDT
2. Motivation
3. Method (2 calculations)
4. Results
5. Summary

What is CDT?

- Non-discrete lattice approach to quantum gravity J. Ambjorn et al. (1998), hep-th/9805108.
- Locally flat n-dimensional simplices form simplicial approximation to manifold (Regge calculus)
- Causality condition: distinguishes between space-like and time-like links on the lattice
- Explicit foliation of the lattice into space-like hypersurfaces of fixed topology





- Triangulations summed and weighted by Einstein-Hilbert action

Causal dynamical triangulations (CDT) cont...

Continuum version

$$Z = \int \mathcal{D}[g] e^{iS_{EH}}, \tag{1}$$

$$S_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g}(-R + 2\Lambda), \tag{2}$$

Discrete version

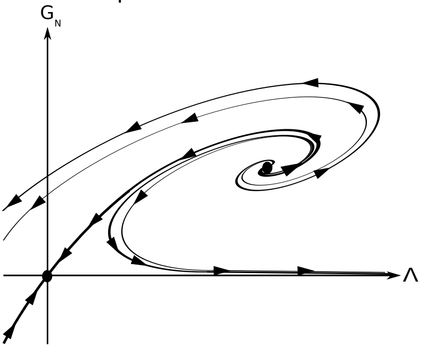
$$Z_E = \sum_T \frac{1}{C_T} e^{-S_E}. \tag{3}$$

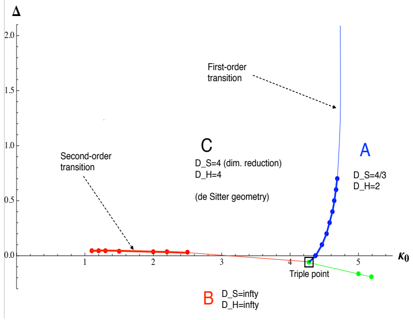
$$S_E^{Regge} = -(\kappa_0 + 6\Delta) N_0 + \kappa_4 (N_{4,1} + N_{3,2}) + \Delta (2N_{4,1} + N_{3,2}) \tag{4}$$

- $N_0 \equiv$ Number of vertices. Two types of simplex: $N_{4,1}$ and $N_{3,2}$
- κ_0 and κ_4 related to bare gravitational and cosmological constants, respectively
- Δ characterises the asymmetry between space and time-like links

Does CDT possess a continuum limit?

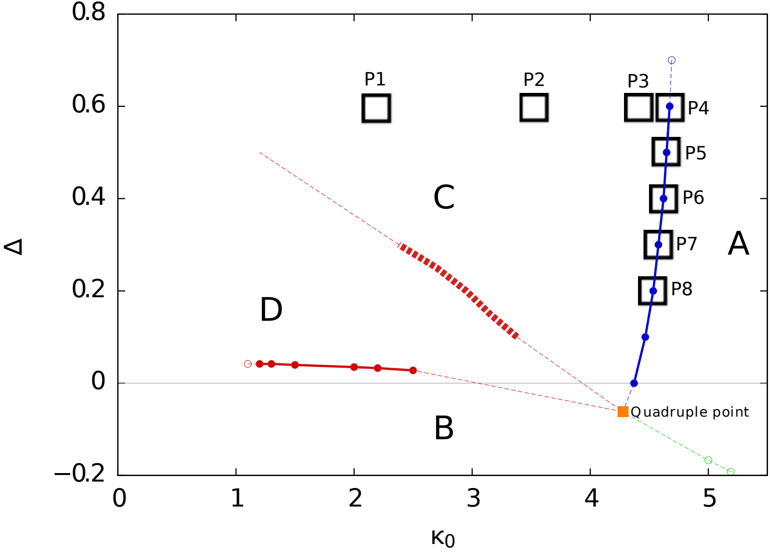
- Nonperturbative quantum gravity via asymptotic safety
- Requires UVFP - which would appear as second order critical point





- Divergent correlation length of UVFP - take $a \to 0$ while keeping observables fixed in physical units
- Important to reliably determine lattice spacing in CDT, and how it changes in parameter space...

