Causal structure of cosmological black holes under scalar-field accretion

Daniel C. Guariento

in collaboration with

E. Abdalla, N. Afshordi, M. Fontanini, A. Maciel, F. Mercati, C. Molina, E. Papantonopoulos

based on

[1212.0155], [1312.3682], [1408.5538], [1502.01003], [1606.01215]

Jul 14 – GR21 – A1

Motivation

- Exact solutions of Einstein equations
- Black holes in the presence of self-gravitating matter
- Two competing effects:
  - Gravitationally bound objects
  - Expanding universe
- Coupling between local effects and cosmological evolution
  - Causal structure
- Quantum gravity
  - Vacuum solutions of modified gravity in the Einstein frame
  - Dynamic solutions of quantum gravity with matter

The McVittie metric

- Cosmological black holes: spatially flat McVittie solution

  [G. C. McVittie, *MNRAS* **93**,325 (1933)]

$$\mathrm{d}s^2 = -\frac{\left[1-\frac{m}{2a\hat{r}}\right]^2}{\left[1+\frac{m}{2a\hat{r}}\right]^2}\,\mathrm{d}t^2 + a^2\left[1+\frac{m}{2a\hat{r}}\right]^4\left(\mathrm{d}\hat{r}^2+\hat{r}^2\,\mathrm{d}\Omega^2\right)$$

  - ☐ $a(t)$, $m(t)$ constant: Schwarzschild metric
  - ☐ $m=0$: FLRW metric

- Unique solution that satisfies (with m constant)

  - ☐ Spherical symmetry
  - ☐ Perfect fluid
  - ☐ Shear-free / CMC foliation
  - ☐ Asymptotic FLRW behavior
  - ☐ Central singularity

McVittie metric features

- Causal structure is more easily seen on non-comoving coordinates
- Areal radius

$$r = a\left(1 + \frac{m}{2\hat{r}}\right)^2 \hat{r}$$

- McVittie in canonical coordinates

[N. Kaloper, M. Kleban, D. Martin, *PRD* **81** 104044 (2010), 1003.4777]

$$\mathrm{d}s^2 = -R^2\,\mathrm{d}t^2 + \left\{\frac{\mathrm{d}r}{R} - \left[\frac{\dot{a}}{a} + \frac{\dot{m}}{m}\left(\frac{1}{R} - 1\right)\right] r\,\mathrm{d}t\right\}^2 + r^2\,\mathrm{d}\Omega^2$$

$$\text{where}\left(R = \sqrt{1 - \frac{2m}{r}}\right)$$

- Past spacelike singularity at $r = 2m$
- Event horizons only defined if $\dot{a}/a$ constant as $t \to \infty$

- McVittie is a solution to a k-essence scalar

[E. Abdalla, N. Afshordi, M. Fontanini, DCG, E. Papantonopoulos, *PRD* **89** 104018 (2014), 1312.3682]

- Lagrangian: $\mathcal{L} = K(X, \varphi) + \kappa\mathcal{R}$
- Unique solution: *cuscuton* field

$$K(X, \varphi) = A(\varphi) + B(\varphi)\sqrt{X}$$

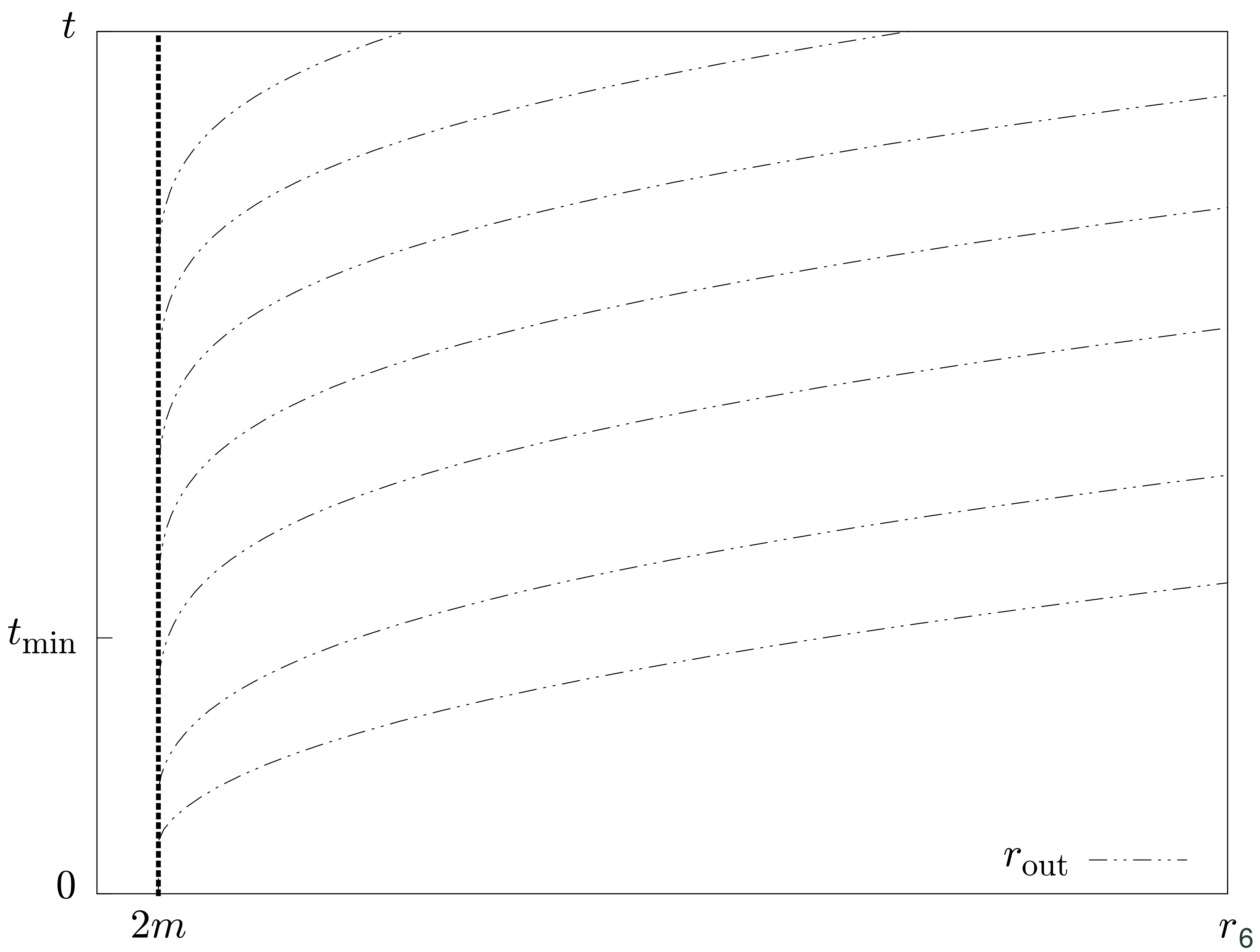
- Field imposes CMC foliation, which the McVittie class satisfies

