Fast and accurate evaluation of black hole Green's functions using surrogate models

Chad Galley, California Institute of Technology

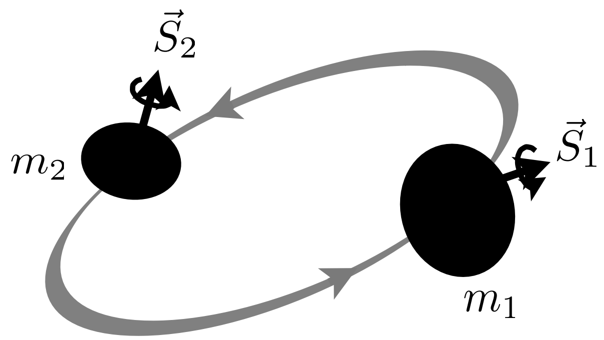
with Barry Wardell (University College Dublin)

GR21, New York City, July 11, 2016

Caltech

Compact binaries

Numerical relativity has proven success approximating compact binary evolutions



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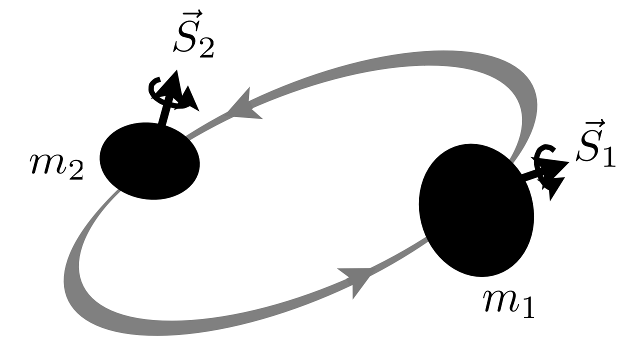

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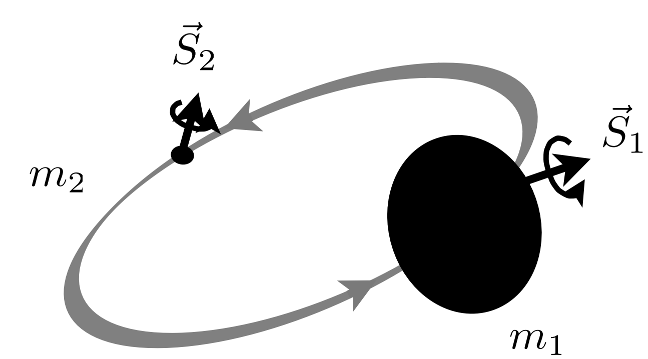

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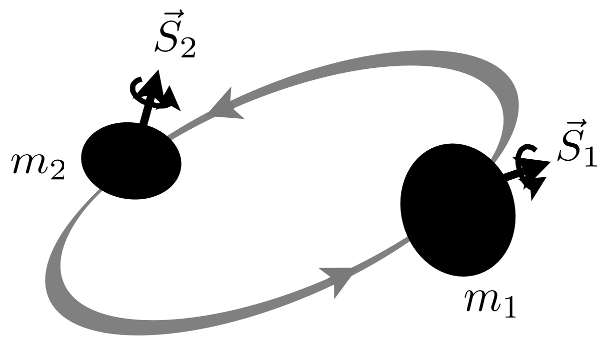

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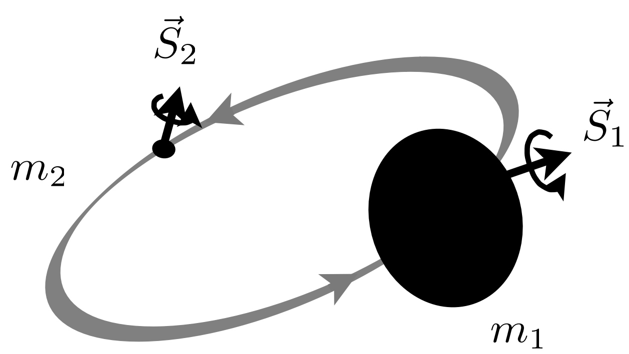

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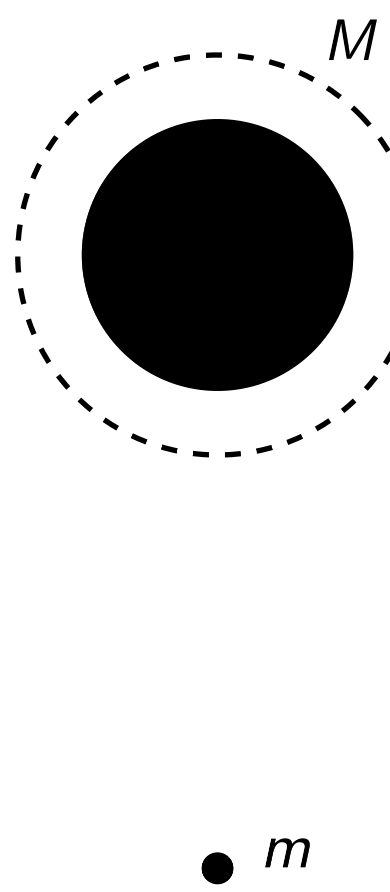

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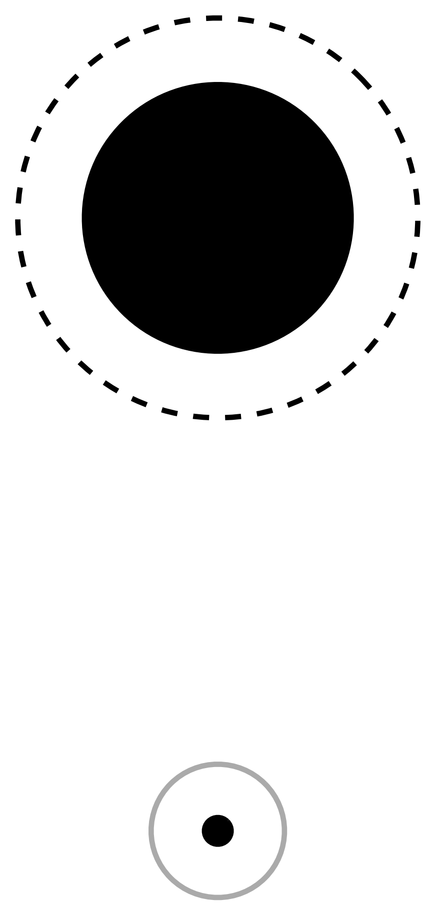

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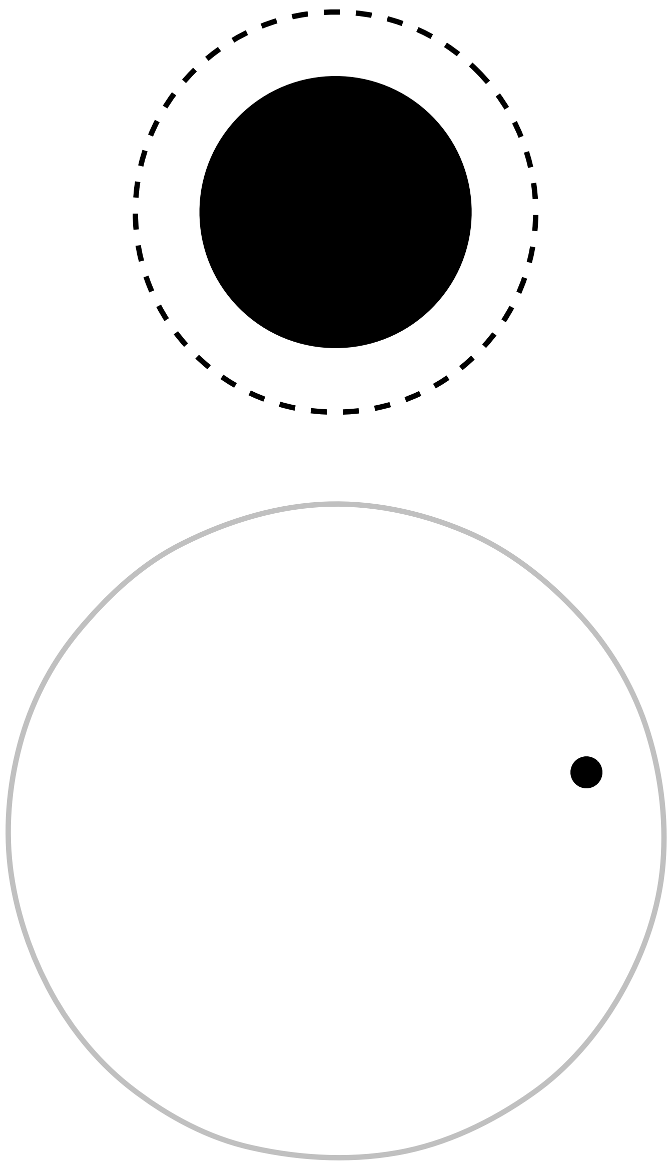

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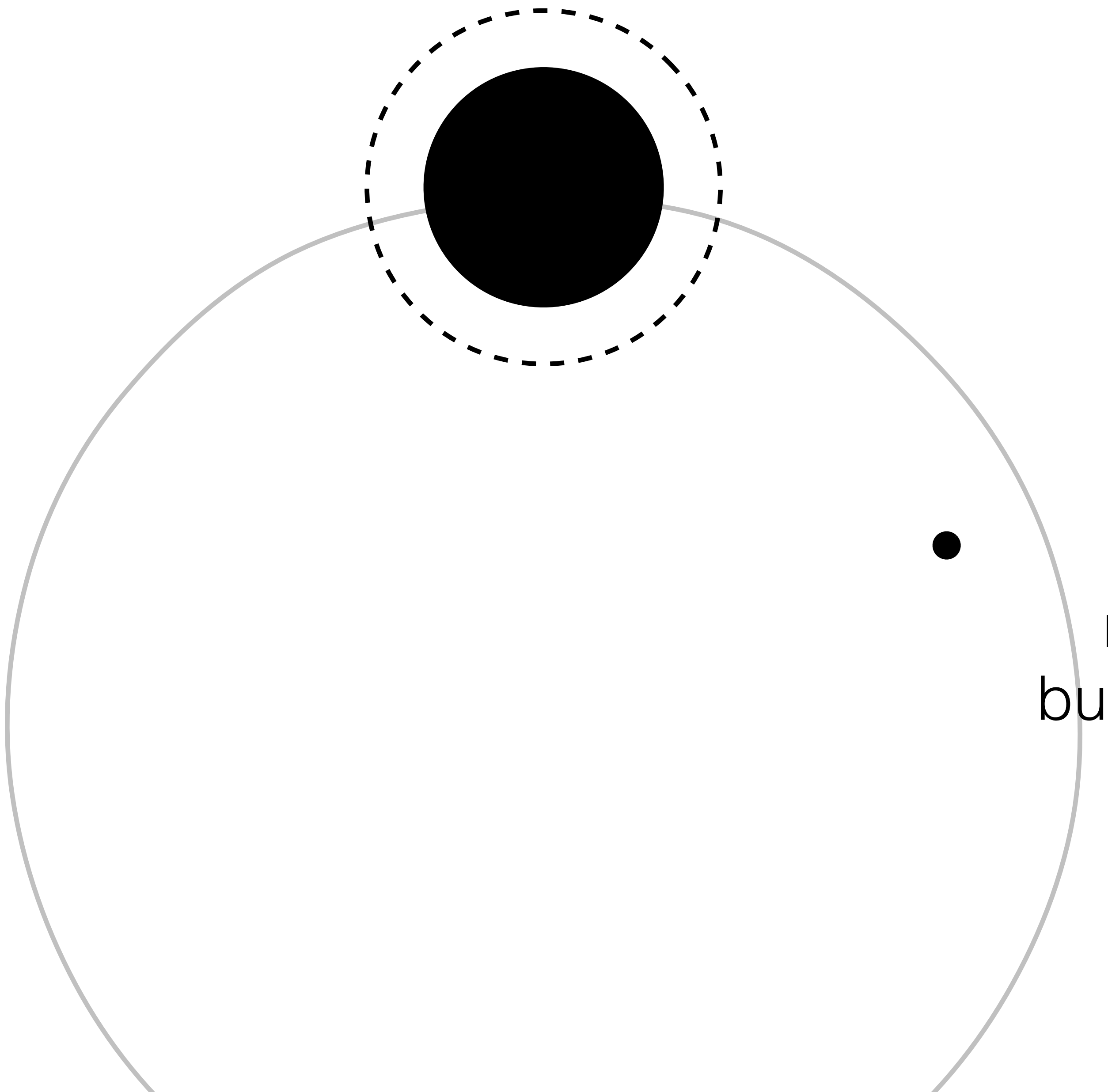

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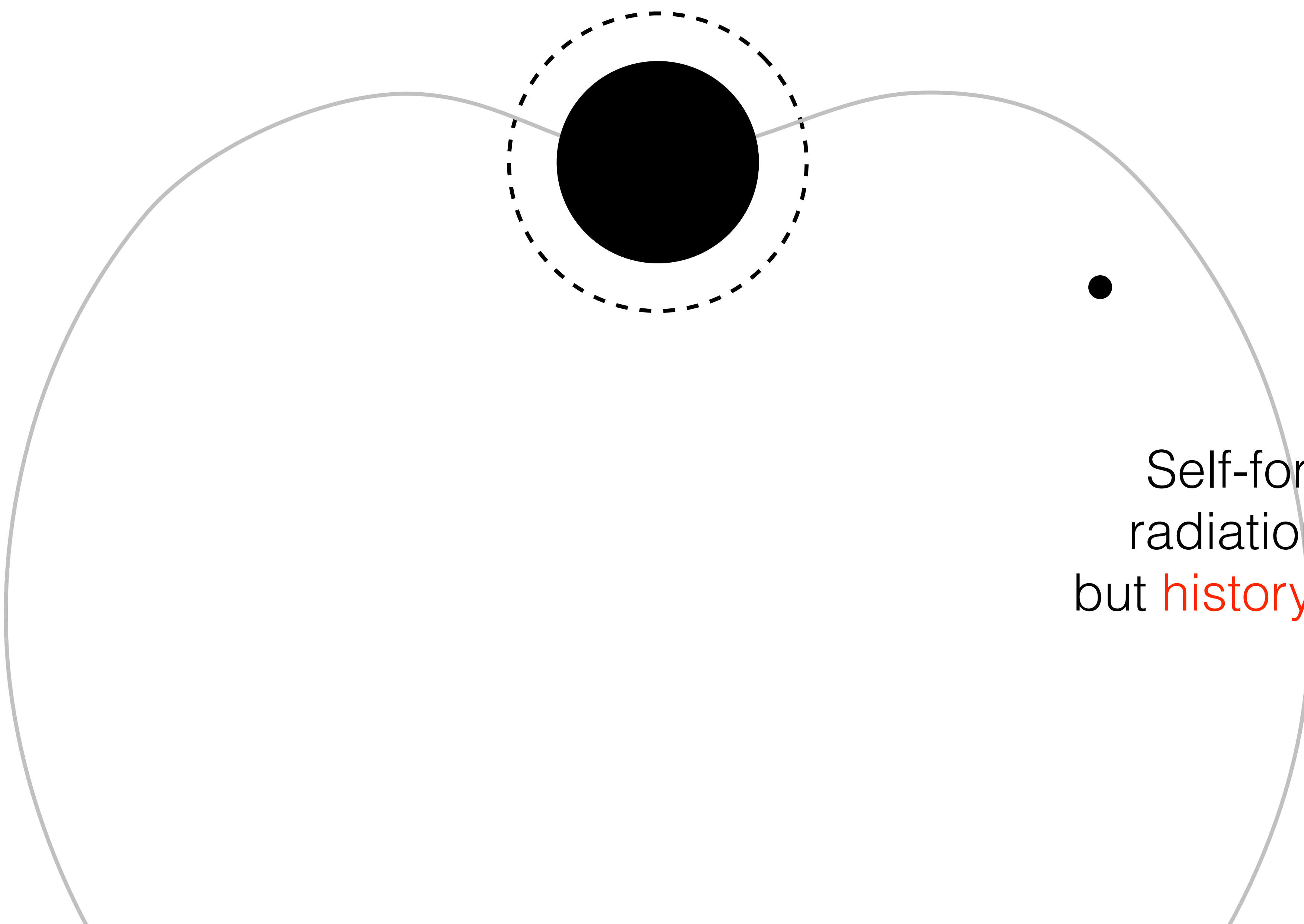

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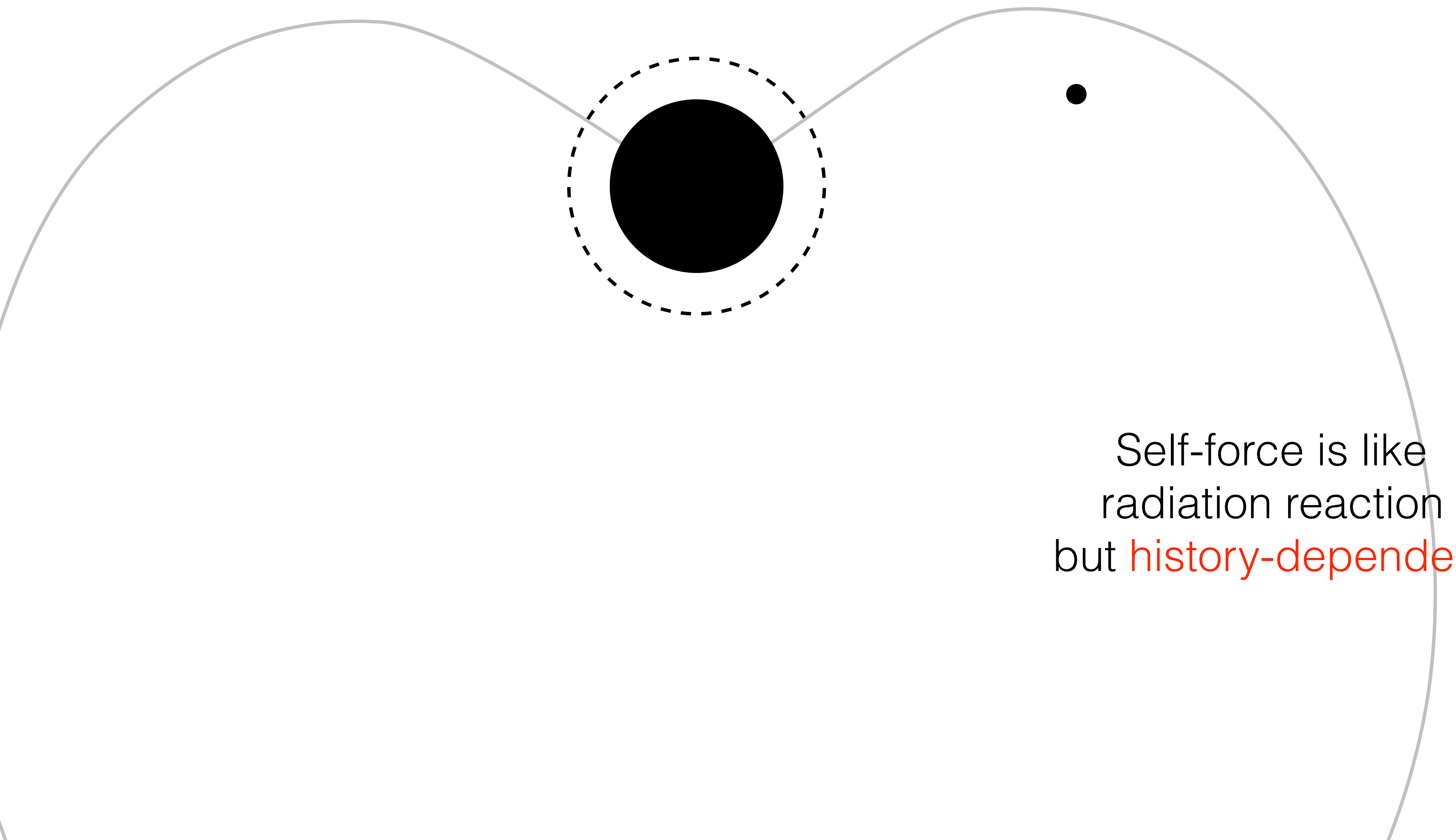

