A new basis for LQG

and application to coarse-graining

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Outlook

- Coarse-graining in the canonical framework
- Fusion basis for 2+1 LQG
- Ribbon operators
- Back to the coarse-graining
- Conclusion

Coarse-graining of spin networks

[Livine 13][Rovelli *et al.* 15][Livine, Charles 16][Dittrich, Geiller 16]

- Coarse-graining of spin networks in terms of **density matrices**:
  $\rightarrow$ change of the dual triangulation

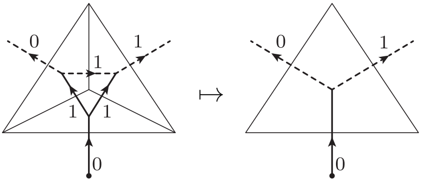
  $\Rightarrow$ Identification of coarser and finer degrees of freedom $\rightarrow \mathcal{H} = \mathcal{H}_{\mathfrak{c}} \otimes \mathcal{H}_{\mathfrak{f}}$

  $\Rightarrow$ Tracing over the finer degrees of freedom

$$\rho^{\mathfrak{c}}\big(\{g^{\mathfrak{c}}\};\{\tilde{g}^{\mathfrak{c}}\}\big) = \int_{\mathcal{G}} \mathbf{d}g^{\mathfrak{f}}\rho^{\mathfrak{f}}\big(\{g^{\mathfrak{f}}\},\{g^{\mathfrak{c}}\};\{g^{\mathfrak{f}}\},\{\tilde{g}^{\mathfrak{c}}\}\big)$$

- Example: Pachner move $3 \rightarrow 1$

$\mathcal{P}_{3\rightarrow 1}$:



- **Violation** of the Gauß constraint
  (curvature induced torsion [Dittrich, Geiller 15])

Graphs on punctured manifolds

[Konig, Kuperberg, Reichardt 10][Hu *et al.* 15][Lan, Wen 15][Dittrich, Geiller 15]

- Spin network basis is **not stable** under coarse-graining

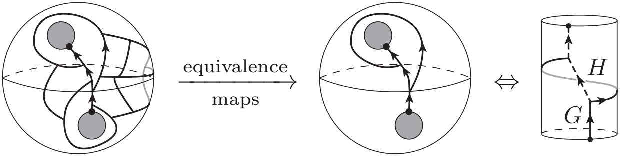
  $\Rightarrow$ Add defects d.o.f $\Rightarrow$ new basis

- Graphs embedded on **punctured surfaces**

  $\rightarrow$ open edges = torsion d.o.f

  $\rightarrow$ non-contractible loops = curvature d.o.f

- Example: 2-punctured sphere Σ_2



$= \psi^{\Sigma_2}_{G,H}$

- Constraints violations = **excitations** (w.r.t BF vacuum)

Gluing of manifolds ⇒ Drinfeld double

[Hu *et al.* 15][Lan, Wen 15][Dittrich, Geiller 15]

- **Gluing** of two 2-punctured spheres

= identification + gauge averaging + imposition of flatness =

$$\boxed{\psi^{\Sigma_2}_{G',H'} \star \psi^{\Sigma_2}_{G,H} = \delta((G')^{-1}H'G', H)\psi^{\Sigma_2}_{G'G,H'}}$$

⇒ Reproduces the **Drinfeld double** multiplication

⇒ $\psi^{\Sigma_2}_{G,H} \leftrightarrow$ Drinfeld double element

- Decomposition into irreducible representations
  [Koornwinder *el al.* 98-99][Dijkgraaf *et al.* 90][Buerschaper *el al.* 09,13]

$$\psi^{\Sigma_2}_{G,H} = \sum_\rho \sum_{I',I} \psi^{\Sigma_2}_{\rho,I'I} \sqrt{d_\rho} D^\rho_{I',I}([G,H])$$

with $\rho = C, R$ (C: conjugacy class, R: representation of the stabilizer)

- $\psi^{\Sigma_2}_{\rho,I'I} \leftrightarrow$ point-particle with mass C and spin R [Noui 06][Noui, Perez 09]

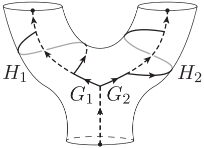
Fusion basis

- $\{\psi^{\Sigma_2}_{\rho,I',I}\}$ forms an **orthonormal** and **complete** set of states for Σ_2

  $\Rightarrow$ Fusion basis states for Σ_2

- 3-punctured sphere:

$$\psi^{\Sigma_3}_{\{G,H\}} =$$

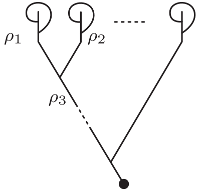


- Fusion basis states obtained by imposing the **fusion rules** between the representations:

  [Koornwinder *el al.* 98-99][Dijkgraaf *et al.* 90]

$$\psi^{\Sigma_3}_{\{\rho,I',I\}} = C^{\rho_1\rho_2\rho_3}_{I_1 I_2 I'_3}\,\psi^{\Sigma_2}_{\rho_1,I'_1,I_1} \otimes \psi^{\Sigma_2}_{\rho_2,I'_2,I_2}$$

- Basis for n-punctured sphere is deduced by **pants decomposition**



Ribbon operators

[Hu *et al.* 15][Lan, Wen 15][Dittrich, Geiller 16][CD, Dittrich 16]

- Configuration space: holonomies that describe **locally flat connections**
- Two kinds of operators:

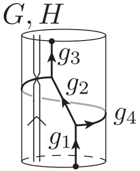
  → Wilson operators $W_\gamma[G]$

  → Translation operators $T_k[H]$ (exponentiated fluxes)

- Kitaev **ribbon operators** on Σ_2: $\mathcal{R}[G,H] = W_{321}[G] \circ T_{4,3}[H]$
  [Kitaev 06]

  ⇒ Action on the vacuum state (BF):

$(\mathcal{R}[G,H]\psi_0^{\Sigma_2}) =$



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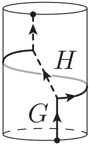


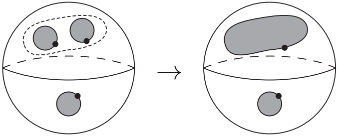
$= \psi_{G,H}^{\Sigma_2}$

- Ribbon operators **generate excitations**

Coarse-graining w/ the fusion basis

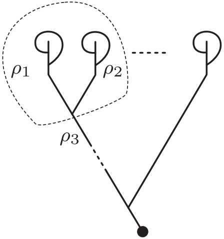
- Coarse-graining of fusion basis states in terms of **density matrices**:
  $\rightarrow$ merging of defects

  $\Rightarrow$ **Fusion** of the corresponding irreducible representations



  $\Rightarrow$ Definition of the fusion basis state on Σ_3

- **Stability** of the states under coarse-graining
- Control the behavior of the states at all ‘scales’ at once (MERA style [Vidal 10])



Conclusion

- **Summary:**

  ⇒ New basis with emphasis on the excitations

  ⇒ Kitaev's ribbon operators generate excitations

  ⇒ Stability under coarse-graining

- **Future directions**:

  ⇒ Generalization of the fusion basis to the 4D case (lifting of the ribbon operators [CD, Dittrich 16])

  ⇒ Homogeneous curvature phase