$$\mathcal{K}^{\alpha}{}_{\alpha} = \frac{1}{\mu^2}\frac{\mathrm{d}V}{\mathrm{d}\phi} = 3H(t)$$

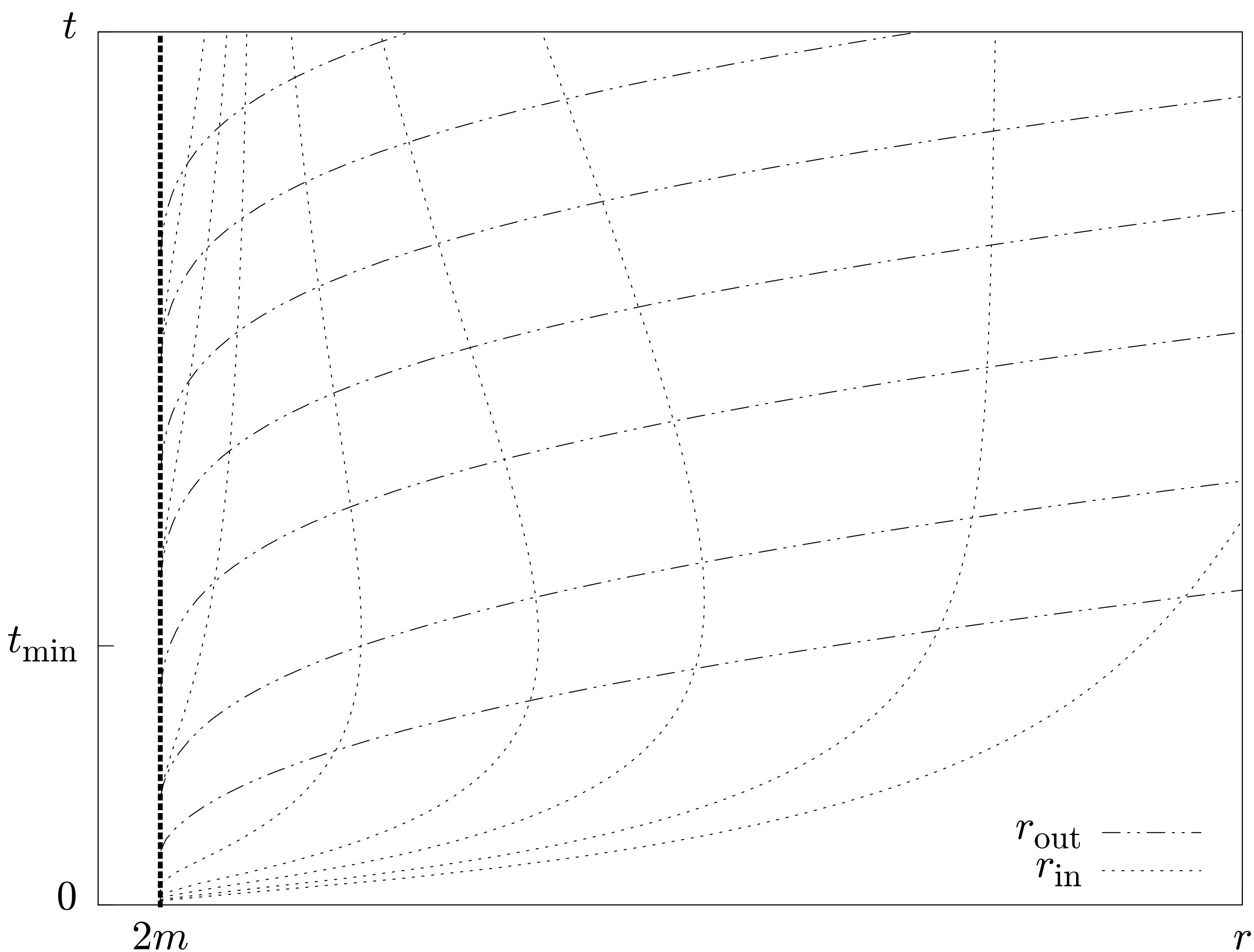
- McVittie is also a solution of *Shape Dynamics*

[DCG, F. Mercati, 1606.01215]