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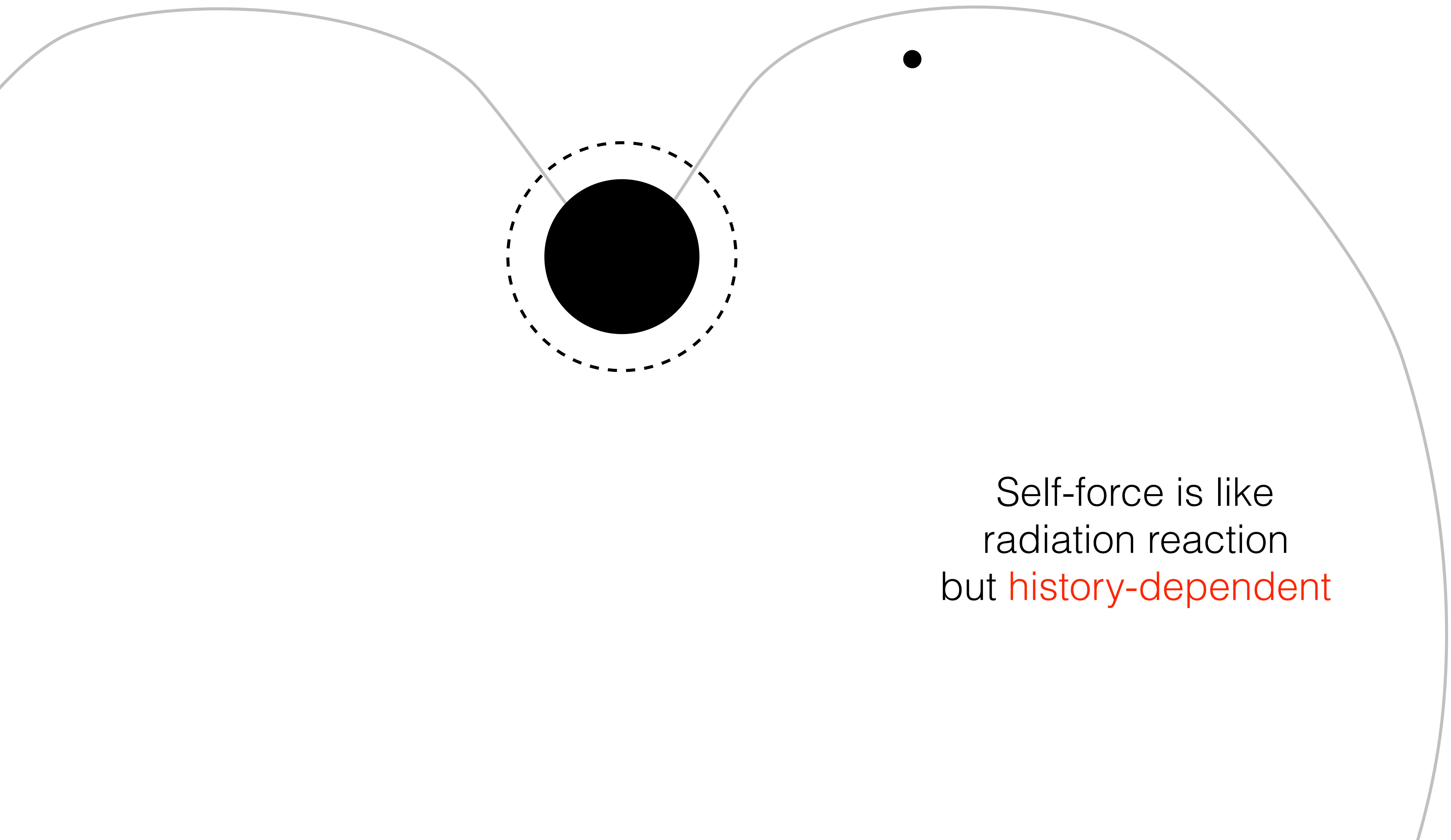

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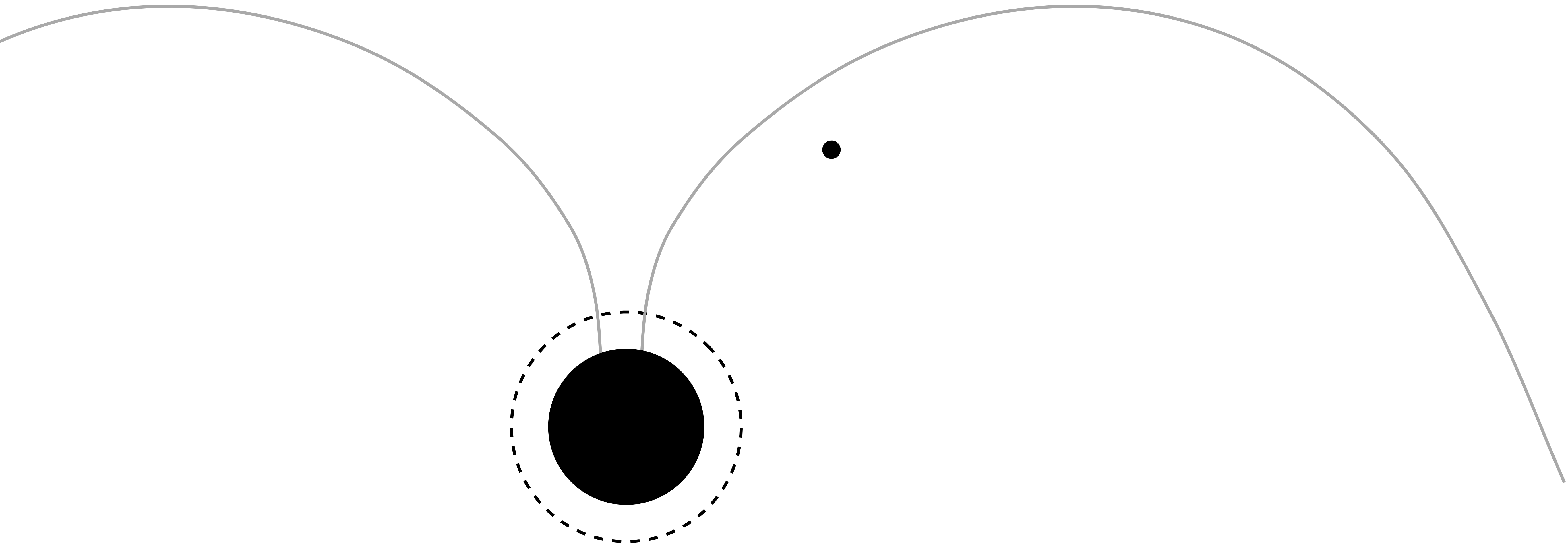

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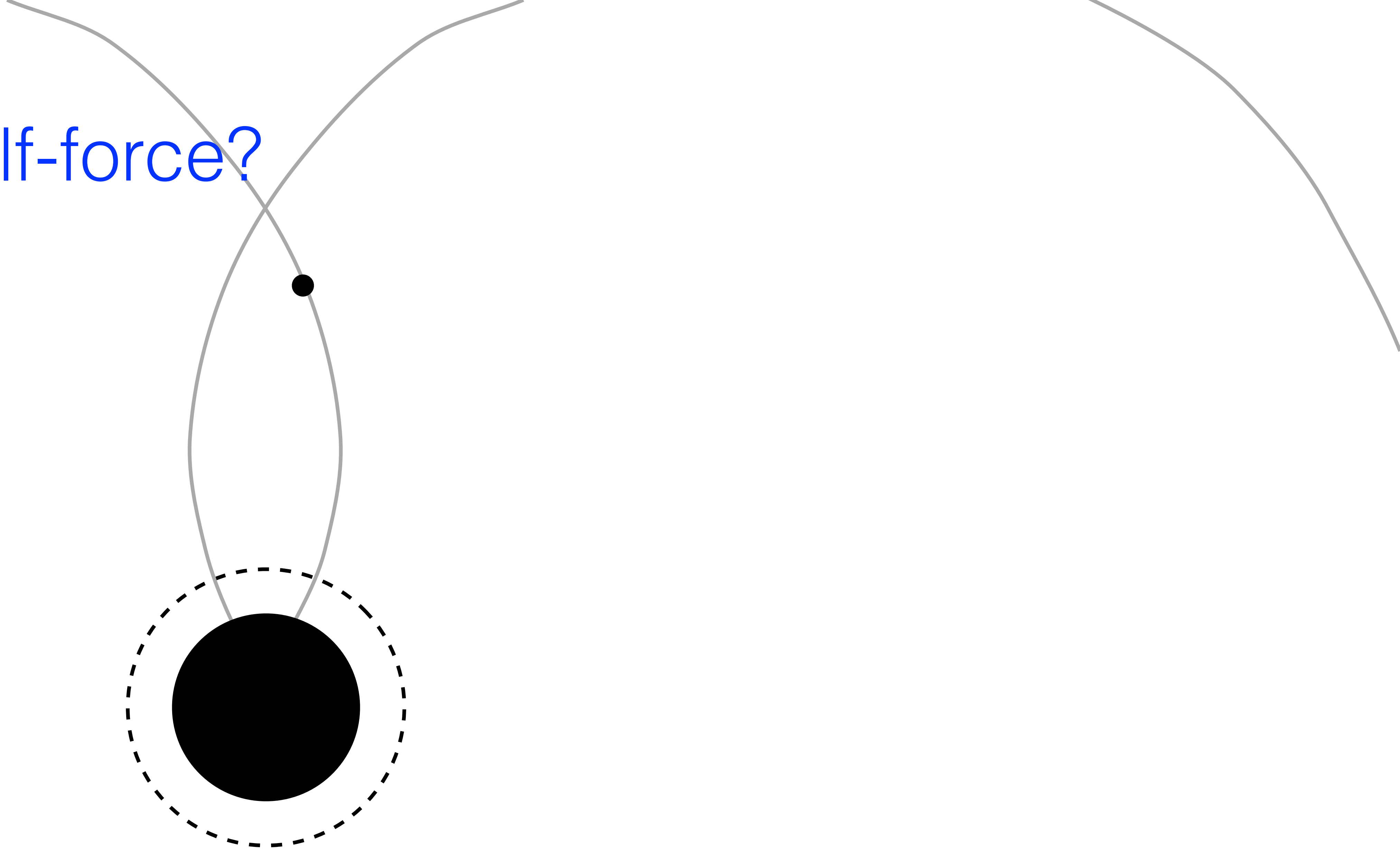

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Self-force

Gravitational self-force is described by the MiSaTaQuWa equation

Mino, Sasaki, Tanaka (96); Quinn, Wald (96)

$$a^\mu = \frac{m}{2} P^{\mu\nu\alpha\beta} \lim_{\epsilon \to 0^+} \int_{-\infty}^{\tau-\epsilon} d\tau' \, \nabla_\nu G^{\mathrm{ret}}_{\alpha\beta\gamma'\delta'}(z^\mu(\tau), z^{\mu'}(\tau')) u^{\gamma'} u^{\delta'}$$

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Scalar self-force is described by the <u>Quinn equation</u>

Quinn (00)

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The Green's function is central to understanding and describing self-force effects

$$\Box_x G(x, x') = \frac{\delta^4(x - x')}{\sqrt{-g(x)}} \quad \Longrightarrow \quad \phi(x) = \int d^4x' \, \sqrt{-g(x')} G(x, x') J(x')$$

- x is field point; x' is source/base point; G is a biscalar/bitensor

Green's functions

Green's functions

What are the advantages of using Green's functions?

- Compute only once for every source
- Nearly all physical quantities of interest are calculated via convolution integrals
- Arbitrary motion for self-force *Wardell, CRG et al (14)*
- Geometric interpretation *Zenginoglu & CRG (12), Wardell, CRG et al (14)*
- Higher-order self-force
- Self-consistent (higher-order) self-forced evolution
- Self-consistent inspiral waveforms
- Arguably straightforward to implement once known

(show movie)

Credit: B. Wardell

Numerical Green's functions

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Numerical Green's functions are globally valid approximations but utilizing analytic approximations at early and late times is extremely helpful for self-force calculations

- Quasi-local expansions *Ottewill & Wardell (08); Wardell's thesis*
- Pade approximants *Casals et al (09)*
- Method of matched expansions *Anderson & Wiseman (05); Casals et al (13)*

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When these analytical approximations (e.g., in Schwarzschild) are available we use numerical Green's functions for intermediate times

What are the disadvantages of using Green's functions?

- Analytically, very difficult to calculate in curved spacetimes
  Casals, Nolan (16)
- Numerically, computationally expensive
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Goal:

Find a way for Green's functions to be efficient and accurate to use for practical self-force and related computations.

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To quickly predict accurate solutions to the Green's function wave equation that are otherwise too slow and too large for practical use

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- *Empirical interpolation (compression in time)*
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The result

An accurate *surrogate model* to generate new Green's function data on demand

Surrogate models for gravitational waveforms have been built successfully for:

- Non-spinning Effective One-Body (*EOBNRv2*)
  Field, CRG, et al PRX (14)
- Spin-aligned Effective One-Body (*SEOBNRv2*)
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- Non-spinning Numerical Relativity (*SpEC*)
  Blackman, Field, CRG et al PRL (15)
- 4d precession, Numerical Relativity (*SpEC*) *(see J. Blackman's talk)*
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However, some steps for building a Green's function surrogate are necessarily different than for waveforms

- Provides one with dynamics, field content, and waveforms
- Source and field points are time-dependent for worldline convolutions

Surrogate building: Initial stuff

see Wardell, CRG et al (14) for details of numerical implementation

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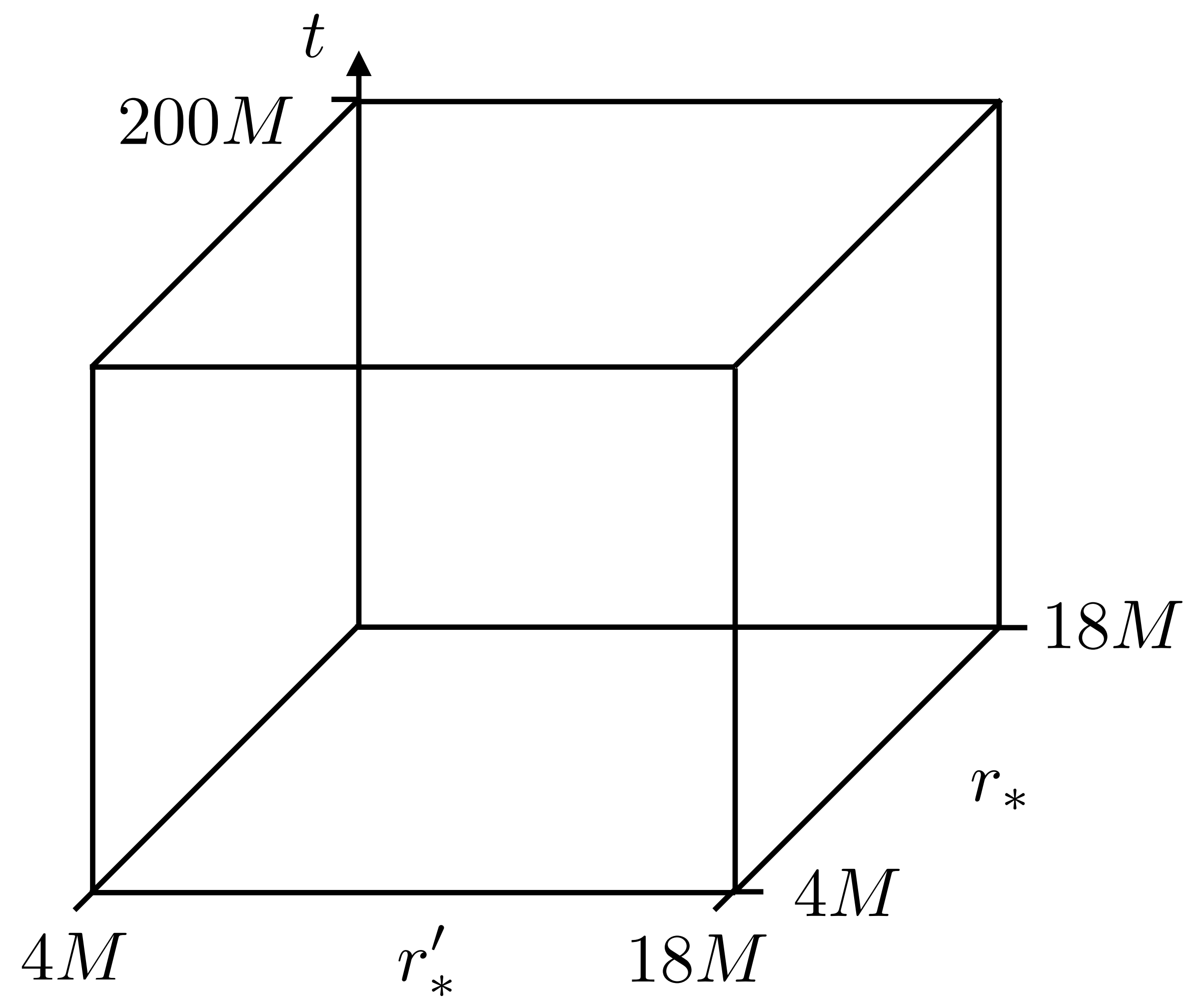
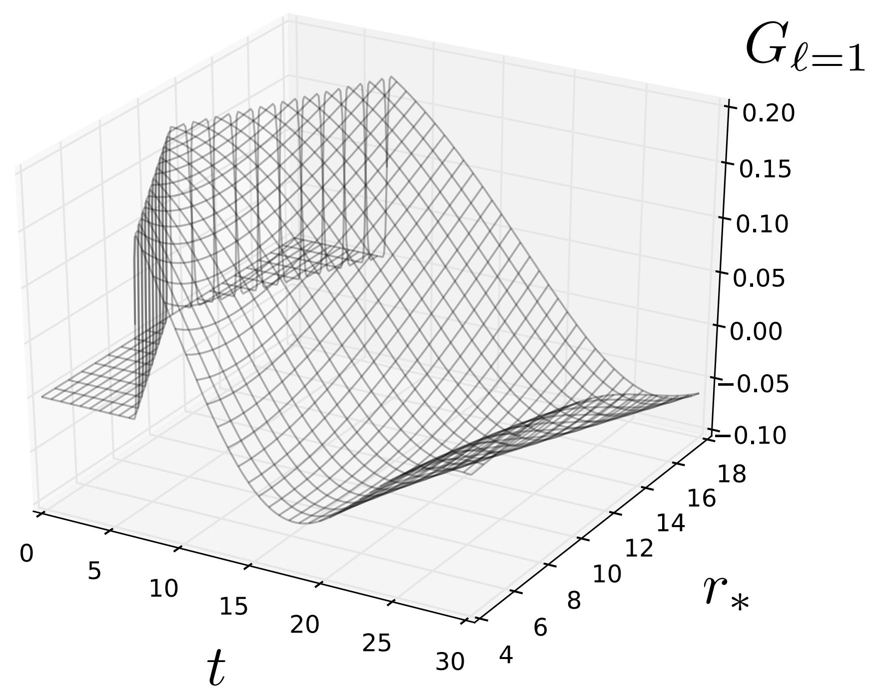


