Effects of anisotropic stresses in cyclic universe models

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Why include anisotropic stress?

- Interaction rates of particles

$$\Gamma = \sigma n v \sim g\alpha^2 T$$

- To remain in equilibrium, $\Gamma > H$
- Before isotropisation, anisotropic universe expands faster
- Harder to maintain equilibrium

Decoupled collisionless particles free stream and exert anisotropic stresses.

Kasner universe in the presence of anisotropic stresses

- The metric

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)dy^2 - c^2(t)dz^2$$

- Friedmann equation

$$3H^2 = \sigma^2 + \rho_{matter},$$

- The shear evolves as,

$$\dot{\sigma}_{\alpha\beta} + 3H\sigma_{\alpha\beta} = \underbrace{\boxed{\mu\mathcal{P}_{\alpha\beta}}}_{\text{anisotropic stress}}$$

The setup

- The generalised metric

$$ds^2 = -dt^2 + h_{ab} d\omega^a d\omega^b$$

- Having isotropic ultra stiff ghost field of density ρ with equation of state $p = (\gamma - 1)\rho$
- and anisotropic pressure ultra stiff field of density μ with equation of state $p_i = (\gamma_i - 1)\mu$
- with $\gamma_\star = (\gamma_1 + \gamma_2 + \gamma_3)/3$ and $\gamma_\star > \gamma$

Phase plane system

- We introduce

$$\sigma_+ \equiv \frac{1}{2}(\sigma_{33} - \sigma_{22}),$$
$$\sigma_- \equiv \frac{1}{2\sqrt{3}}(\sigma_{22} - \sigma_{33}).$$

- Write EFE in terms of expansion normalised variables

$$\Omega \equiv \frac{-\rho}{3H^2}, \quad Z \equiv \frac{\mu}{3H^2}, \quad \Sigma^2 \equiv \frac{\sigma^2}{3H^2}, \quad K \equiv -\frac{{}^{(3)}R}{6H^2}.$$

- Linearise expansion normalised EFE around the FL point

$$\Sigma_+ = 0,\ \Sigma_- = 0,\ N_1 = 0,\ N_2 = 0,\ N_3 = 0,\ \Omega = 0,\ Z = 0$$

Stability analysis

We find the following eigenvalues

- $\frac{3}{2}(2-\gamma)$ of multiplicity 2
- $\frac{3\gamma-2}{2}$ of multiplicity 3
- $3(\gamma-\gamma_{\star})$ of multiplicity 1
- Using the condition $\gamma_{\star} > \gamma > 2$, FL equilibrium point stability cannot be determined

- We have as before ρ and μ for isotropic and anisotropic pressure fields which follow the same equations of state as before.
- the 3 scale factors in the 3 directions are expressed as,

$$a(t) \equiv \mathrm{e}^{\alpha(t)},\ b(t) \equiv \mathrm{e}^{\beta(t)},\ c(t) \equiv \mathrm{e}^{\delta(t)}$$

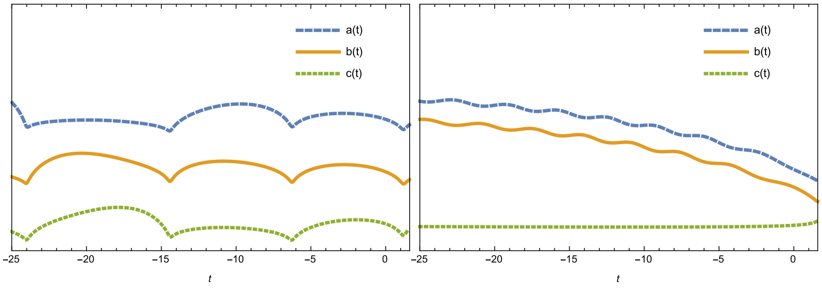
- Define

$$\begin{aligned} x &\equiv \alpha'(t) - \beta'(t), \\ y &\equiv \alpha'(t) - \delta'(t), \\ H &\equiv \frac{1}{3}\left(\alpha'(t) + \beta'(t) + \delta'(t)\right). \end{aligned}$$

- Choose initial conditions satisfying the Friedmann constraint for the variable system

$$\{x, y, H, \alpha, \beta, \delta, \rho, \mu\}$$

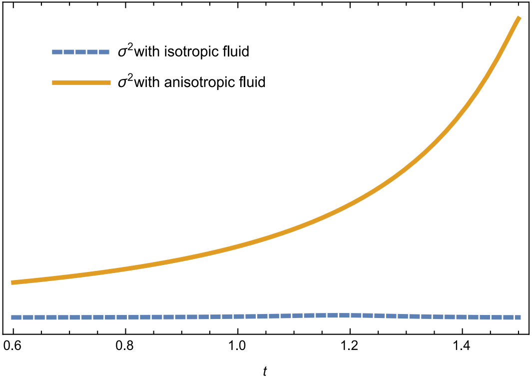
Scale factor evolution



- The scale factors with just an isotropic pressure ghost field bounce and start to expand.
- The scale factors with the anisotropic pressure field included seem to contract towards a singularity.

Evolution of the shear

If we look at the evolution of the shear, we find,



And so we conclude...

- In the initially contracting Bianchi Class A models, FL is no longer an attractor in the asymptotic past
- In the Bianchi IX equations, including an ultra stiff anisotropic pressure field causes the scale factors to contract towards a collapse near the singularity.
  They bounce with only an isotropic ghost field present.
- The shear remains small and nearly constant in the isotropic case but increases without bound when the anisotropic pressure field is included.

Conclusion

Including anisotropic stress does not always result in isotropisation, even if the field is ultra-stiff on average.

Thank you

References

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