Light cones and apparent horizons



Light cones and apparent horizons



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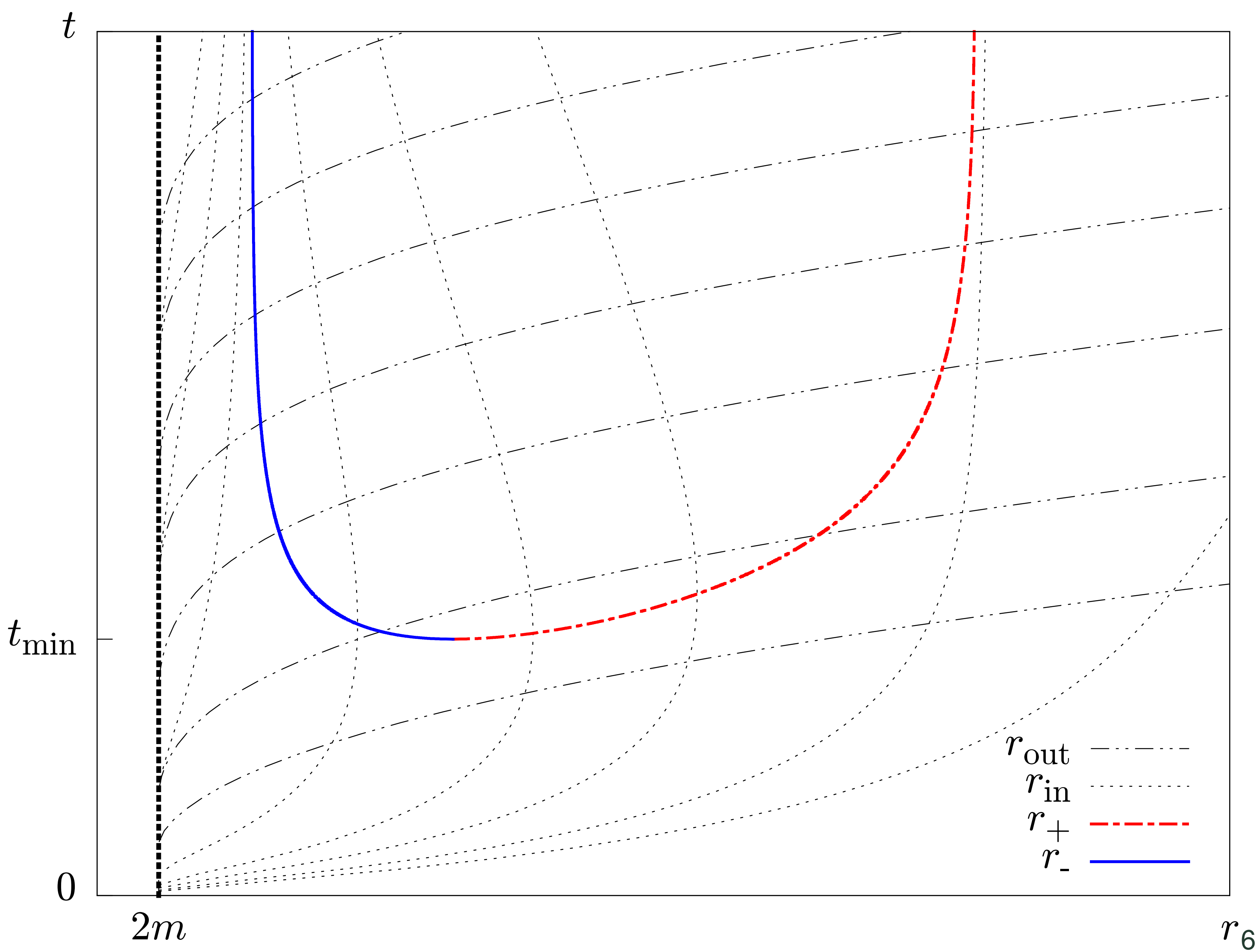
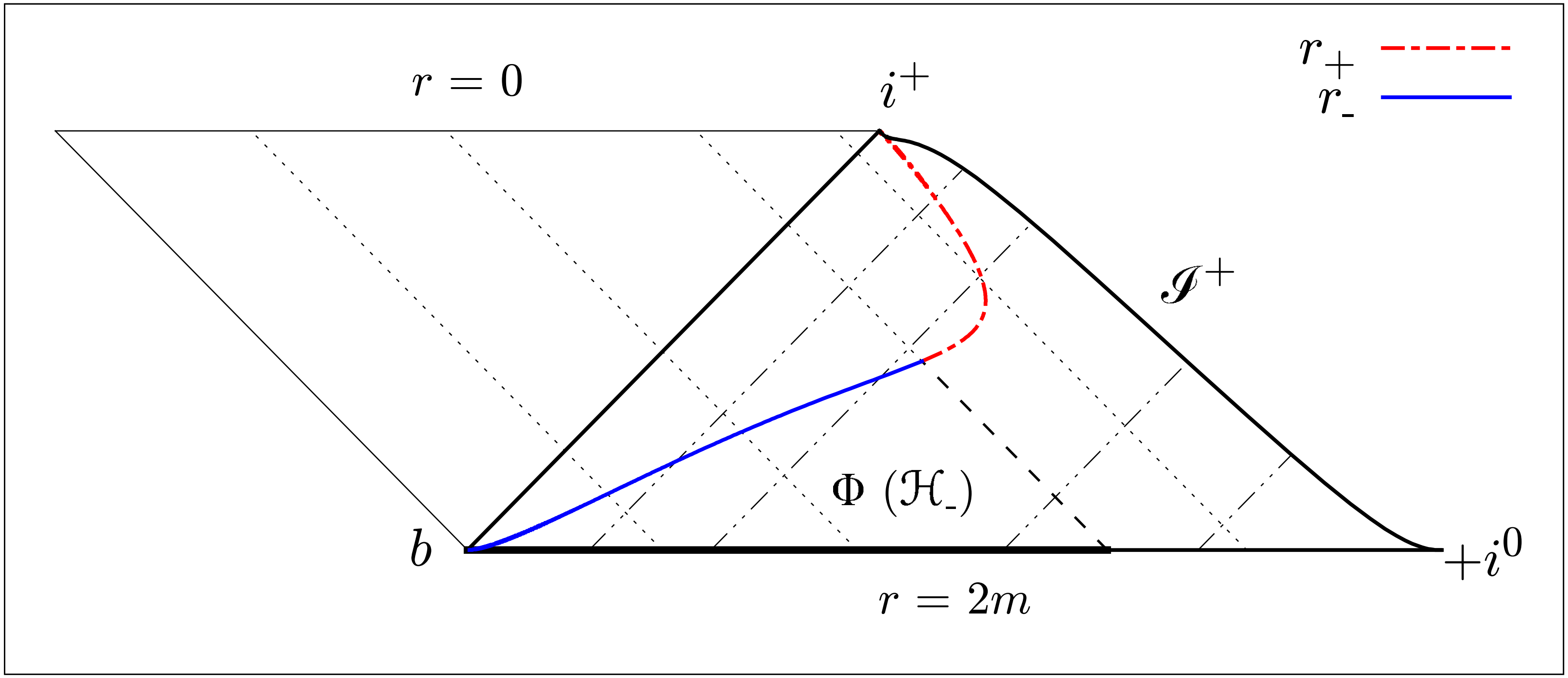
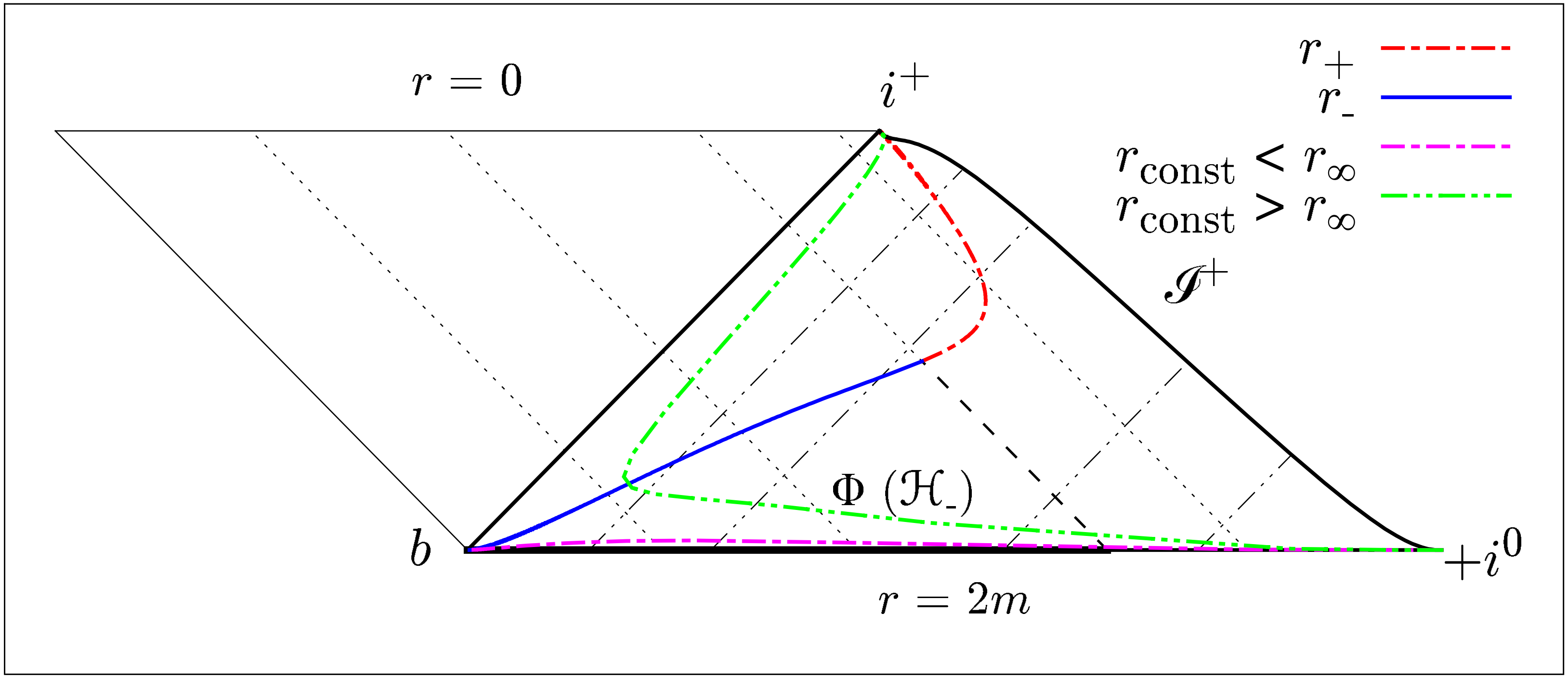