- Reduce known features by analytically time-shifting each series by light travel time from source point to field point, $t \to t - |r_* - r'_*|$



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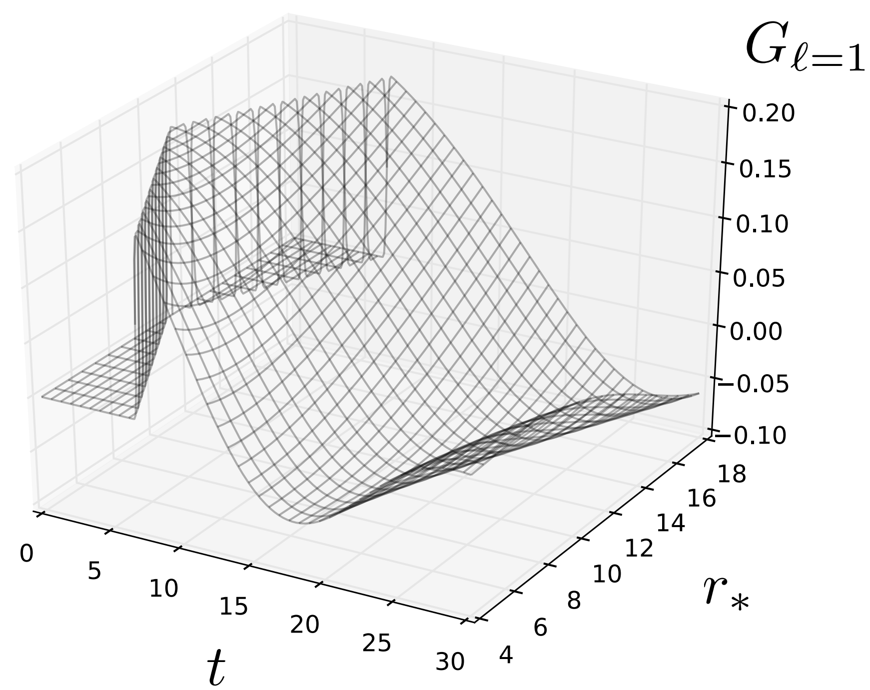

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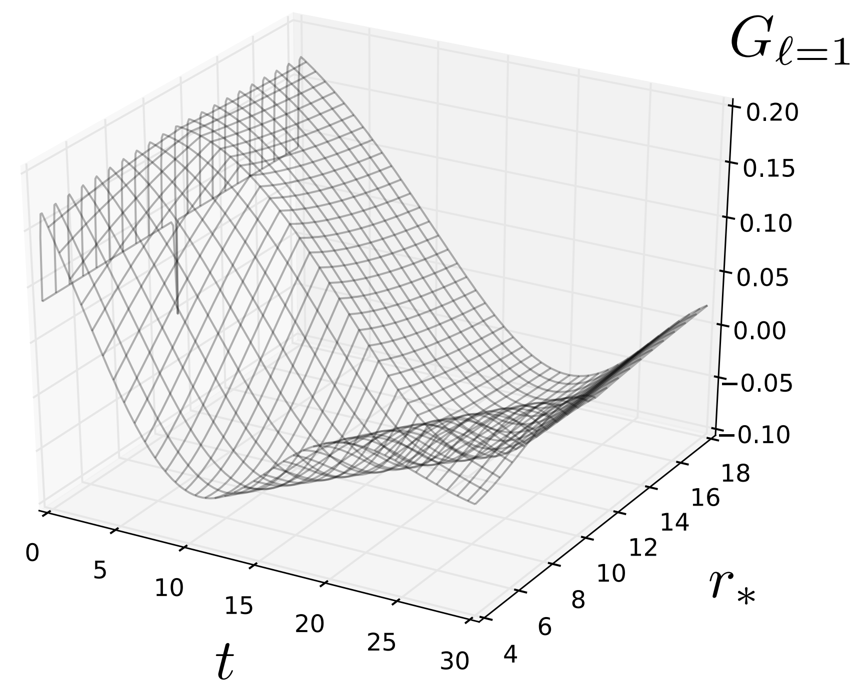

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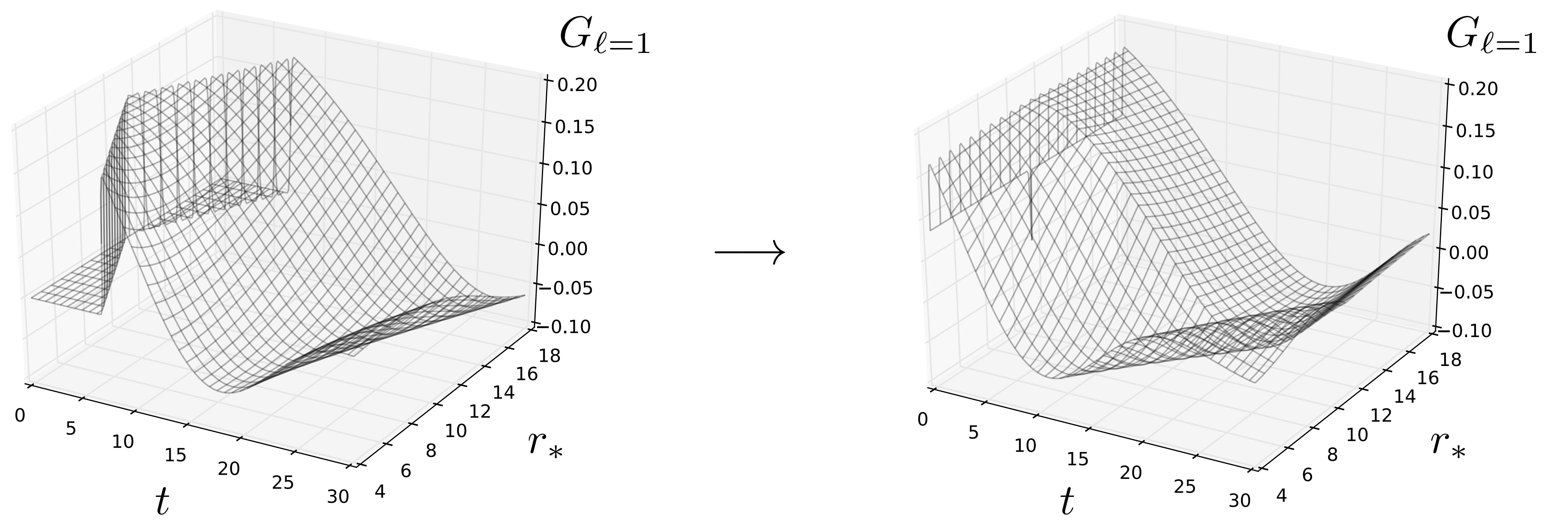


- In addition, to reduce high-frequency noise we introduce a smoothing factor *Wardell, CRG et al (14)*

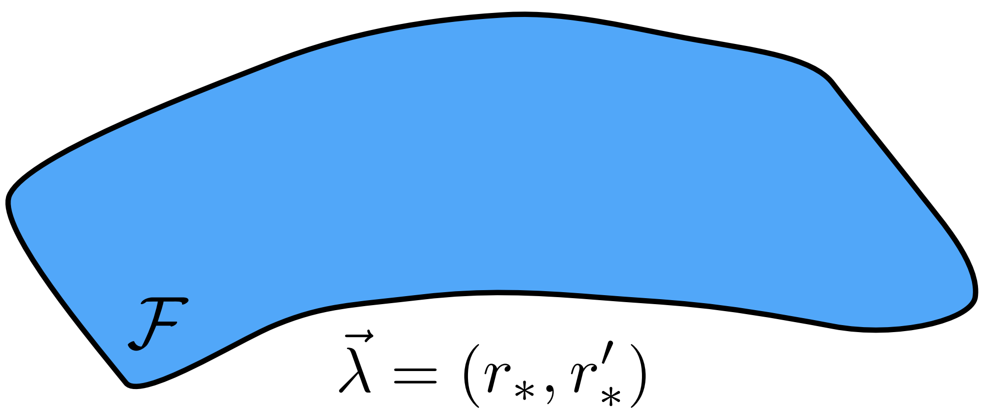
$$G(x^\alpha, x'^\alpha) \approx \frac{1}{rr'} \sum_{\ell=0}^{\ell_{\max}} P_\ell(\cos\theta)(2\ell+1) e^{-\ell^2/2\ell_{\rm cut}^2} G_\ell(t-t'; r_*, r'_*)$$

$\ell_{\max} = 100$ $\qquad\qquad$ $\ell_{\rm cut} = \ell_{\max}/5$

1) Reduced basis via greedy algorithm

Can find a linear approximation space that is nearly optimal

Set of functions $\mathcal{F}$



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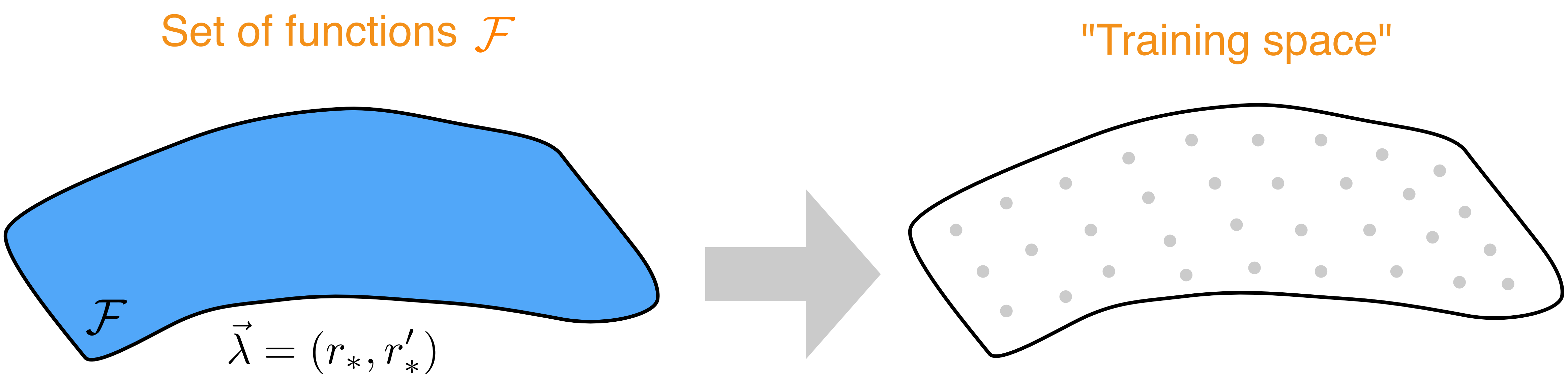

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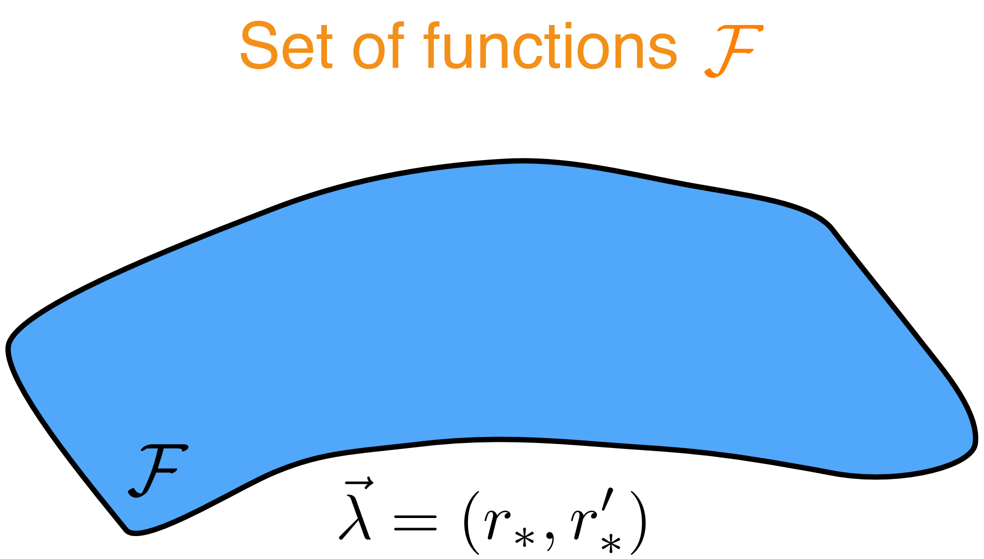

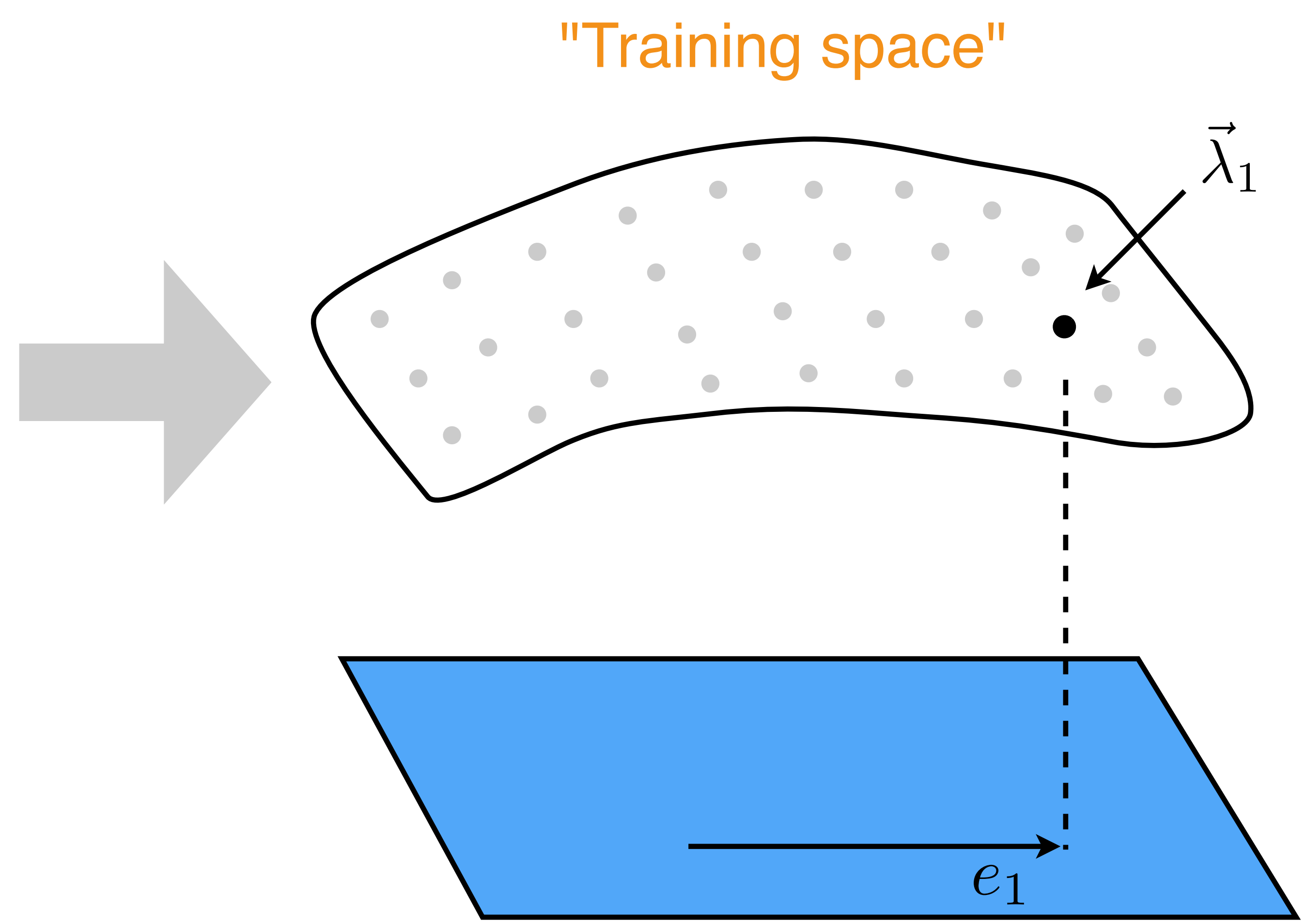

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$$e_1 = G_\ell(t; \vec{\lambda}_1), \; C_1 = \{e_1\}$$