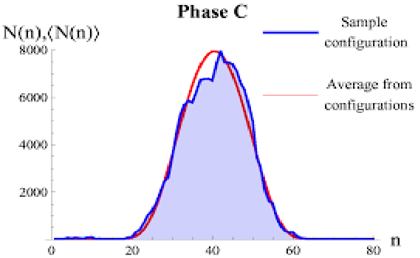
Updated phase diagram

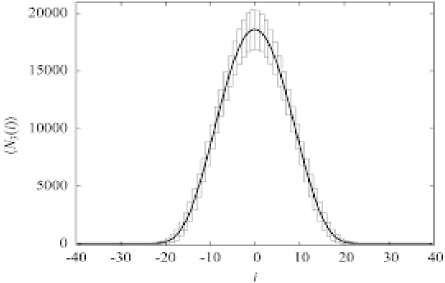


Method 1: Fluctuations about de Sitter space

- In phase C distribution of 3-volume as function of time has an expectation value that closely matches Euclidean de Sitter space ($T_{uni} = \pi s_0 \left(N_4^{(4,1)}\right)^{1/4}$)

$$\langle N_3(t) \rangle = \frac{3}{4} \frac{N_{(4,1)}^{3/4}}{s_0} \cos^3 \left(\frac{t}{s_0 N_{(4,1)}^{1/4}} \right) . \quad (5)$$





- Quantum fluctuations $\delta N_3(t) \equiv N_3(t) - \langle N_3(t) \rangle$ around de Sitter space consistent with effective action (Γ depends on amplitude of quantum fluctuations)

$$S_{eff} = \frac{1}{\Gamma} \sum_t \left(\frac{\left(N_3(t+1) - N_3(t)\right)^2}{N_3(t+1) + N_3(t)} + \mu N_3(t)^{1/3} - \lambda N_3(t) \right) , \quad (6)$$

Method 1: Fluctuations about de Sitter space

Seek continuum expressions to compare with CDT parametrisations...

- Assume phase C represents a universe described by Euclidean de Sitter space with superimposed quantum fluctuations for a spatially isotropic and homogeneous metric
- The E-H action for this metric becomes a minisuperspace (MS) action which can be parametrised by a spatial volume observable $V_3(\tau) = 2\pi^2 a^3(\tau)$ via

$$S_{MS} = \frac{1}{24\pi G} \int d\tau \sqrt{g_{\tau\tau}} \left(\frac{g^{\tau\tau} \left(\partial_\tau V_3(\tau)\right)^2}{V_3(\tau)} + \tilde{\mu} V_3(\tau)^{1/3} - \tilde{\lambda} V_3(\tau) \right), \tag{7}$$

- For the MS action the semiclassical spatial volume profile is given by

$$\langle V_3(\tau) \rangle = 2\pi^2 R^3 \cos^3 \left(\frac{\sqrt{g_{\tau\tau}}\ \tau}{R} \right) = \frac{3}{4} \frac{V_4}{\tilde{s}_0 V_4^{1/4}} \cos^3 \left(\frac{\sqrt{g_{\tau\tau}}\ \tau}{\tilde{s}_0 V_4^{1/4}} \right), \tag{8}$$

- where $V_4 = 8\pi^2 R^4/3$ is the volume of the 4-sphere and R its radius. $\tilde{s}_0 = 3/(8\pi^2)^{1/4}$.

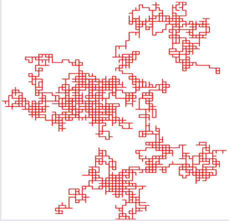
- Comparison via dimensional analysis yields $l_P^2 \equiv G \propto \Gamma a_{abs}^2$
- Proportionality factor derived in J. Ambjorn et al. (2009), hep-th/0807.4481.

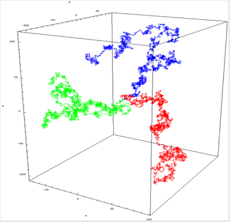
$$\boxed{a_{abs} = \sqrt{\frac{3\sqrt{6}}{\sqrt{C_4 s_0^2 \Gamma}}}\, l_P}$$

Method 2: Rescaling of the spectral dimension

What is the spectral dimension?

- The spectral dimension D_S defines the effective dimension of a fractal geometry via a diffusion process
- D_S is related to the probability of return, $P_r(\sigma)$, for a random walk over an ensemble of triangulations after σ diffusion steps





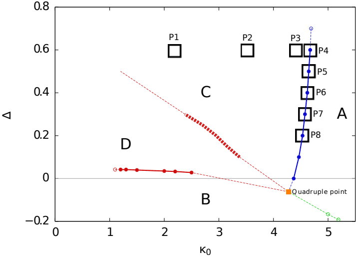
- In asymptotically flat space the spectral dimension is defined via

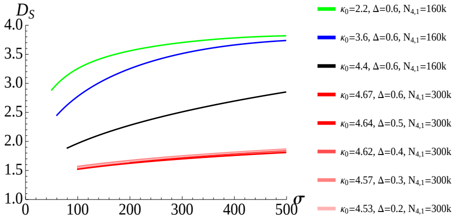
$$D_S = -2\frac{d\log\langle \mathrm{P_r}(\sigma)\rangle}{d\log\sigma} \quad (9)$$

- $\boxed{\text{Calculate } D_S(\sigma) \text{ (distance scale} \propto \text{number of diffusion steps)}}$