Penrose diagrams



[A. M. da Silva, M. Fontanini, DCG, *PRD* **87** 064030 (2013), 1212.0155]



[A. M. da Silva, M. Fontanini, DCG, *PRD* **87** 064030 (2013), 1212.0155]

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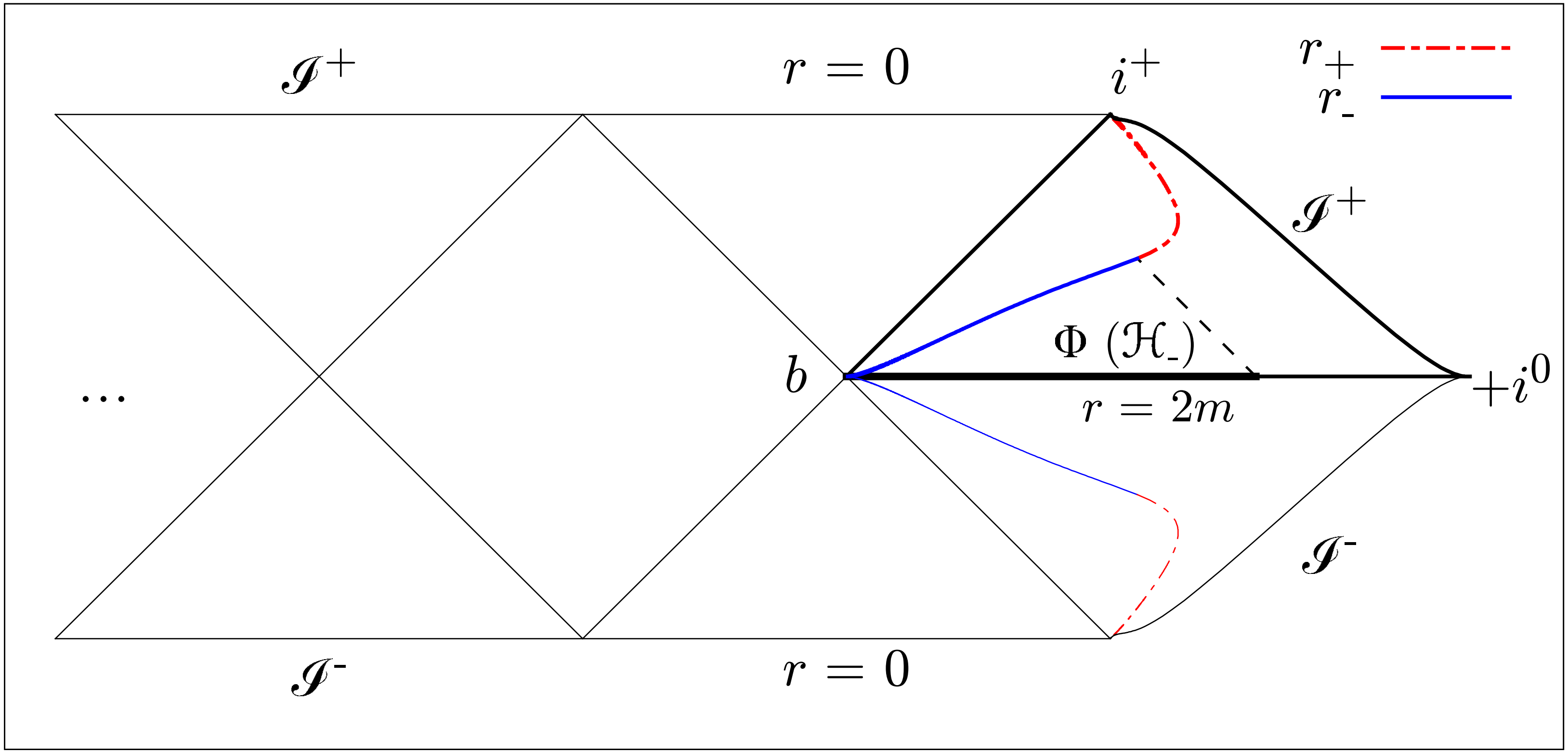

[A. M. da Silva, M. Fontanini, DCG, *PRD* **87** 064030 (2013), 1212.0155]

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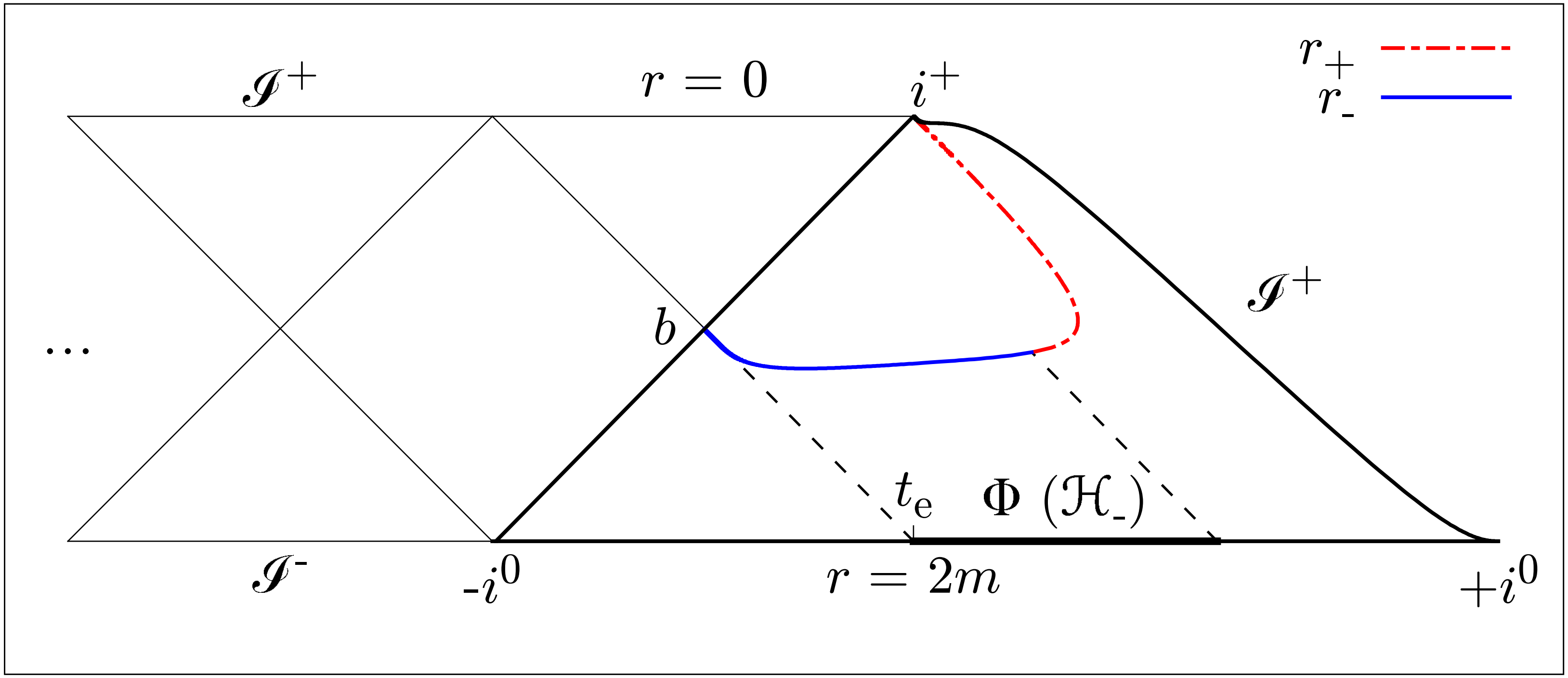


[A. M. da Silva, M. Fontanini, DCG, *PRD* **87** 064030 (2013), 1212.0155]

Horndeski action

- We can also have a field as source for McVittie with $\dot{m} \neq 0$

[N. Afshordi, M. Fontanini, DCG, *PRD* **90** 084012, 1408.5538]