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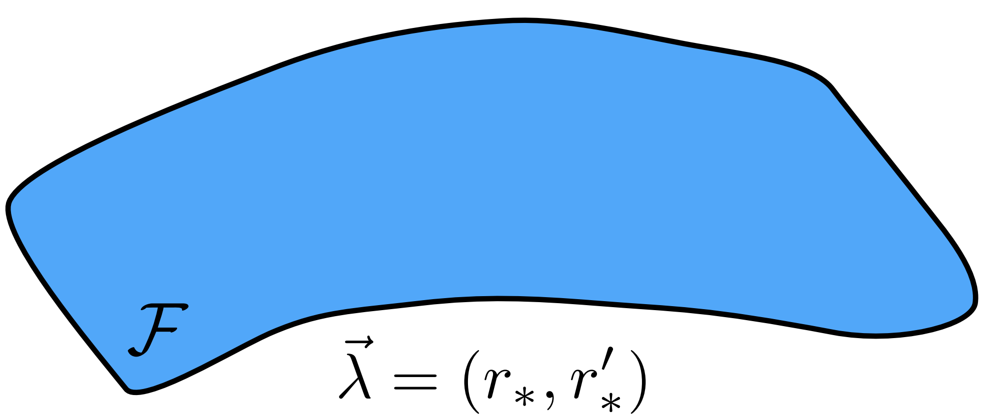
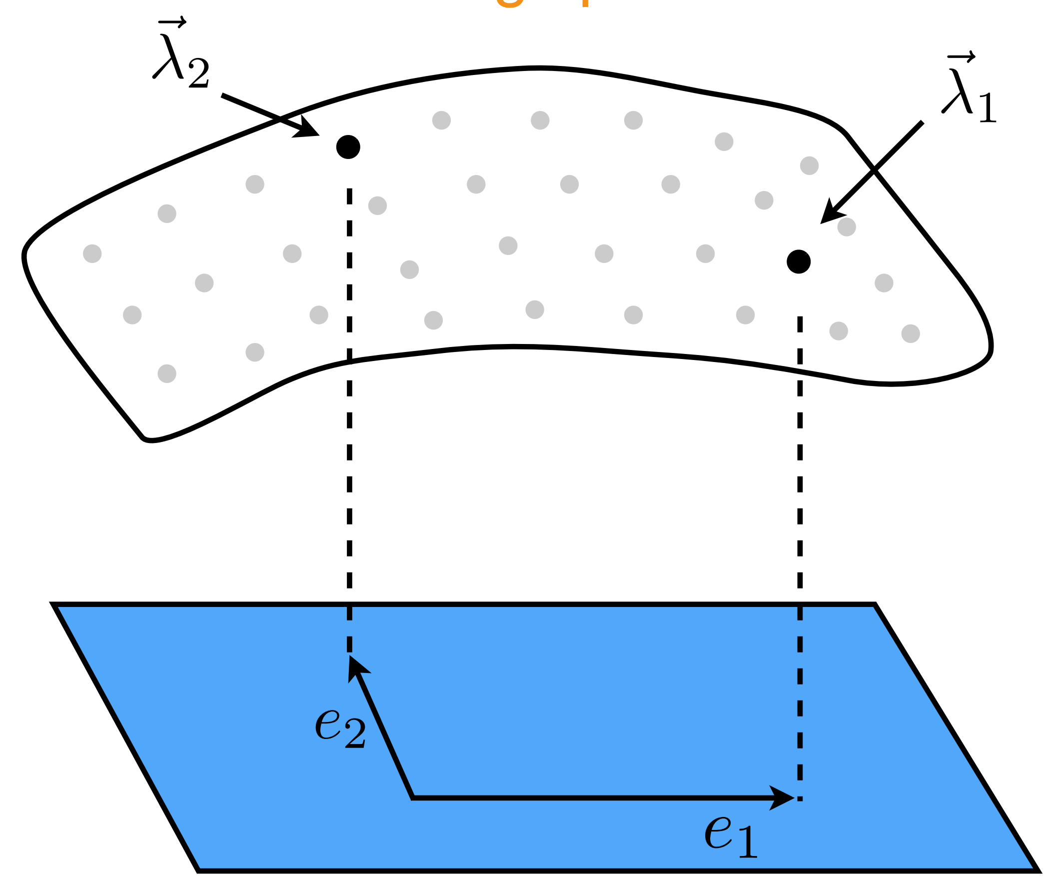


"Training space"



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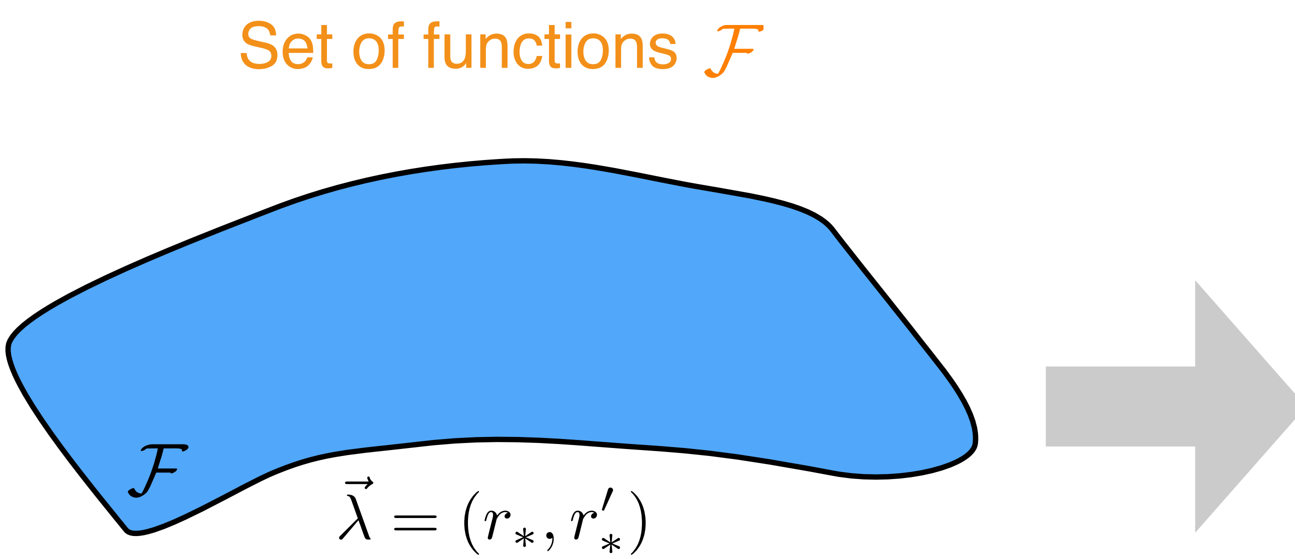

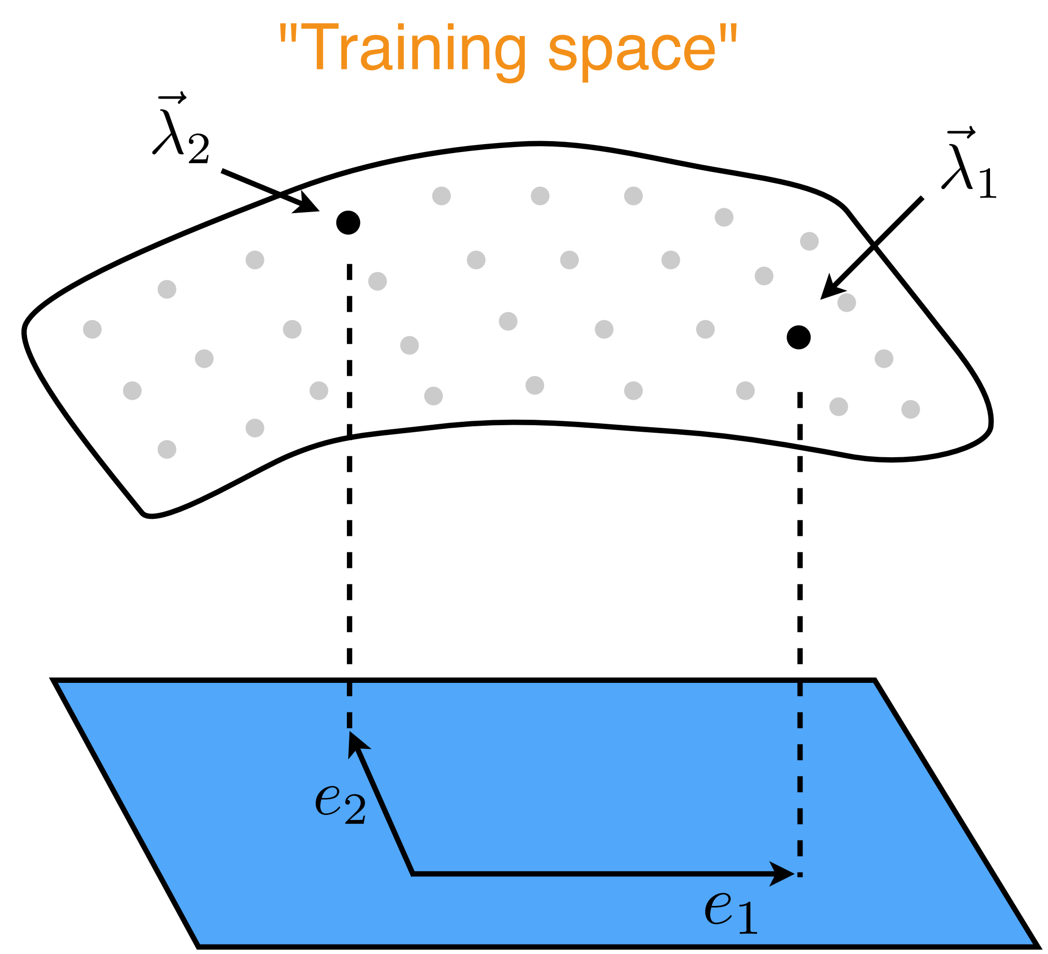

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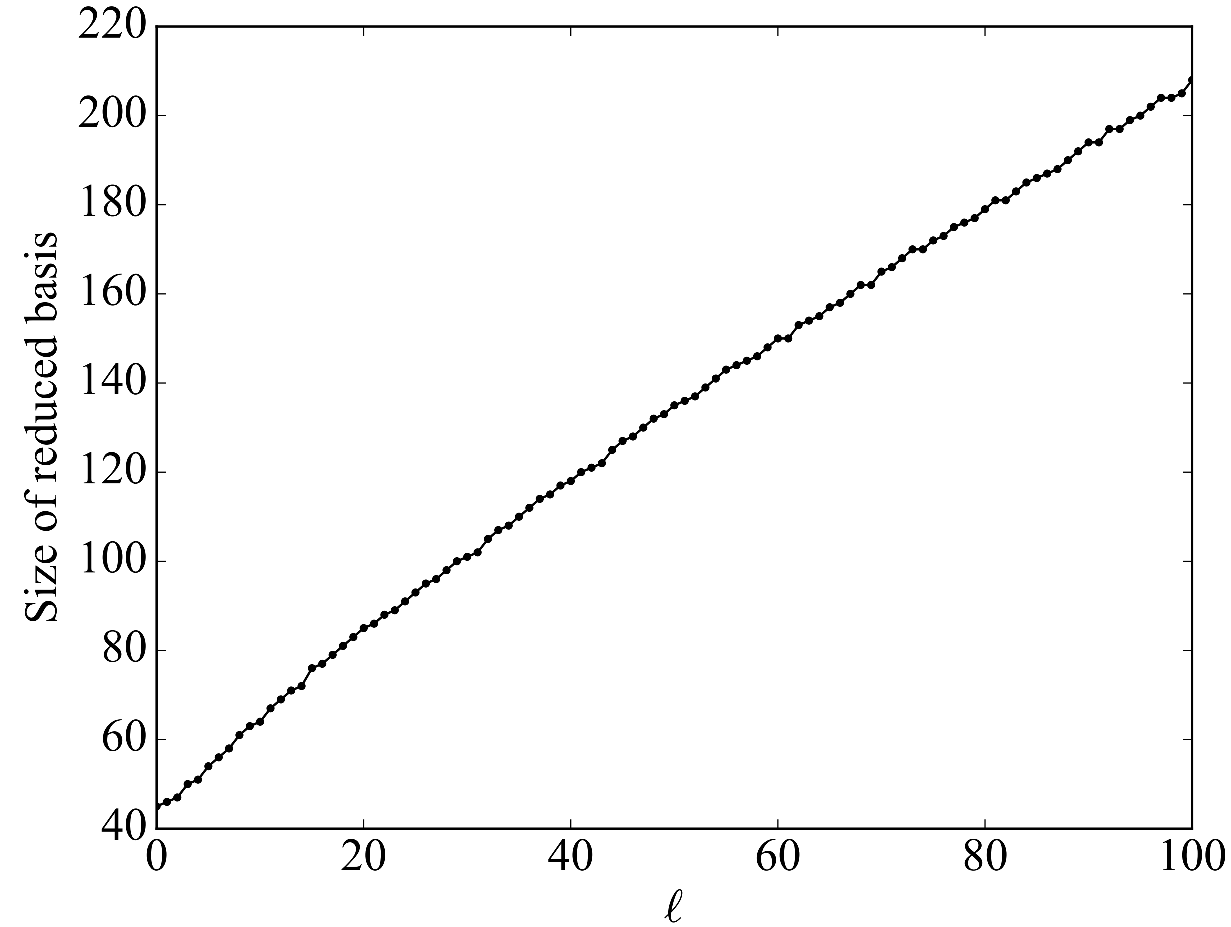
3) Orthogonalization to get basis vector e_2

$$C_2 = \{e_1, e_2\}, \; C_1 \subset C_2$$

Basis size grows nearly linearly with mode number



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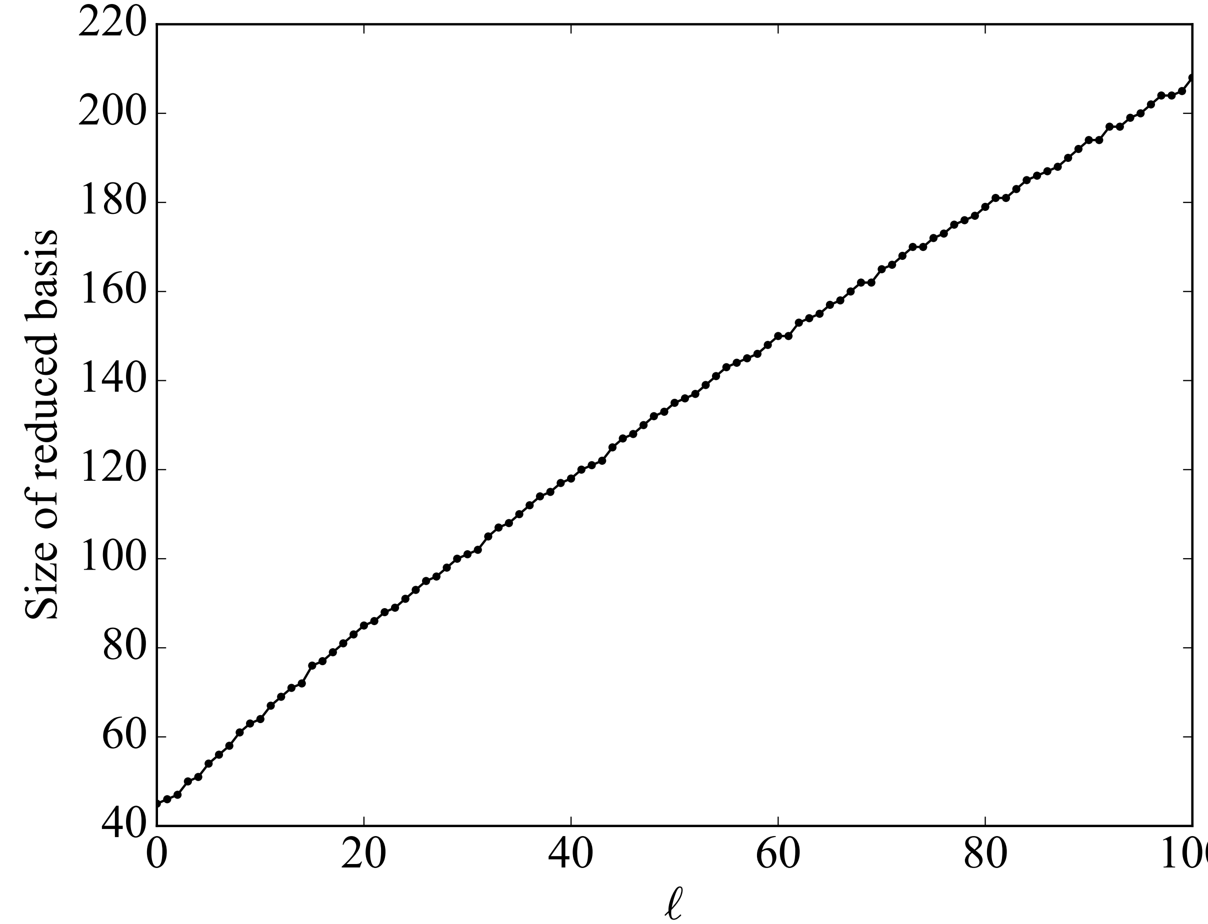

