B-Modes and Inflation

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REFERENCES:
arXiv: 1607.03523 [gr-qc] .

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Inflationary account for the seeds of cosmic structure and their imprint in the CMB is extremely successful however the expectation that the scalar modes will be accompanied by similarly generated tensor modes, has so far yielded no positive results. People are using this as a means to constrain the inflationary models.

Looking at a series of conceptual difficulties in the standard scenario we have been led to a different approach. As it turns out it leads to a rather different expectation for the tensor modes while it gives very similar expectations for the scalar ones.

The point is that from this perspective one has no right to use the (at present levels) non-observation (at present levels) of tensor modes to constrain the inflationary models !!.

The usual treatment is based on quantum treatment of perturbations Ψ, $\delta\phi$ and h_{ij} (using $v = a(\delta\phi + \frac{\dot{\phi}_0}{\mathcal{H}}\Psi)$).
Then computes $\langle\hat{\phi}(x)\hat{\phi}(y)\rangle$ (for various fields) (in the B-D vacuum state) and takes them to represent the primordial inhomogeneities which evolved into all the structure in our Universe.

THE THEORY FITS VERY WELL WITH THE OBSERVATIONS.

However, let us consider the following: The analysis starts with a H&I region, (both in the part that could be described at the “classical level", and the quantum level) that grows into our causal Universe. We end up with a situation which is not H&I : It contains the primordial inhomogeneities which will result in our Universe’s structure, and the conditions that permit our own existence.

How does this happen if the dynamics of the closed system does not break those symmetries?

Issue is recognized by various authors:
T. Padmanabhan (in Section 10.4, page 364 of " Structure Formation in the Universe", indicates that one must work with *certain classical objects mimicking the quantum fluctuations*, and that this is not easy to do and to justify). Also in S. Weinberg page 476 "Cosmology", S. Weinberg we find *"... the field configurations must become locked into one of an ensemble of classical configurations with ensemble averages given by quantum expectation values... It is not apparent just how this happens...."*, while V. Mukhanov page 348 of "Physical Foundations of Cosmology", clearly acknowledges that the problem is not resolved simply by invoking decoherence: *" .. However decoherence is not enough to explain the breakdown of translational invariance.."* and states that one might have to rely on something like the Many Worlds Interpretation.

In fact the issue can be related to one considered by N.F.Mott in 1929. I.e. α decay and the "measurement problem", but, in an aggravated form (no observers in the early universe !!).

Moreover, in the standard treatment both the metric and the field backgrounds are treated classically and both the metric and inflaton field perturbations are quantized.
However, it is not completely clear that quantizing the metric perturbation is the correct thing to do.
Even if one agrees that gravitation itself is quantum mechanical in nature, that does not mean that the metric degrees of freedom are the ones that need to be treated quantum mechanically.
Various arguments suggest that space-time geometry might emerge from deeper, non-geometrical degrees of freedom; and, just as one does not directly quantize, say, the heat equation, it might be incorrect to quantize the metric.
Furthermore Canonical quantization leads to timeless theories. The recovery of fully covariant spacetime notions is nontrivial. Often it is only achieved in an approximated sense when passing to a semiclassical treatment.
Perhaps it is OK to quantize just the the perturbations of metric but , then again it might not.

OUR APPROACH: The situation we face here is unique (Quantum + Gravity (GR) + Observations).

Need to point to a physical process that occurs in time as explaining the emergence of the seeds of structure. After all, emergence (in this context) means : Something that was not there at a certain time is there at a later time.
We need to explain the breakdown of the symmetry of the initial state: Collapse can do this.

Collapse Theories: There is important existing work in this direction (GRW, Pearle, Diosi, Penrose, etc) but in this talk, and for simplicity, we will NOT use any of those.

We consider adding, to the standard inflationary paradigm, a quantum collapse of the wave function as a self-induced instantaneous process.

The starting point will be the semi-classical Einstein's equation,

$$R_{\mu\nu} - (1/2)g_{\mu\nu}R = 8\pi G\langle \hat{T}_{\mu\nu}\rangle, \quad (1)$$

which treats gravitation classically and all other fields quantum mechanically.

We assume such a framework to be a valid approximation at the regime under consideration, which lies well after the full quantum gravity regime has been left behind.

To this, we add an explicitly collapse in the quantum state of matter fields. Non-trivial because while the collapse occur, the semiclassical equation breaks down.

(We have formalized a treatment incorporating this in proposal known as SSC) (*JCAP*. **045**, 1207, (2012) arXiv:1108.4928 [gr-qc]).

Take the standard inflationary account, the state of the universe before the time at which the seeds of structure emerge is given by the homogeneous and isotropic Bunch-Davies vacuum and the homogeneous and isotropic classical FRW spacetime.