  - Additional terms in the action must look like heat flow
  - Most general scalar action: Horndeski

- First term added to the k-essence action: *kinetic gravity braiding*

[C. Deffayet, O. Pujolàs, I. Sawicki, A. Vikman, *JCAP* (2010) 026, 1008.0048]

$$S_\varphi = \int \mathrm{d}^4x\sqrt{-g}\,[K(X,\varphi) + G(X,\varphi)\Box\varphi]$$

Reduces to McVittie/Cuscuton when $G = 0$ ($\dot{m} = 0$)

Field equations

- Energy-momentum tensor of the KGB term

$$T_{\mu\nu} = \left(K - G_{;\alpha}\varphi^{;\alpha}\right) g_{\mu\nu} + \left(K_{,X} + \Box\varphi G_{,X}\right) \varphi_{;\mu}\varphi_{;\nu} + 2G_{(;\mu}\varphi_{;\nu)}$$

- Solution of the field equations

$$G = g_0(\varphi) \ln X + g_1(\varphi)$$
$$K = f_1(\varphi) + f_2(\varphi)\sqrt{X} + 2X\left[(2 - \ln X)g_0' - 24\pi g_0^2\right]$$

- Connected via disformal transformations to *cuscuton* and *beyond Horndeski* actions

Causal structure

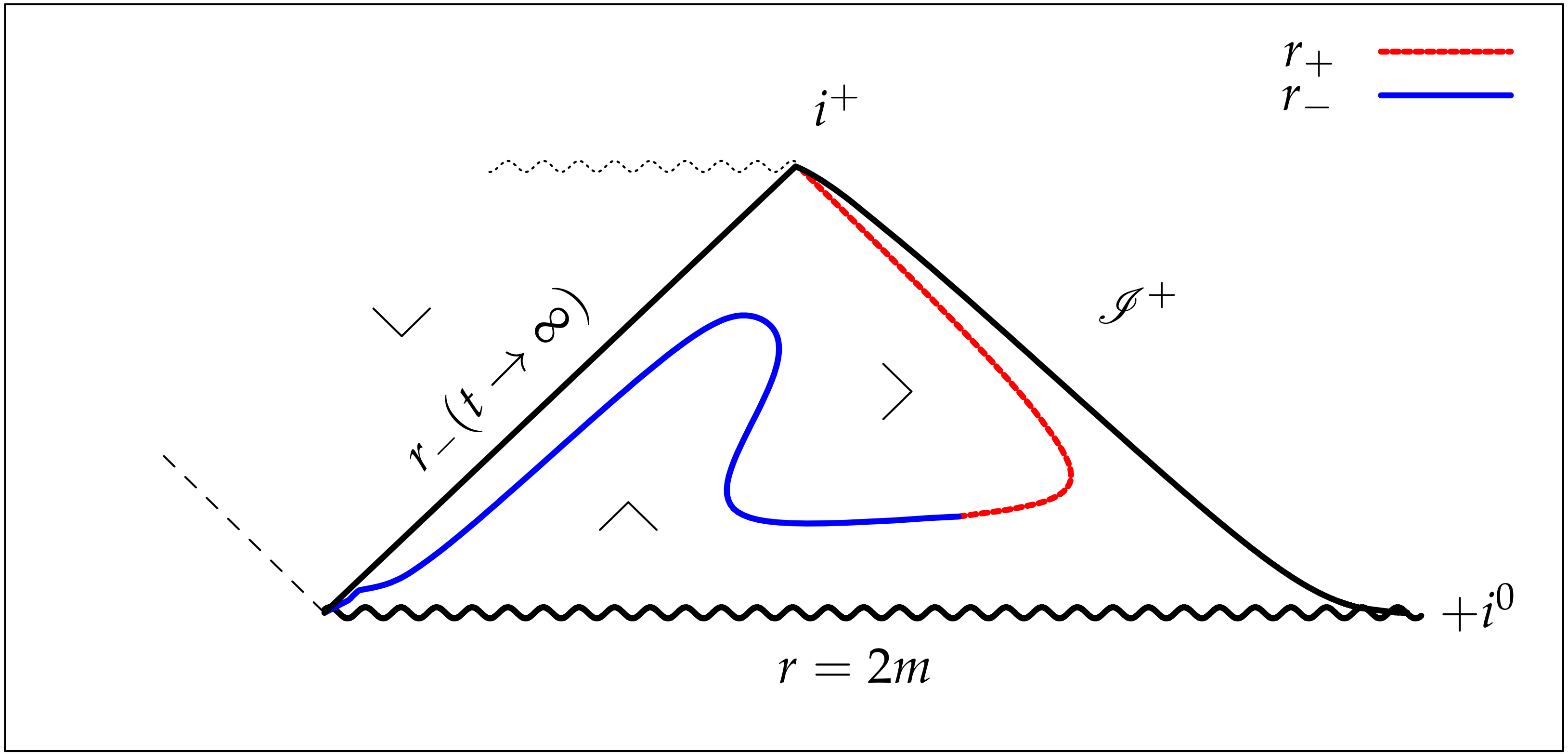
- Radial null geodesics [A. Maciel, DCG, C. Molina, *PRD* **91** 084043 (2015), 1502.01003]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -R_\infty \left[\alpha \frac{r - r_\infty}{r_\infty} - \xi(t)\right] + o(\delta)\,,$$

- Causal structure depends on whether geodesics cross the apparent horizon in the bulk

	$\dot{\xi}(t) \to 0^-$	$\dot{\xi}(t) \to 0^+$
$\left\lvert\int^\infty e^{(\alpha H_0 - \sigma)u}\xi(u)\,\mathrm{d}u\right\rvert < \infty$	black hole and white hole	black hole and white hole
$\left\lvert\int^\infty e^{(\alpha H_0 + \sigma)u}\xi(u)\,\mathrm{d}u\right\rvert \to \infty$	black hole only	white hole only