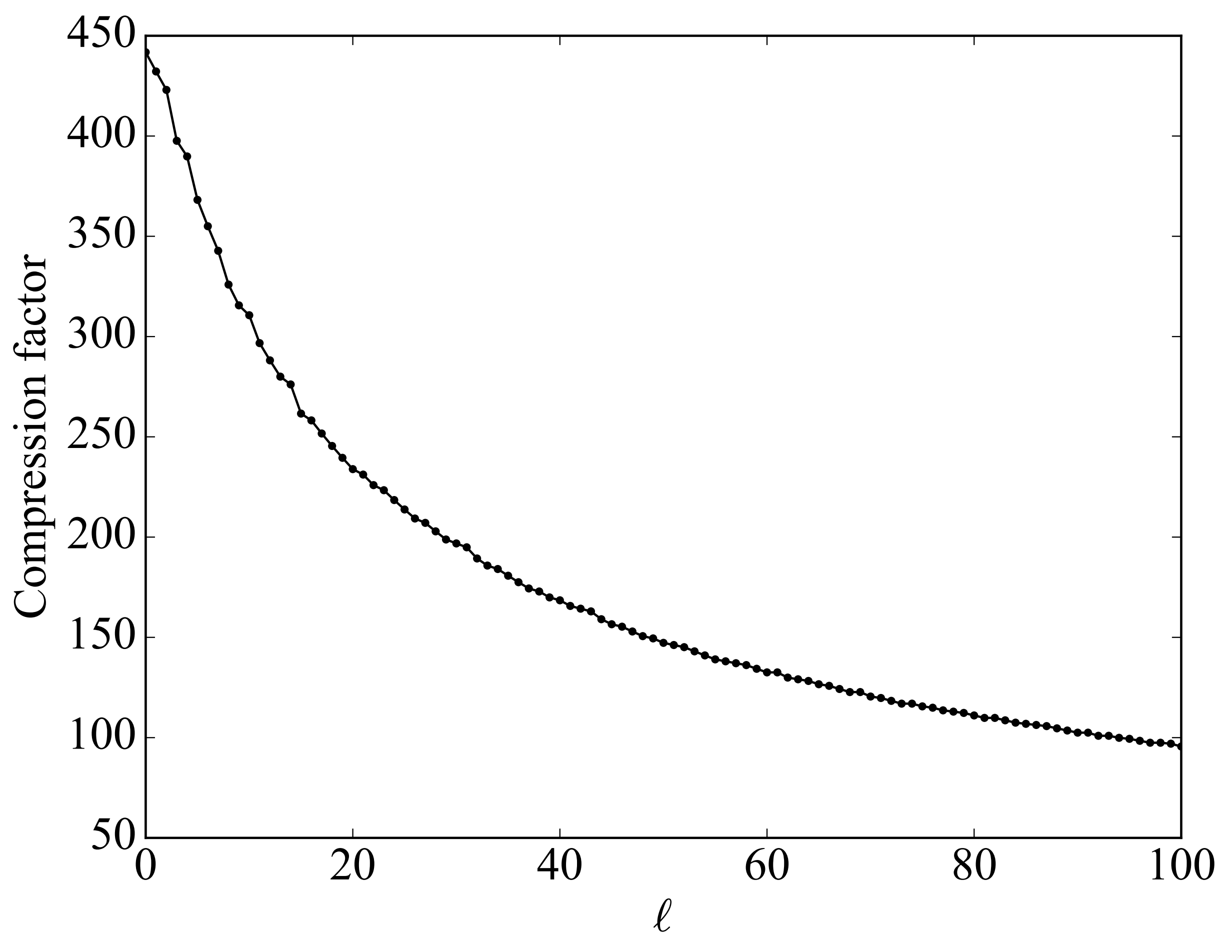

Total compression factor:

$$C_{\rm total} = (\ell_{\rm max} + 1) \left(\sum_{\ell=0}^{\ell_{\rm max}} \frac{1}{C_\ell} \right)^{-1}$$

$$\approx 151$$

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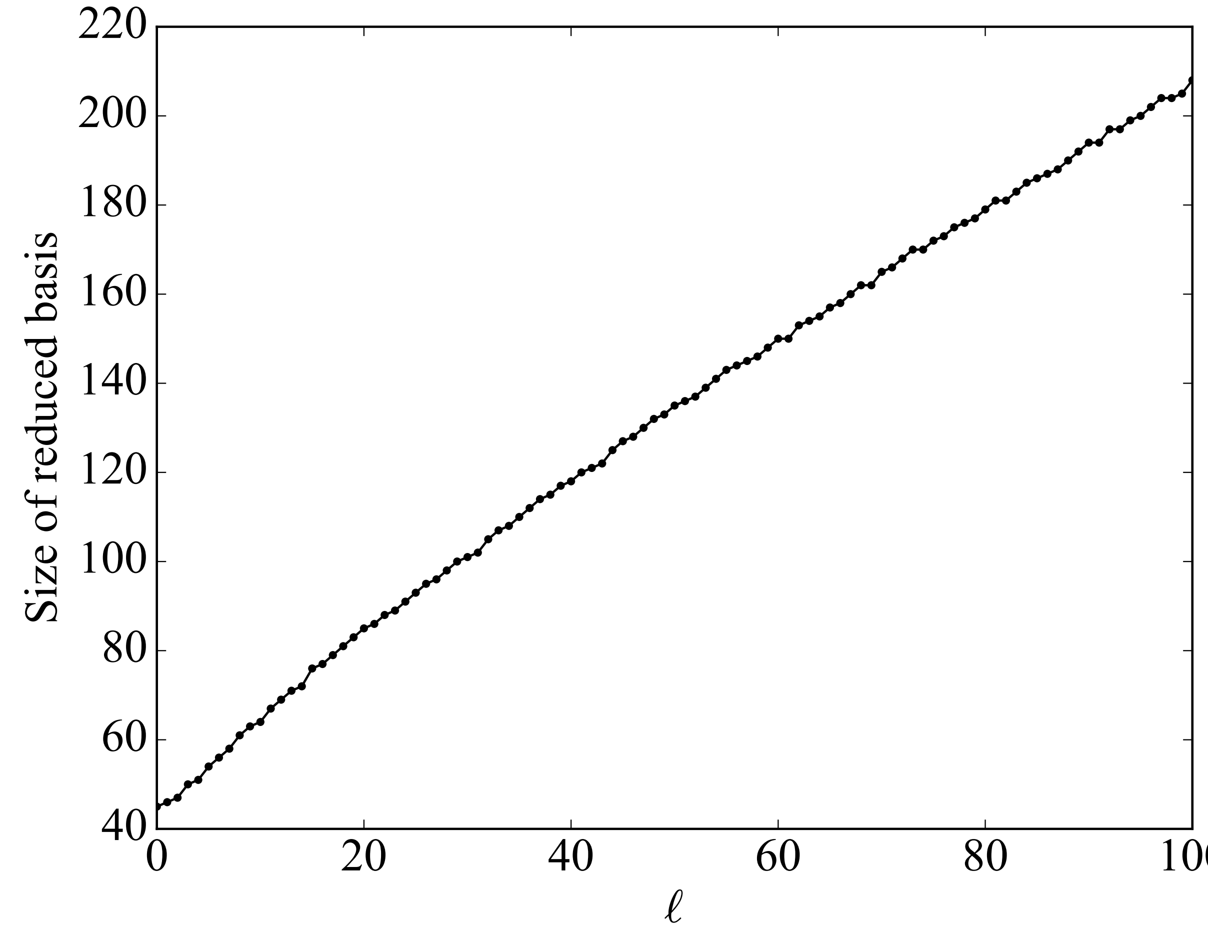

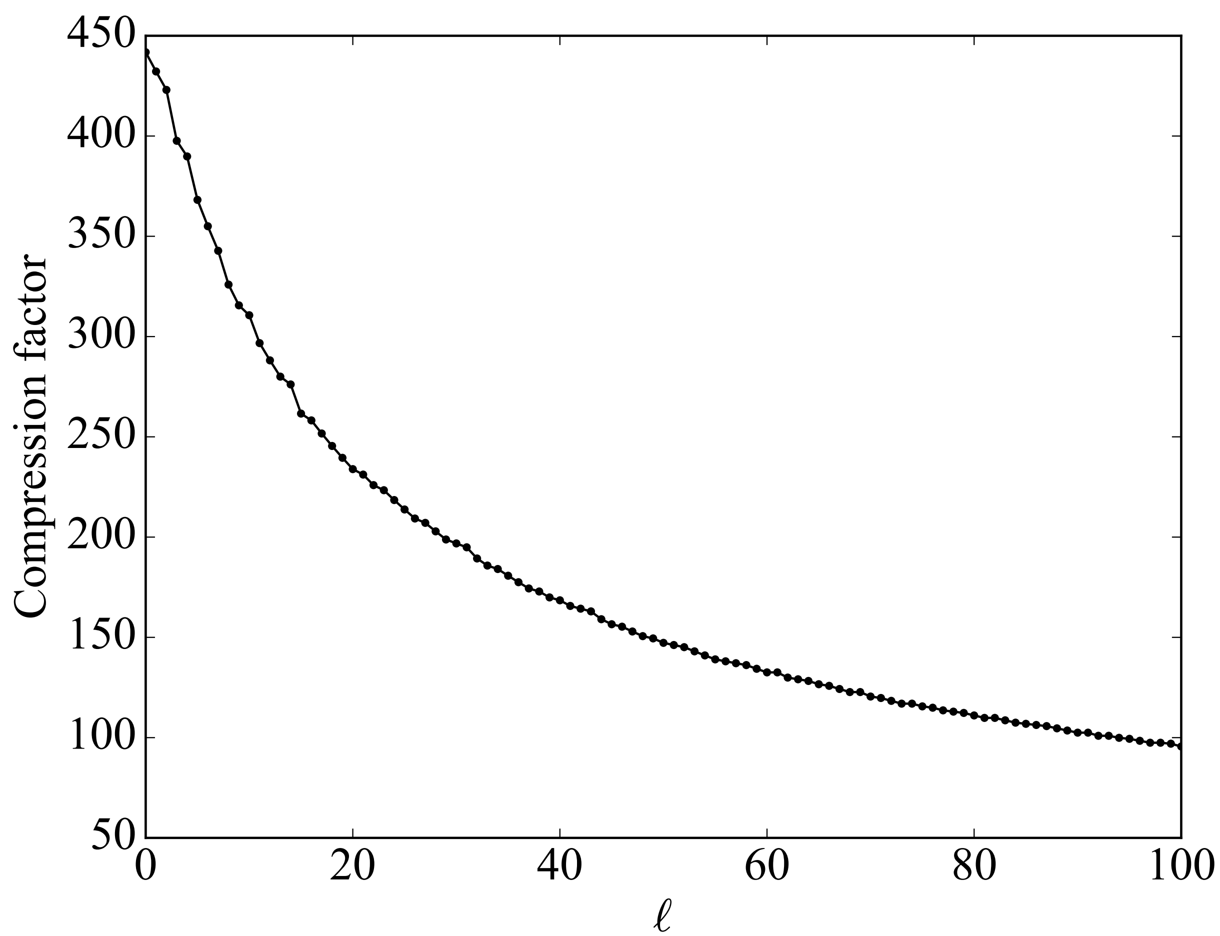

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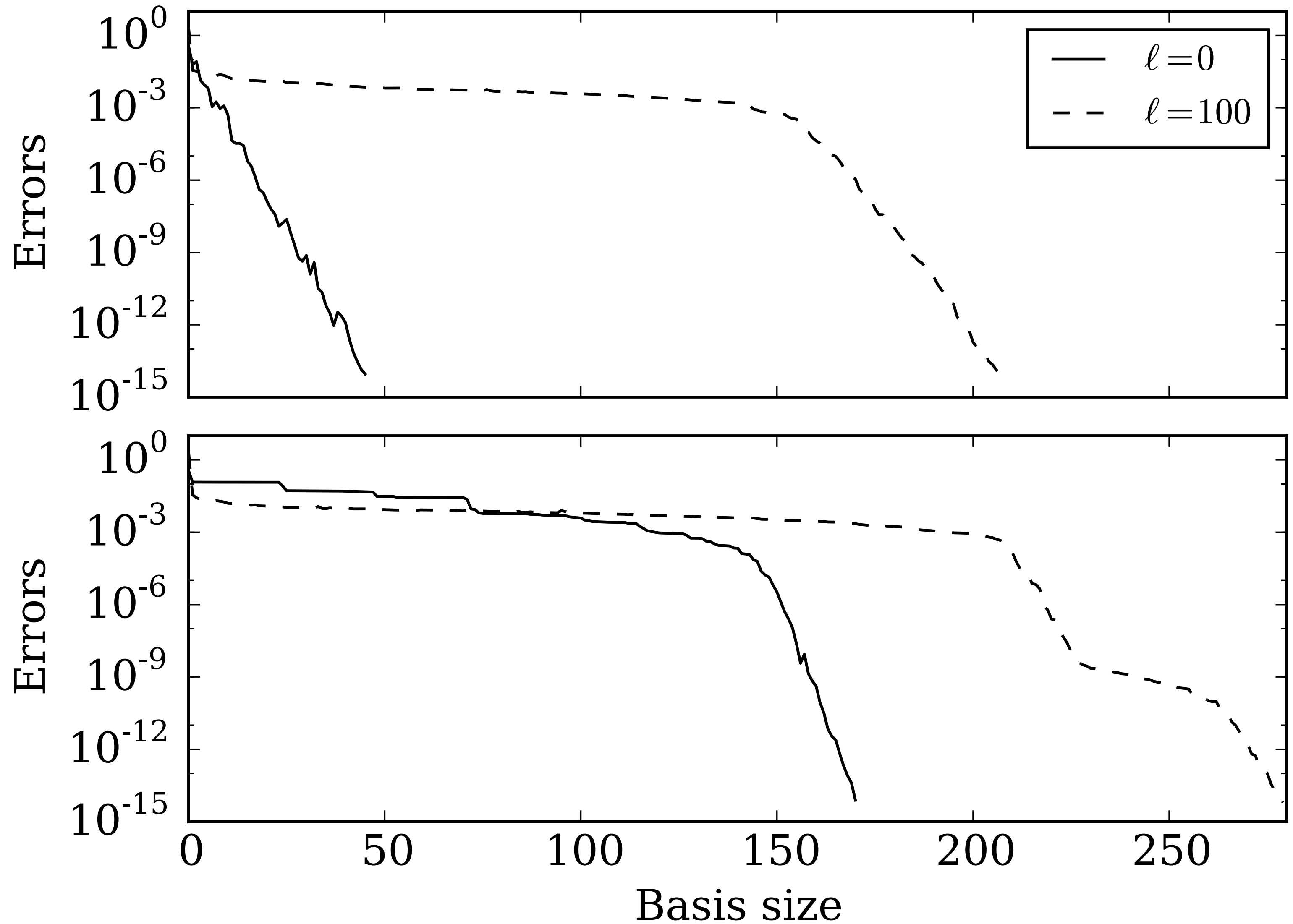
Less than 1% of the data is needed to capture all features up to numerical round-off errors

Shifting by time-of-arrival



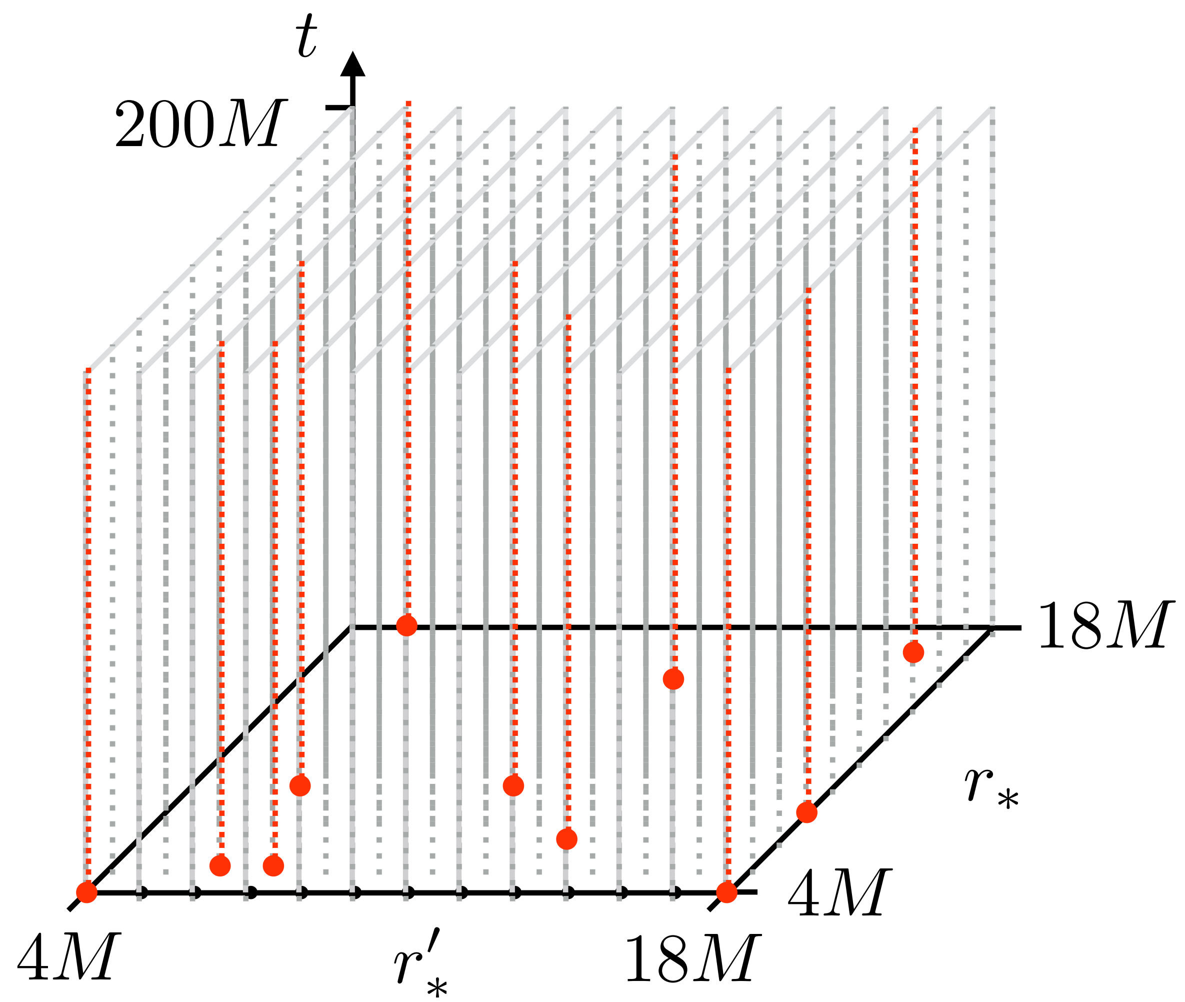
Not shifting by time-of-arrival

2) Empirical interpolation

Barrault+ (04)
Maday+ (09)