However, as the state of the matter field undergoes a spontaneous and stochastic quantum collapse into a new state, which need not share these symmetries.

After the collapse, the semiclassical Einstein equation is again assumed to hold and, since $\langle \hat{T}_{\mu\nu} \rangle$ for the new state may not have the symmetries of the pre-collapse state, we are led to a geometry that, generically, will no longer be homogeneous and isotropic.

SPECIFIC TREATMENT:

As we said, space-time is thus treated in classical language, and in our case (working in a specific gauge) the metric is:

$$ds^2 = a^2(\eta)\{-(1+2\Psi)d\eta^2 + [(1-2\Psi)\delta_{ij} + h_{ij}]dx^i dx^j\},$$

In the practical approach, the field is split $\phi = \phi_0 + \delta\phi$: The homogeneous scalar background $\phi_0(\eta)$ and a perturbation $\hat{\delta\phi}$ to be treated with QFT. In the previous, more precise treatment, the former corresponds to a specific mode (or combination) of the quantum field.

During Inflation (slow roll regime), we will have $a(\eta) \approx (\frac{-1}{H_I\eta})^{1+\epsilon}$.

Set $a_{today} = 1$, and end of inflation at $\eta = \eta_0$ (negative and very small in absolute terms).

The perturbations of the scalar field lead to perturbations of the energy-momentum tensor, which, through the semiclassical Einstein equation, impact on the metric. At lowest order, this leads to the following equation for the Newtonian potential

$$\nabla^2 \Psi = 4\pi G \dot{\phi}_0 \langle \delta\dot{\phi} \rangle, \tag{2}$$

where we have made use of the specific form of the scale factor during inflation in the slow role regime.
The Fourier components of Ψ are given by

$$\tilde{\Psi}(\eta, \vec{k}) = -k^2 4\pi G \dot{\phi}_0 \langle \hat{\pi}_{\vec{k}}(\eta) \rangle \tag{3}$$

This expression then exhibits the fact that unless the quantum state differs from the initial vacuum, the Newtonian Potential vanishes.

That is $\langle 0|\hat{\delta\phi}_k(\eta)|0\rangle = 0$ and $\langle 0|\hat{\pi}_k(\eta)|0\rangle = 0$.

The collapse will modify the state and, thus, the expectation values of the operators $\hat{\delta\phi}_k(\eta)$ and $\hat{\pi}_k(\eta)$. Consider the rules of how the “collapse happens" THE SIMPLEST, MOST NAIVE SCHEME:

Single Instantaneous collapse (per mode) to a new state $|\Theta\rangle$. Just after the collapse, the expectation value of the momentum operator (or/and the field operator) in each mode is related to the uncertainties of the pre-collapse state.

$$\langle\hat{\delta\phi}_{\vec{k}}^{R,I}(\eta_k^c)\rangle_\Theta = x_{\vec{k}}^1\sqrt{(\Delta\hat{\delta\phi}_{\vec{k}}^{R,I})_0^2}, \langle\hat{\pi}_{\vec{k}}^{R,I}(\eta_k^c)\rangle_\Theta = x_{\vec{k}}^2\sqrt{(\Delta\hat{\pi}_{\vec{k}}^{R,I})_0^2},$$

where η_k^c is the *time of collapse* for each mode.

The $x_{\vec{k}}^{1,2}$ are numbers selected randomly from a Gaussian distribution centered at 0 and with unit dispersion. Our Universe corresponds to a **single realization** of these random variables.

The prediction for the observable quantities is recovered from consideration of a formal random walk obtained in reconstructing the full $\Psi(\eta, \vec{x})$ and from it the coefficients on the sky of its spherical harmonic decomposition. The analysis follows the standard treatment except for the fact that one needs to evaluate the ensemble average (rather than the quantum two-point function) over the states resulting from the collapse:

$$\overline{\langle \delta\hat{\pi}_{\vec{k}}(\eta)\rangle\langle \delta\hat{\pi}_{\vec{k'}}(\eta)\rangle}. \tag{4}$$

This result for scalar power spectrum (including modifications from the post reheating regime) agrees with the usual estimate for the scalar perturbation power spectrum.

Similarly, the equation of motion for the tensor perturbations is

$$(\partial_0^2 - \nabla^2)h_{ij} + 2(\dot{a}/a)\dot{h}_{ij} = 16\pi G\langle(\partial_i\delta\phi)(\partial_j\delta\phi)\rangle^{tr-tr} \tag{5}$$

where the superscript $tr - tr$ stands for the transverse traceless part of the expression.