- Black hole only



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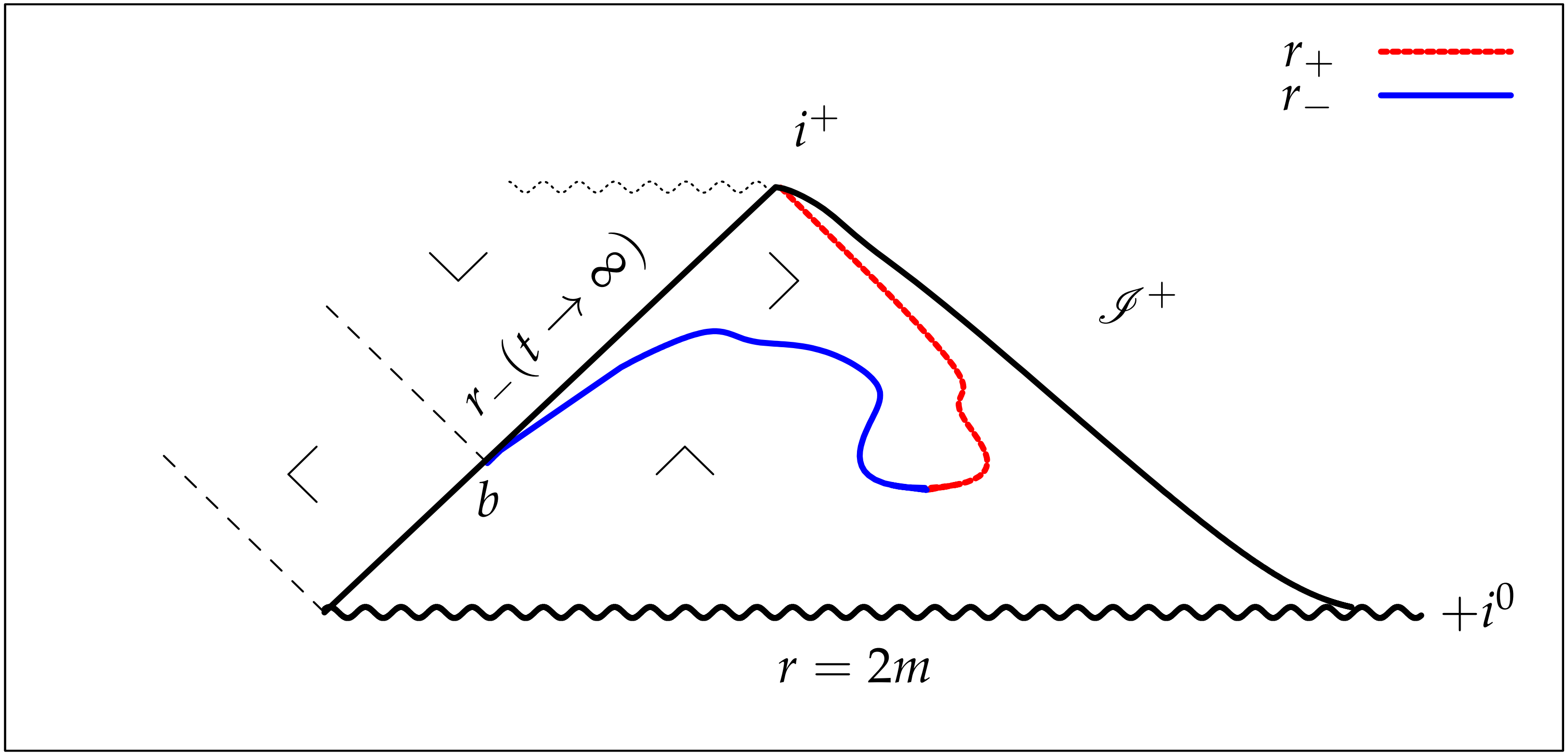
Black hole and white hole



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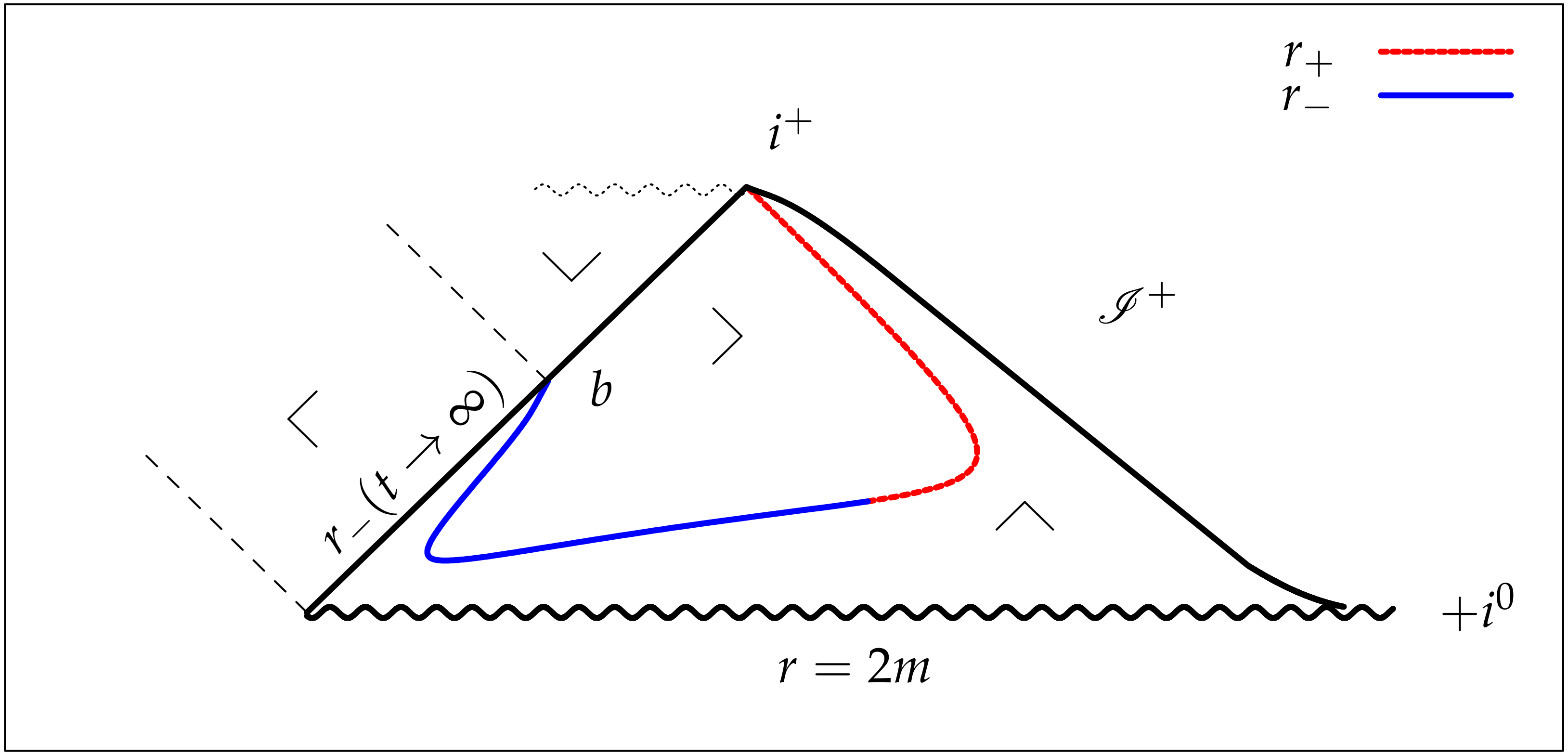