RB approximation:

$$G_\ell(t;\vec{\lambda}) \approx \sum_{i=1}^{N_\ell} C_i^\ell(\vec{\lambda}) e_i^\ell(t)$$



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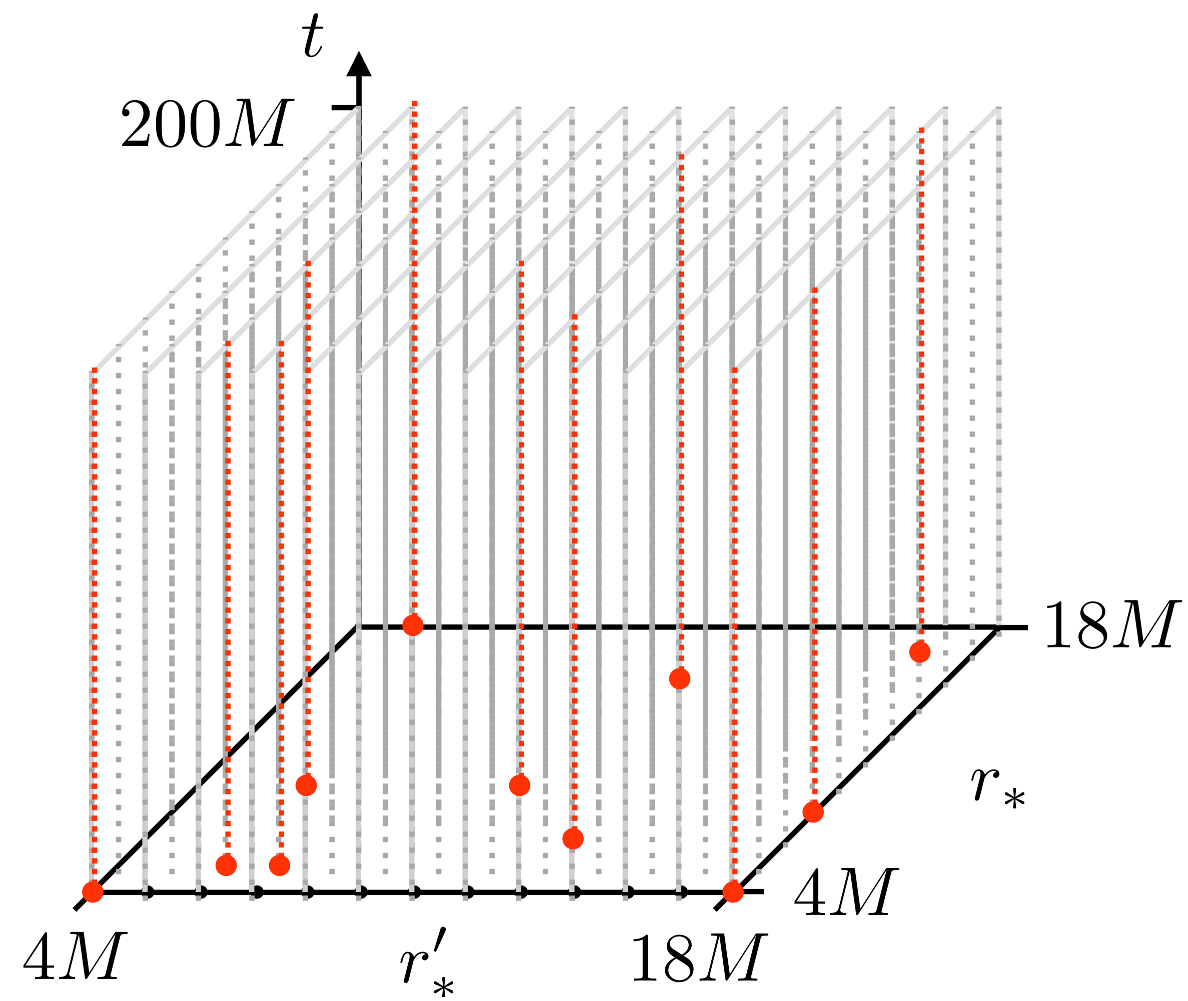

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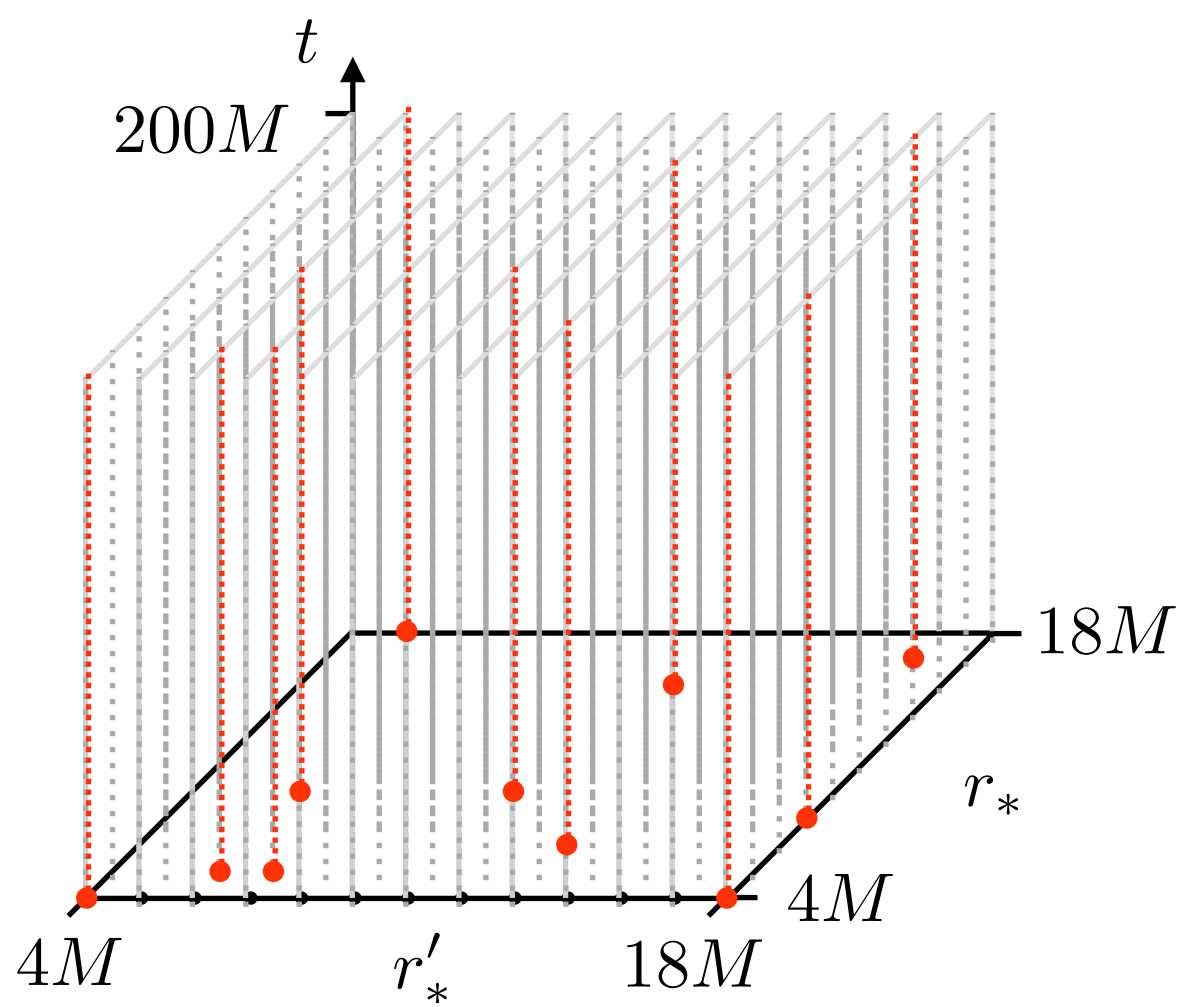

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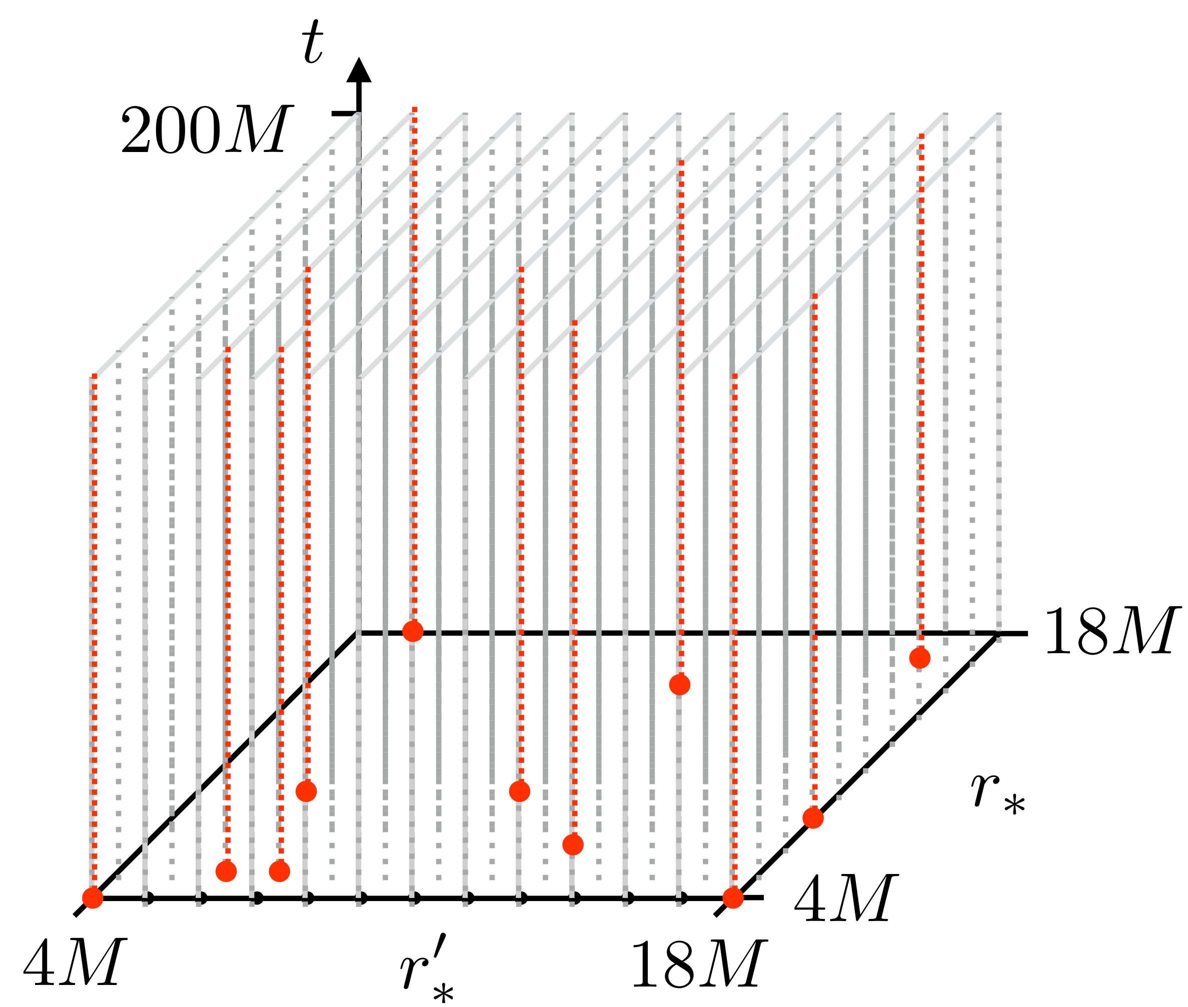
Find the interpolation nodes through another greedy algorithm that minimizes the interpolation error



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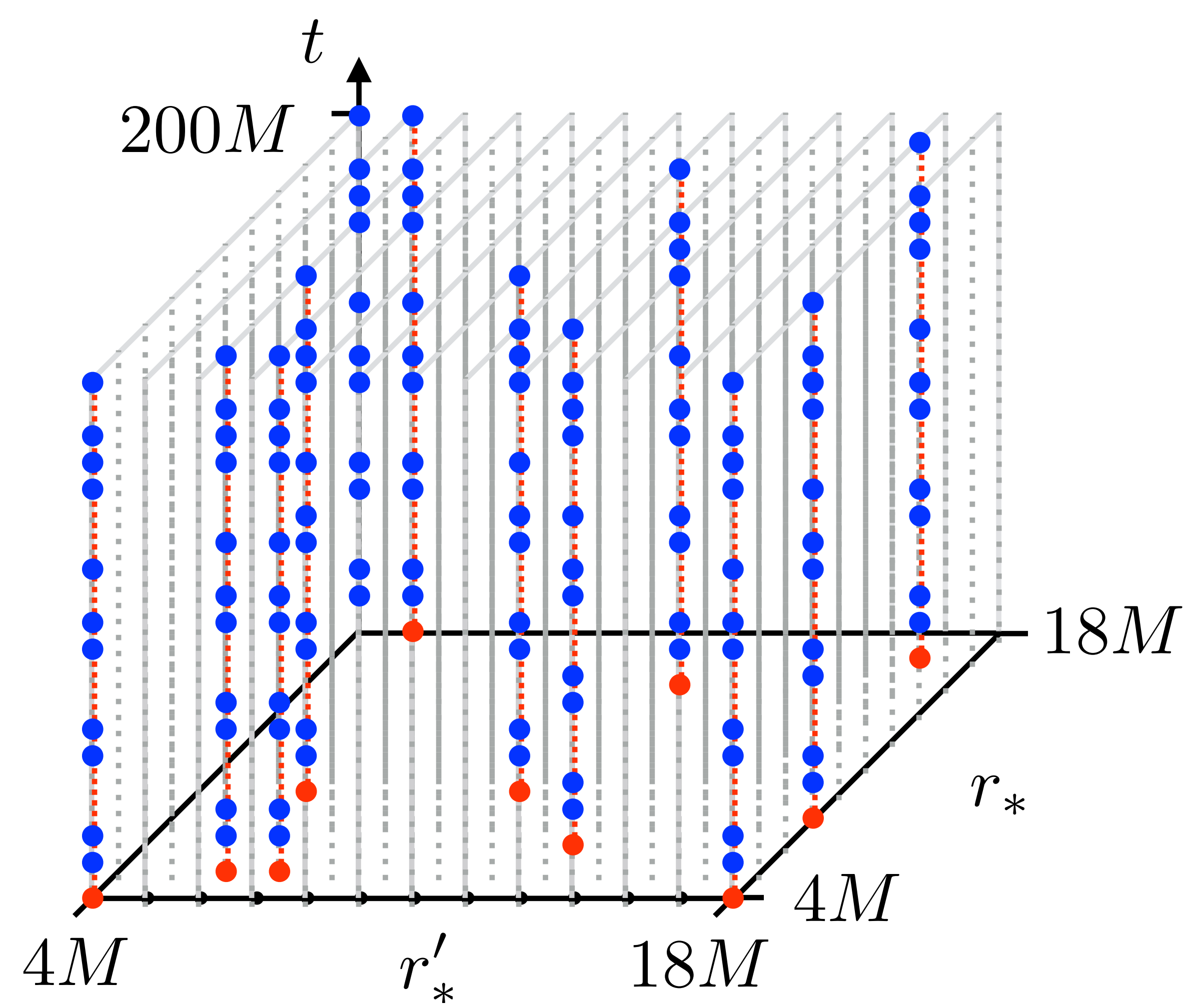
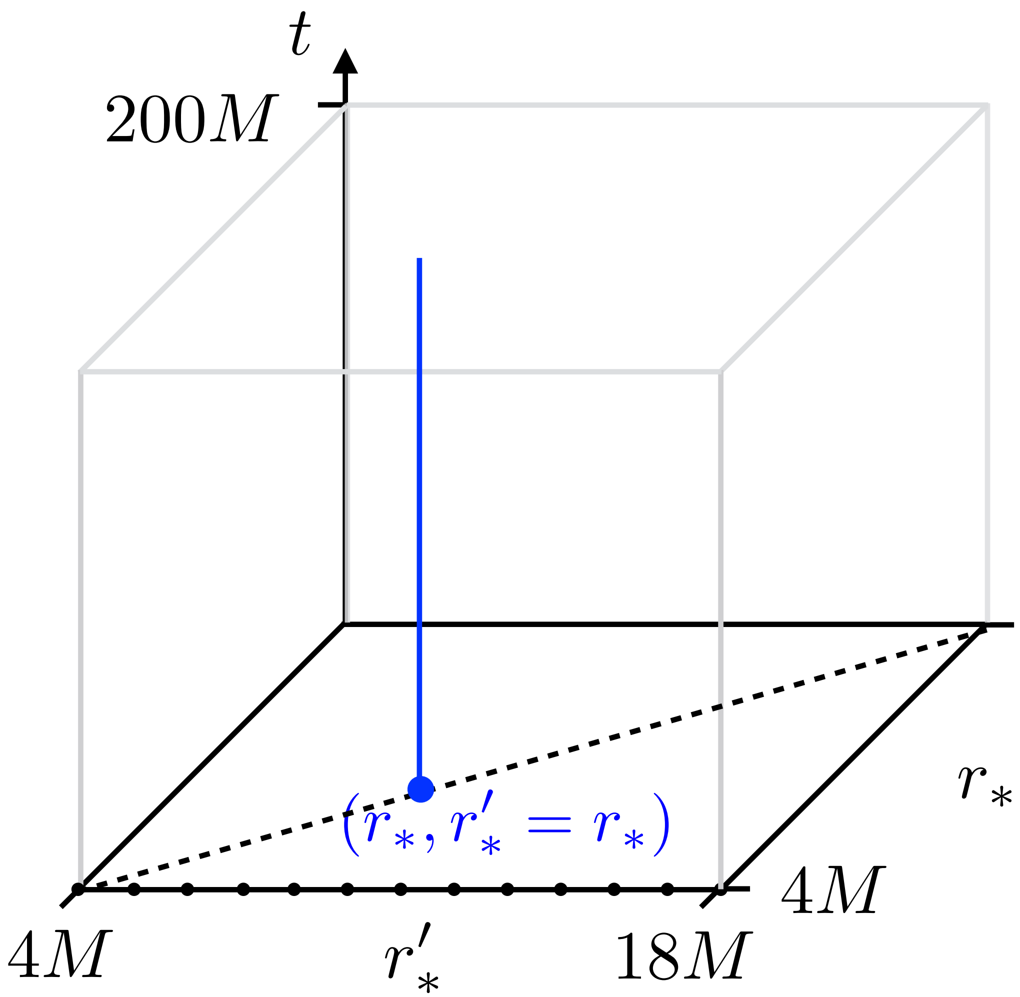