Note that, even though both (the renormalized values for) $\langle \delta\dot{\phi} \rangle$ and $\langle (\partial_i \delta\phi)(\partial_j \delta\phi) \rangle$ vanish when evaluated in the vacuum, they will become non-vanishing in the quantum state of the field that results from the spontaneous collapse.

The expressions above show the basic difference, within this scheme, between the scalar and tensor metric perturbations. While the former are seeded by linear terms in perturbations of the scalar field, the latter are seeded by quadratic terms in such perturbations.
This difference, represents a radical departure from the standard approach, with crucial consequences: as long as we are in a regime where perturbation theory makes sense, the second term will be much smaller than the first.

In fact, the expression for the Fourier components $\tilde{h}_{ij}(\vec{k})$ is now obtained by solving the evolution non- homogeneous differential equation, with zero initial data

$$\ddot{\tilde{h}}_{ij}(\vec{k},\eta) + 2(\dot{a}/a)\dot{\tilde{h}}_{ij}(\vec{k},\eta) + k^2\tilde{h}_{ij}(\vec{k},\eta) = S_{ij}(\vec{k},\eta) \qquad (6)$$

with zero initial data and sourced by

$$S_{ij}(\vec{k},\eta) = 16\pi G \int d^3k e^{i\vec{k}\vec{x}} \langle \partial_i \delta\phi(\eta,\vec{x}) \partial_j \delta\phi(\eta,\vec{x}) \rangle^{tr-tr}. \qquad (7)$$

One must use the renormalized expression and evaluate it in the post-collapse state. This is done using the field and momentum conjugate expectation values and the simplifying assumption that the post-collapse state is a coherent state.

Focusing on a particular wave polarization and direction ($h_{12}(k\hat{x}_3, \eta)$ and dropping the indices) the solution of the above equation can be written explicitly as:

$$\begin{aligned} \tilde{h}(\vec{k}, \eta) &= -if^{+}(k, \eta) \int \frac{f^{-}(k, \eta')S(\vec{k}, \eta')}{H^2 \eta'^2} d\eta' \\ &\quad + if^{-}(k, \eta) \int \frac{f^{+}(k, \eta')S(\vec{k}, \eta')}{H^2 \eta'^2} d\eta' \end{aligned} \tag{8}$$

where

$$f^{\pm}(k, \eta) = \frac{H}{\sqrt{2k}}(\eta \pm \frac{i}{k})e^{\pm ik\eta} \tag{9}$$

Putting all this together, we obtain an expression for $h_{12}(k\hat{x}_3, \eta)$ that depends on the random numbers characterizing the post-collapse state.

Averaging over the random numbers, one is led to an expression that characterizes the Power spectrum for the tensor modes, in terms of:

$$\overline{S_{12}(k\hat{z},\eta)S_{12}(k'\hat{z},\eta)} = \delta(k-k')\frac{8G^2H^4}{(2\pi)^{10}}\int d^3p\frac{p_1^2p_2^2}{p^3(p+q)^3} \quad (10)$$

This is formally divergent, however we must introduce a cut-off (the last scale exiting the horizon during inflation: $a_{end-inf}p^{UV}/2\pi = H_I$, or more realistically the scale of diffusion dumping). The dominant term in the expression is then

$$\overline{S_{12}(k\hat{z},\eta)S_{12}(k'\hat{z},\eta)} = \delta(k-k')cG^2H^4P_{UV}. \quad (11)$$

Thus the prediction for the power spectrum of tensor perturbations is:

$$P_h^2(k) \sim (1/k^3)(V/M_{Pl}^4)^2(P_{UV}/k) \quad (12)$$

substantially smaller than the standard prediction for $P_h^2(k)$.

Look at very large scales !!

Note that, even if one quantizes also the metric perturbations, the current approach leaves room for predictions for r that differ dramatically from those of the standard accounts.
The point is that, in order for actual inhomogeneities to emerge from the uncertainties in the state of the quantum fields, one needs some physical process capable of breaking symmetries of quantum states (such as the Bunch-Davies vacuum).
That is the spontaneous collapse of the quantum state, (as described by some of the dynamical reduction theories) and, in the present context, it is far from clear that such process would affect in the same fashion the matter fields and the geometrical variables.

In fact, it is quite possible to consider schemes where matter fields, would undergo collapse with one rate, and metric perturbations would do so with a different one, or even schemes where the metric perturbations, although quantized, do not undergo spontaneous collapse by themselves.

In conclusion the use of bounds on the detection of B-modes in the polarization of the CMB, to constrain or rule out inflationary models is based on a very particular theoretical framework which, despite its popularity, is far from unique, and which, moreover, contains a serious shortcoming, as it cannot really account for the emergence of primordial inhomogeneities.

THANK YOU