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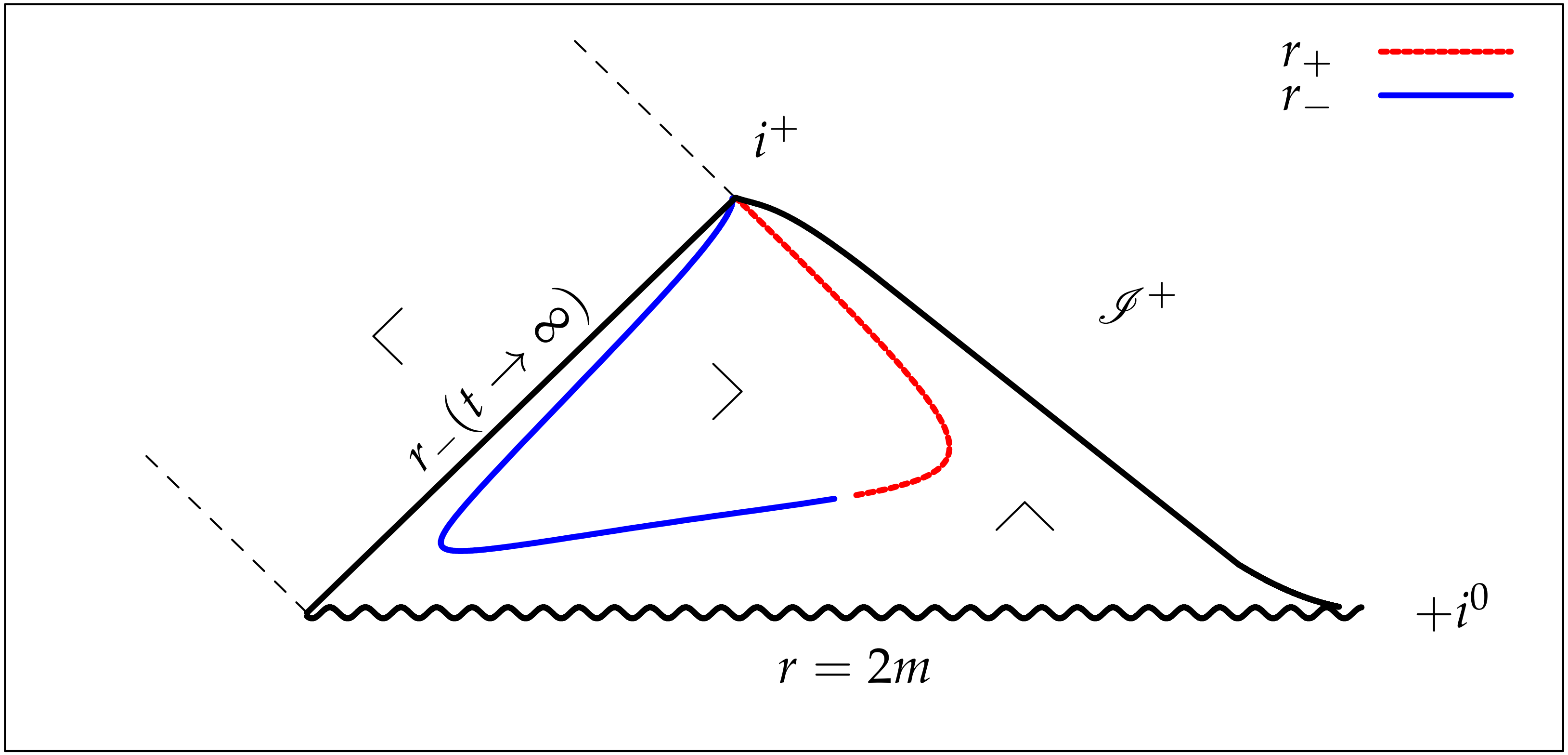
- White hole only



[A. Maciel, DCG, C. Molina, *PRD* **91** 084043 (2015), 1502.01003]

Next steps

- Causal structure of charged and non-flat McVittie metrics
  - Apparent horizons
  - Cauchy horizons
  - Past singularity
- Other shear-free members of the family of solutions
  - Kustaanheimo-Qvist, Stefani class, Weyman class
- Degrees of freedom of the field: is it a higher order cuscuton?
- Stability analysis
- Nature of the past singularity
  - Not present in Shape Dynamics
- Solution to other modifications of General Relativity

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Thank you!