3) Parametric fitting — r^*= constant

Mostly interested in computing Green's function on worldlines for self-force



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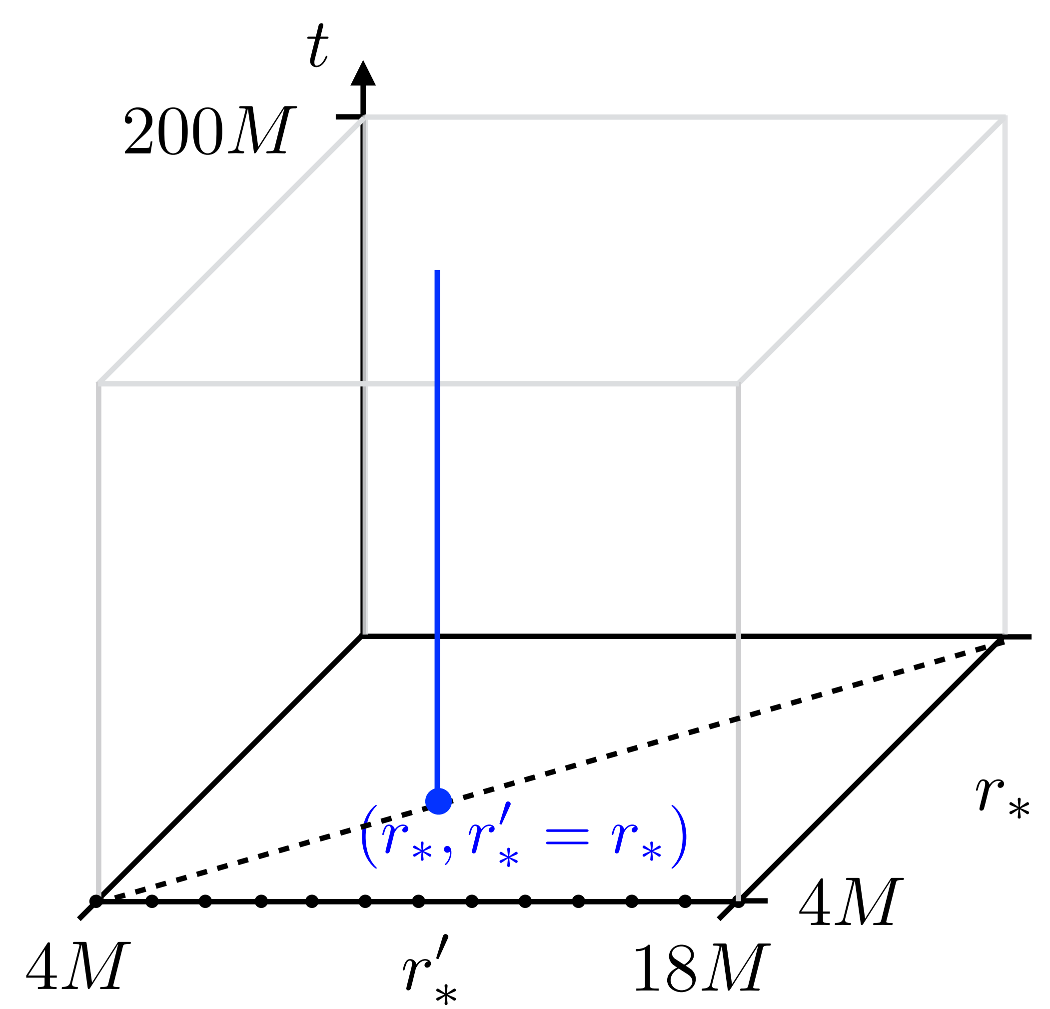
The surrogate is simple to evaluate on worldlines with r_* = constant

- static worldlines
- circular geodesics
- accelerated circular worldlines



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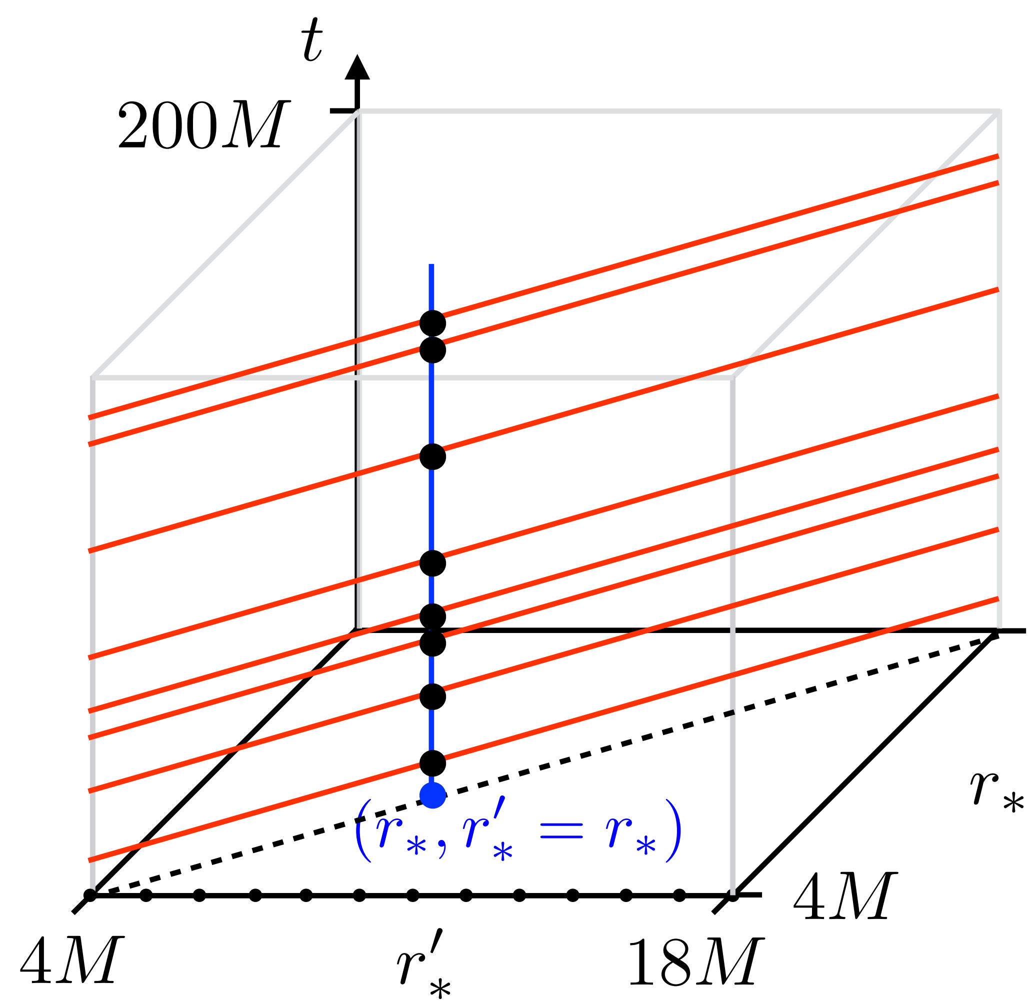

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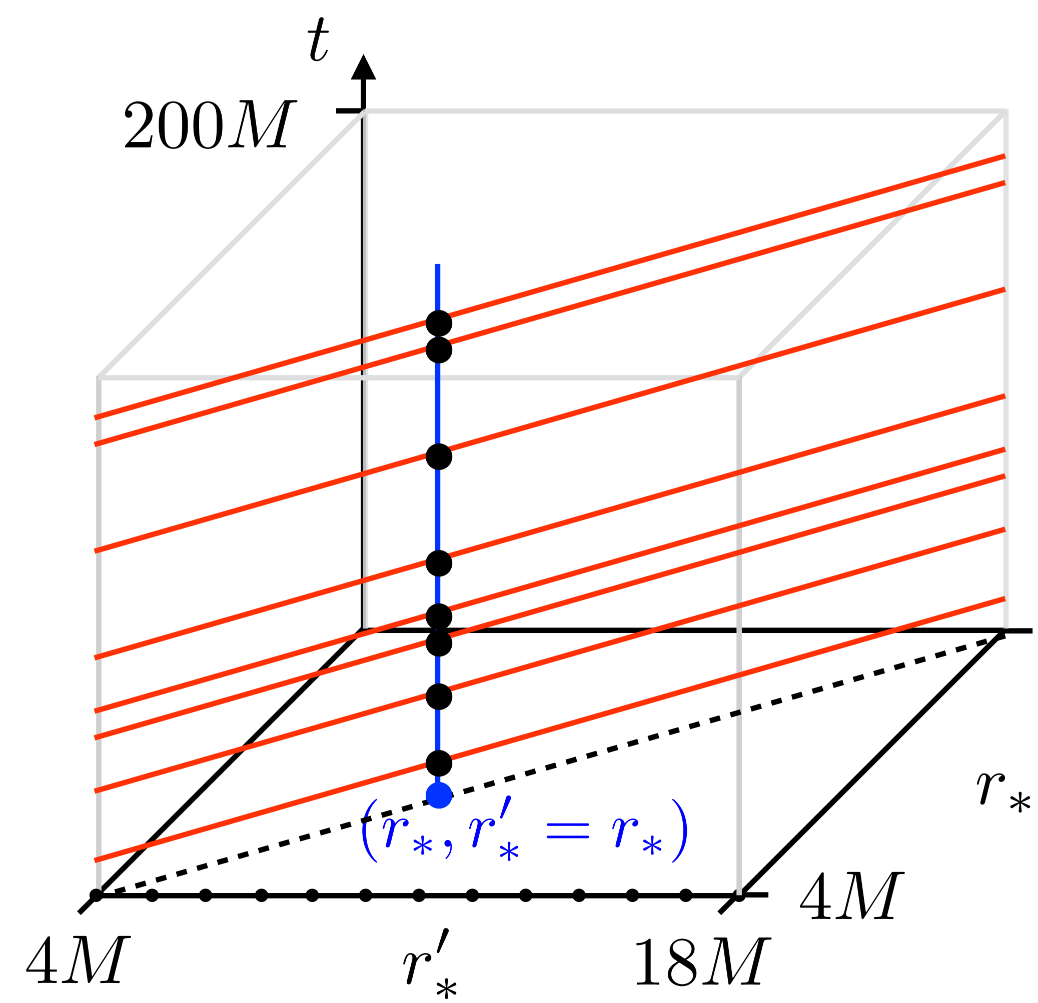

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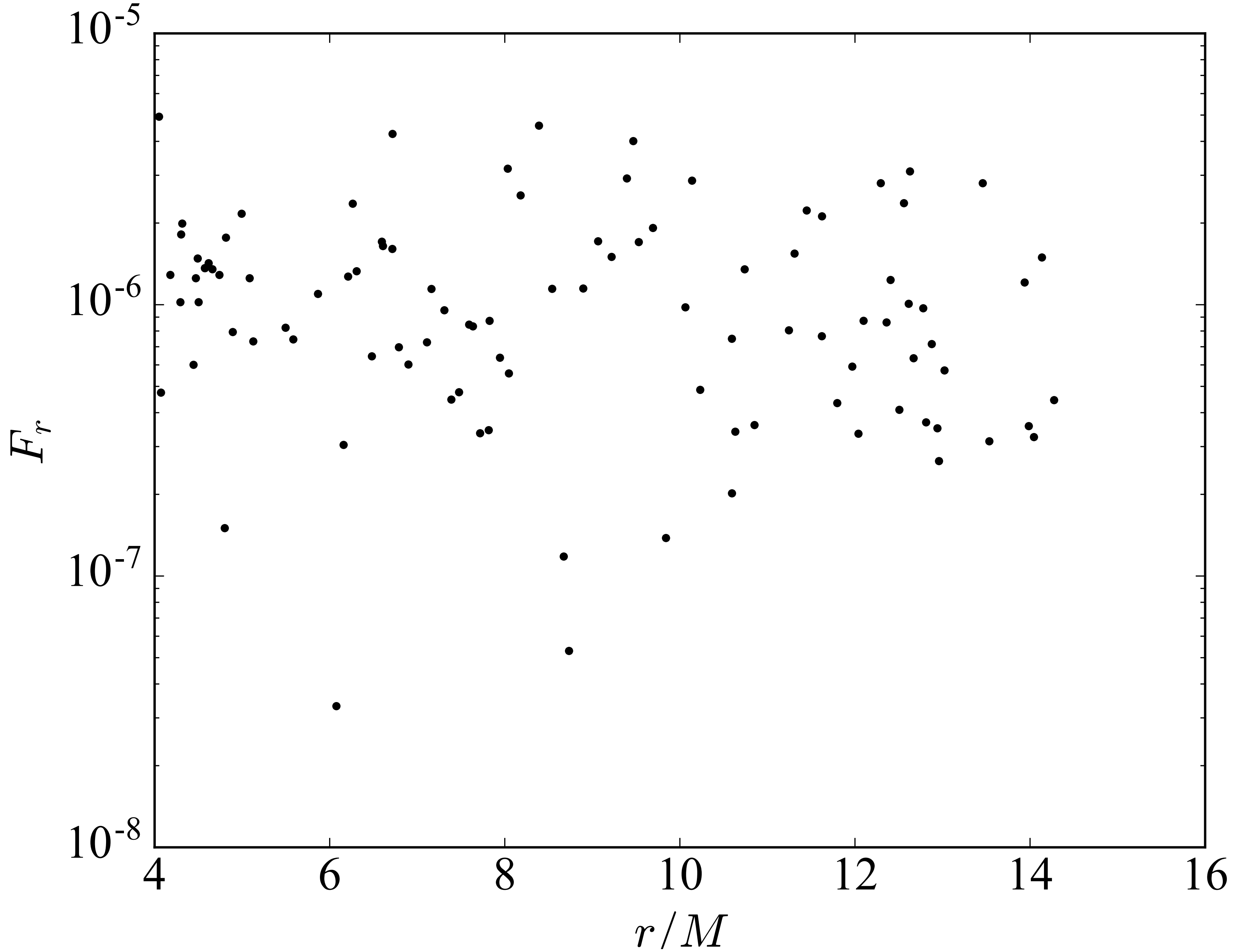
<u>Total compression</u>: 30GB /192MB = 156

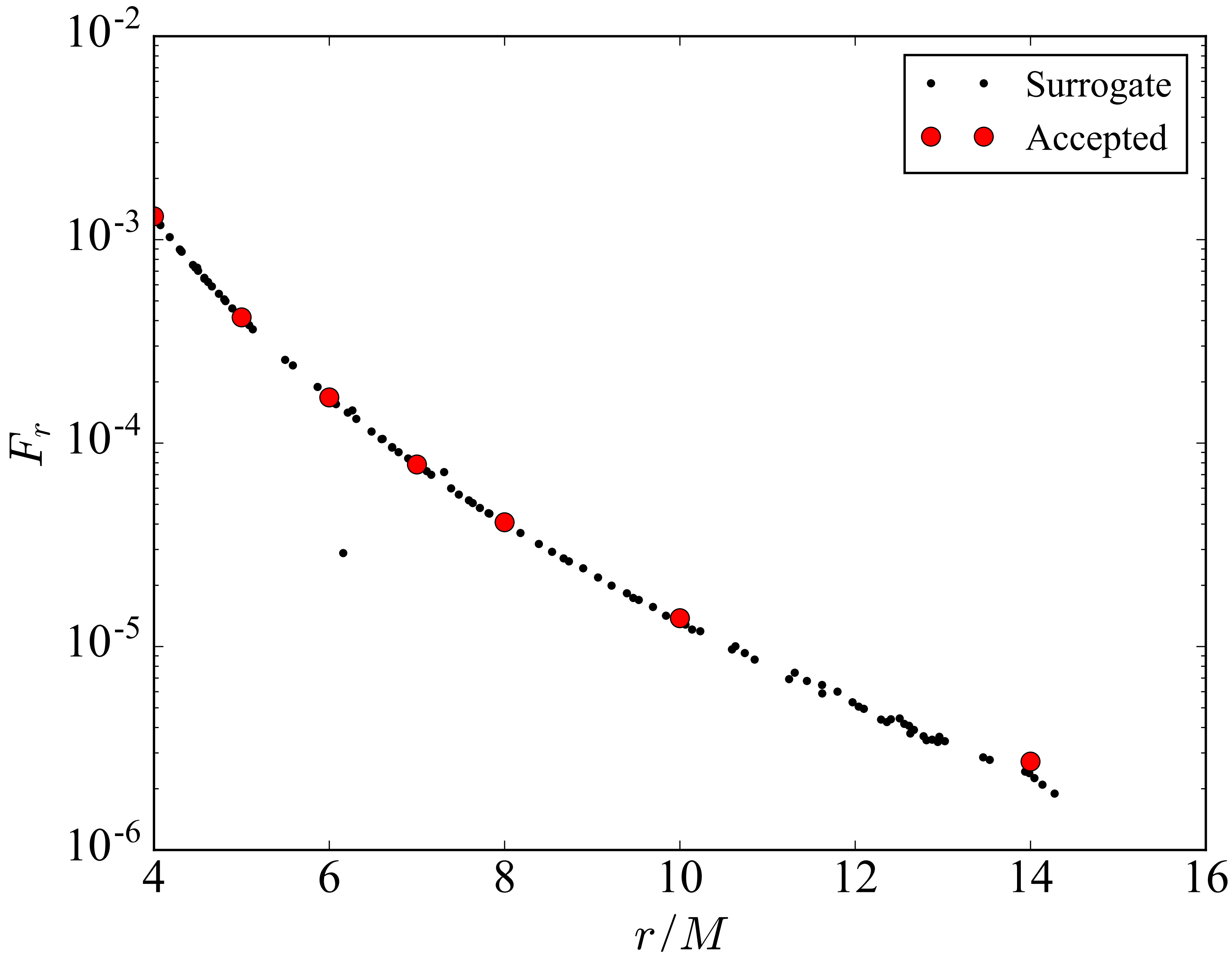
<u>Speed-up</u>: 380s / 0.5s = 760*x*

Static self-force

Analytically derived and found to be zero *Wiseman (00)*



Self-force on circular geodesics



Accepted or “truth” values computed in *Diaz-Rivera et al (04)*

3) Parametric fitting — generic case

More general worldlines

$$z^\mu(t) = \big(t, r(t), \pi/2, \gamma(t)\big)$$

destroy the affine nature
of the model (i.e., separability)

$$G_\ell(t; r_*(t), r'_*) \approx \sum_{j=1}^{N_\ell} B^\ell_j(t) G_\ell(T_j; r_*(t), r'_*)$$