Method 2: Rescaling of the spectral dimension

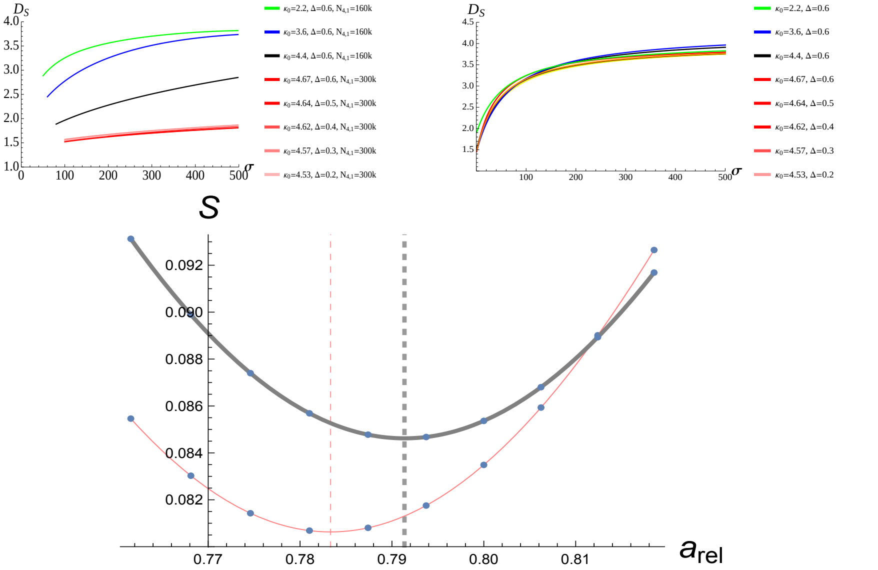
- Moving toward the C-A transition we notice that the curves flatten out





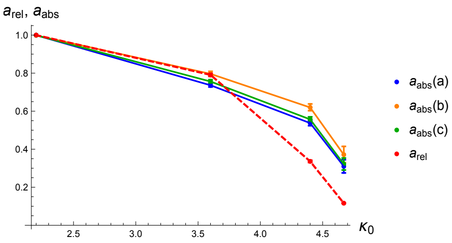
- Implies as one increases κ_0 lattice spacing a decreases: $\rightarrow$ takes a greater number of steps σ before the same dimension is obtained
- Rescale diffusion time σ by factor a_{rel}^2 to obtain "best overlap" $\left(D_S(\sigma) = a - \frac{b}{c+\sigma/a_{rel}^2}\right)$
- $\boxed{a_{rel} \text{ proportional to change in lattice spacing}}$

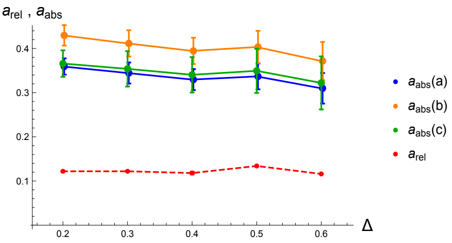
Method 2: Rescaling of the spectral dimension



Maximise κ_0 to take continuum limit...?

(κ_0, Δ)	a_{rel}	a_{abs} **(a)**	a_{abs} **(b)**	a_{abs} **(c)**
$(2.20, 0.6)$	1	1	1	1
$(3.60, 0.6)$	0.791 ± 0.008	0.74 ± 0.01	0.80 ± 0.01	0.76 ± 0.01
$(4.40, 0.6)$	0.336 ± 0.006	0.54 ± 0.01	0.62 ± 0.02	0.56 ± 0.01
$(4.67, 0.6)$	0.116 ± 0.001	0.31 ± 0.03	0.37 ± 0.04	0.32 ± 0.03
$(4.64, 0.5)$	0.134 ± 0.001	0.34 ± 0.03	0.40 ± 0.04	0.35 ± 0.03
$(4.62, 0.4)$	0.118 ± 0.003	0.33 ± 0.02	0.40 ± 0.03	0.34 ± 0.02
$(4.57, 0.3)$	0.122 ± 0.001	0.34 ± 0.02	0.41 ± 0.03	0.35 ± 0.02
$(4.53, 0.2)$	0.122 ± 0.001	0.36 ± 0.02	0.43 ± 0.02	0.37 ± 0.02





Outlook and summary

Outlook

- Some valid issues raised by *Cooperman* that need addressing (see arXiv:1604.01798)
- Important to determine order of new C-D transition... stay tuned!

Summary

- Calculated change in lattice spacing using 2 independent methods for 8 different points in parameter space

So what have we learned from our search for a continuum limit?

That maximising κ_0 in phase C may be an important piece of the puzzle...

Thank you!