As a result, fitting is done on-the-fly
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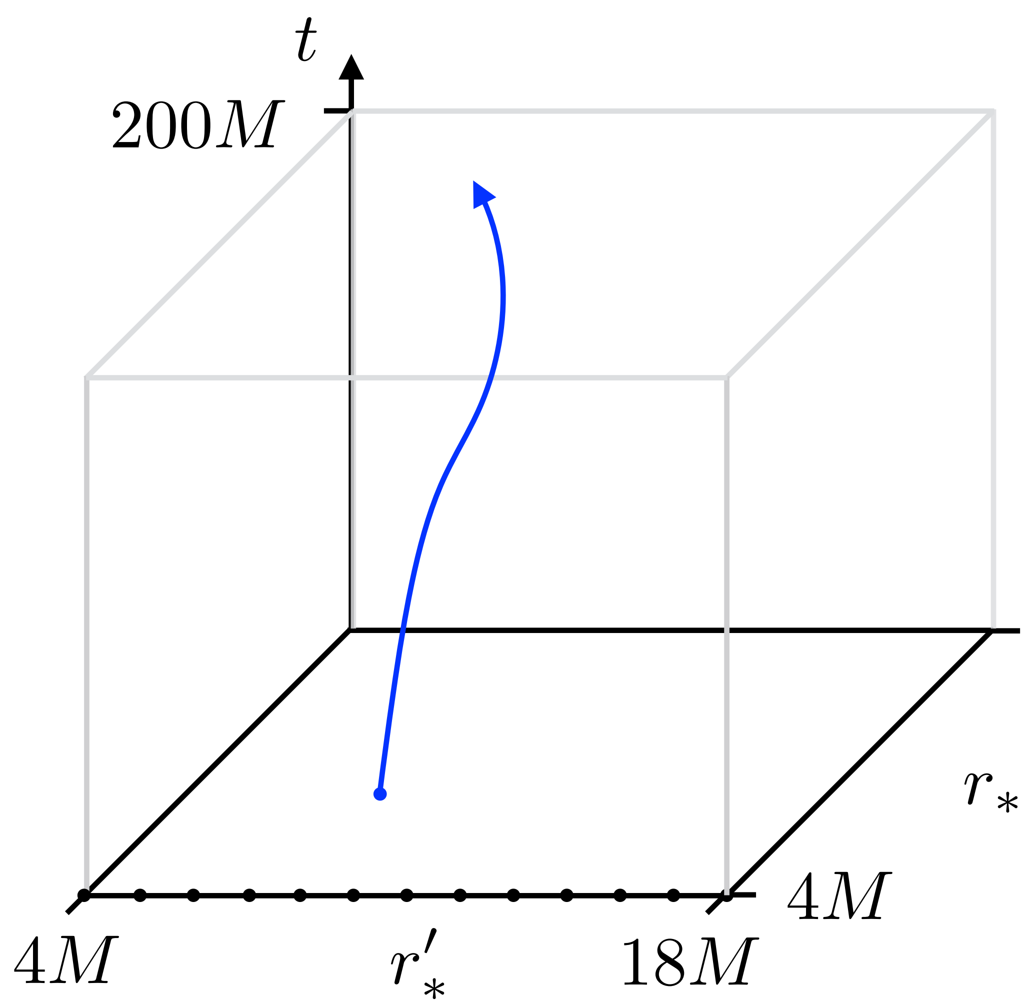

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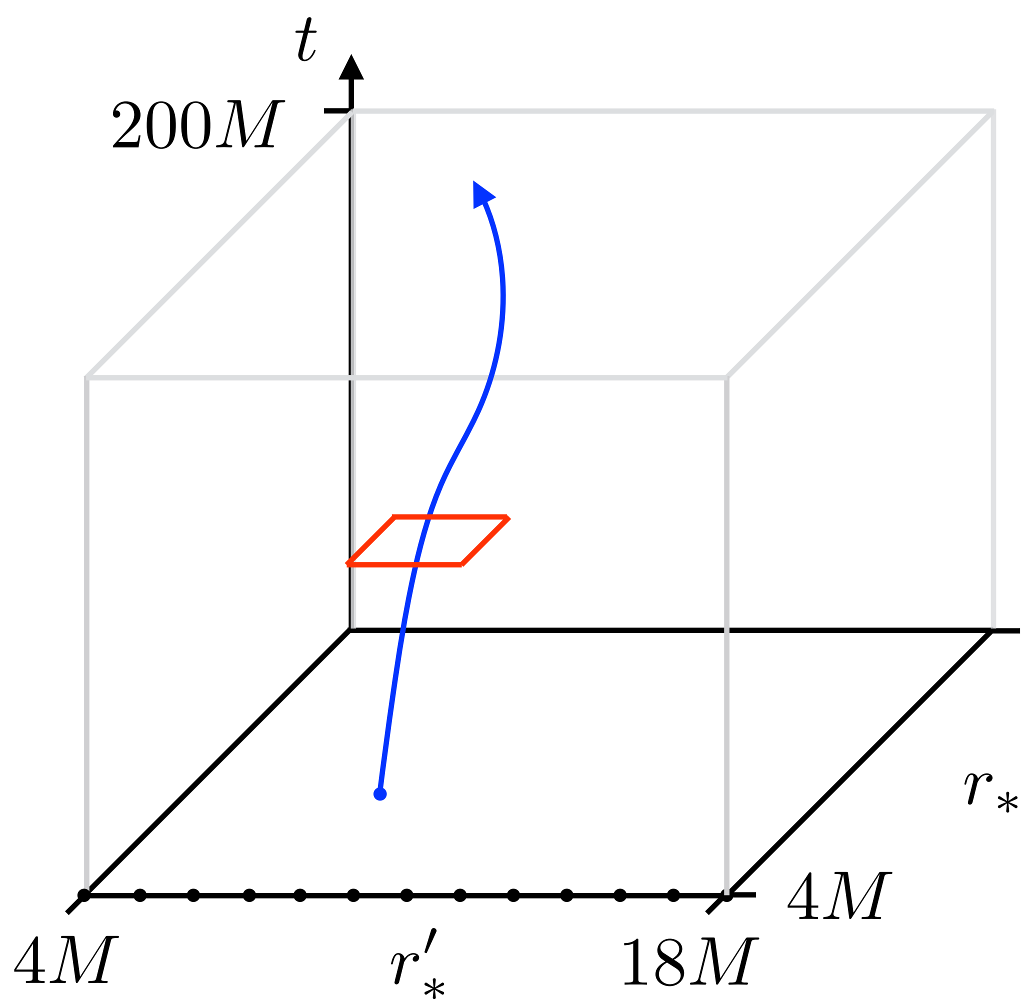
At a given time step, t_k, reconstruct
the Green's function data in a small
patch around the worldline

Fit with spline and interpolate to $r_*(t_k)$

Repeat for all time steps



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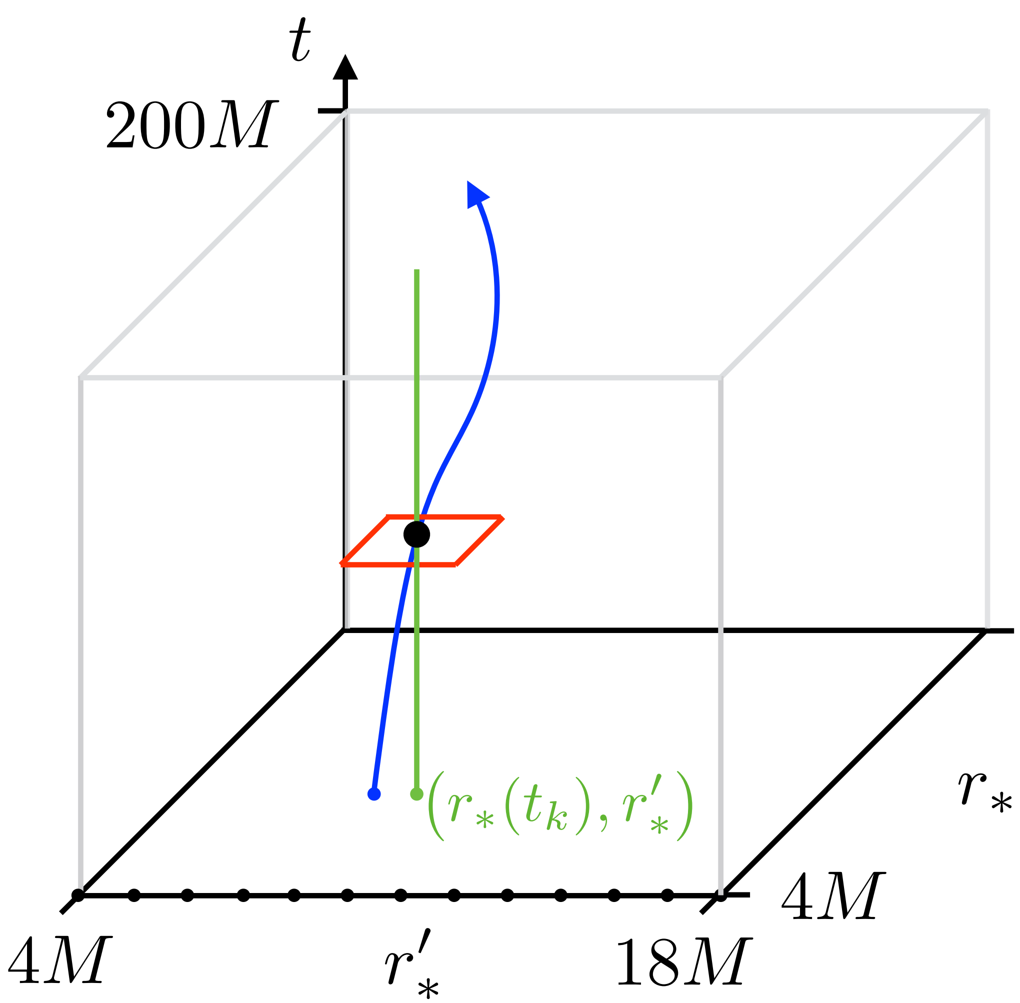

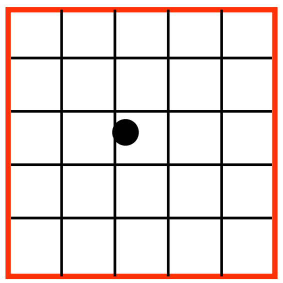

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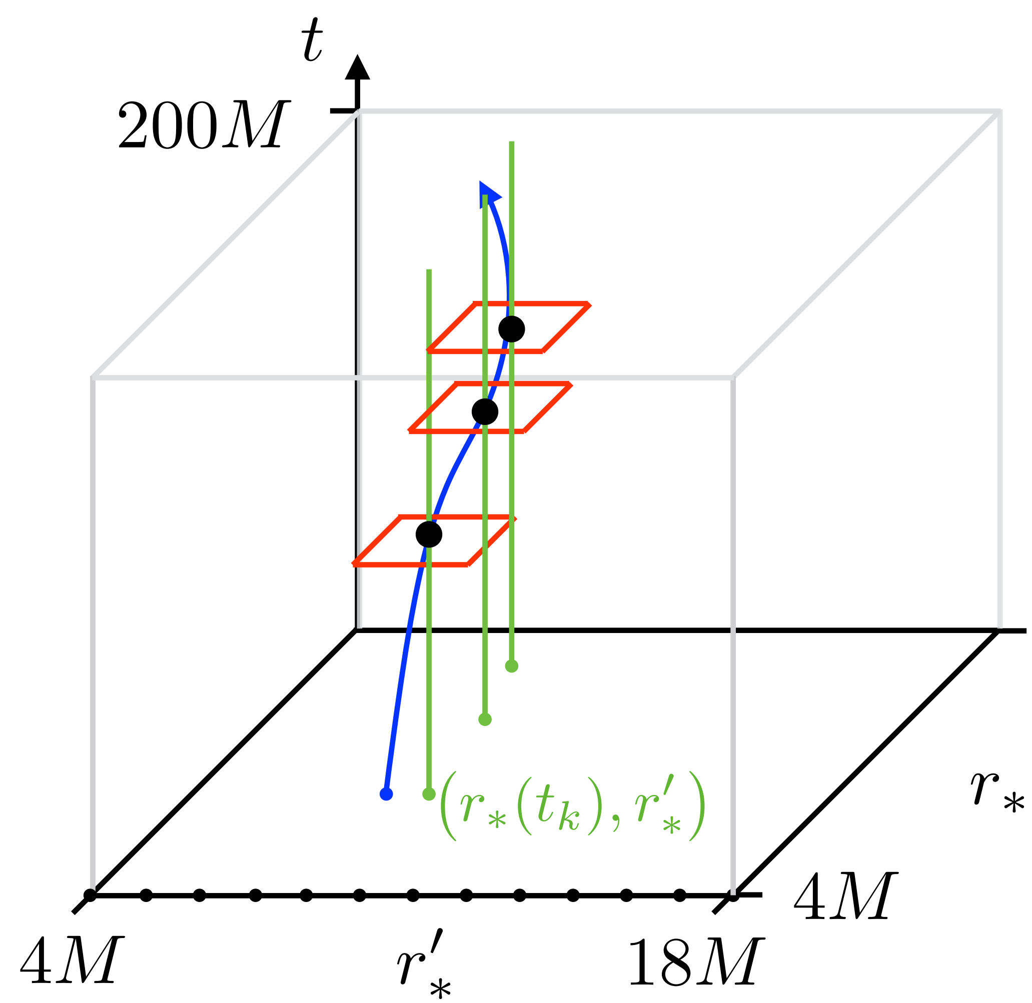

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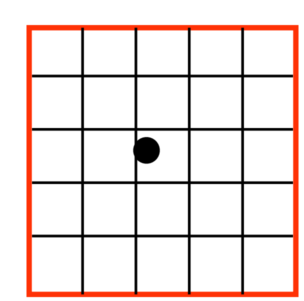

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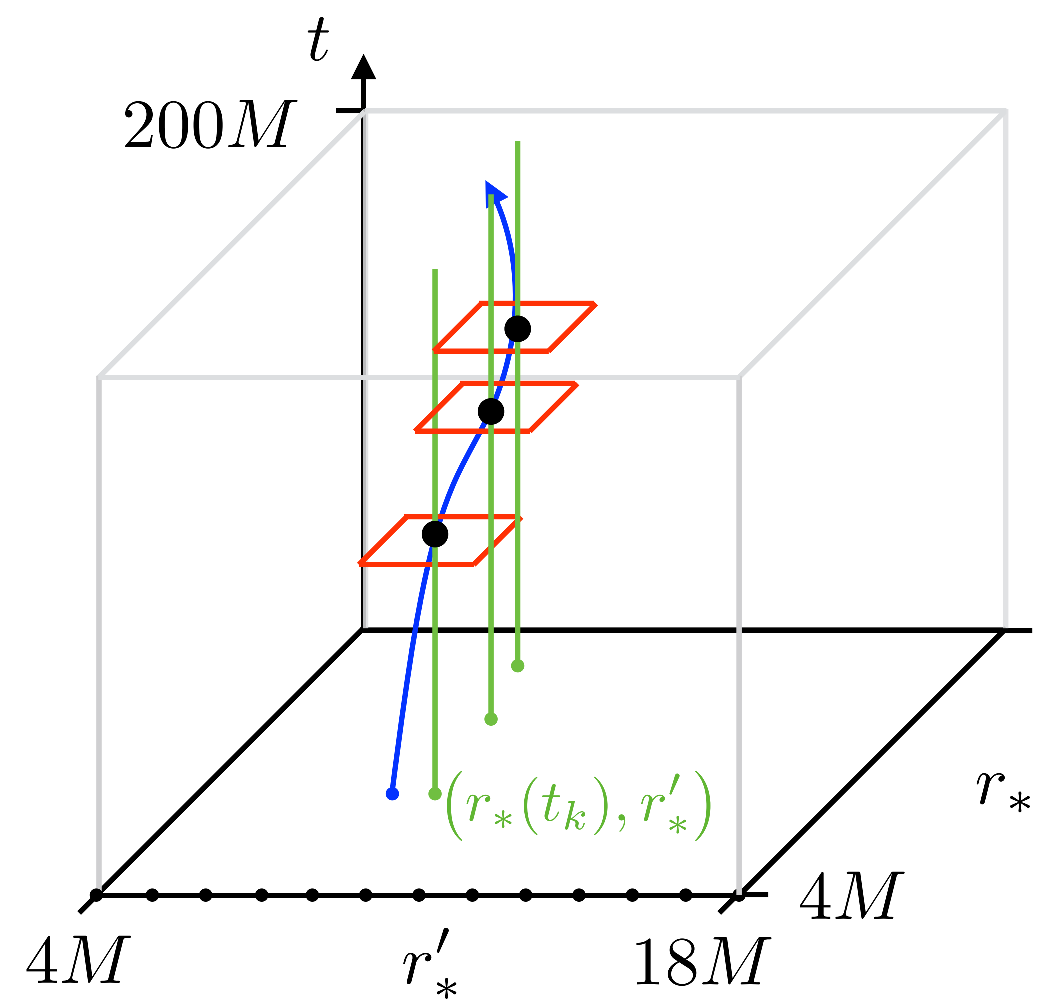

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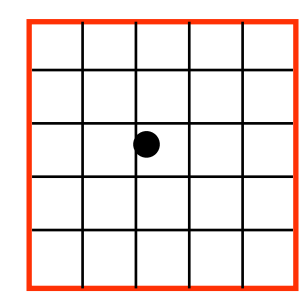
Slower and need to store to disk all data at each T_j…



Repeat for all time steps

Surrogate accuracy, speed-up, and size

Eccentric geodesic orbit ($e = 0.5$, $p = 7.2$)

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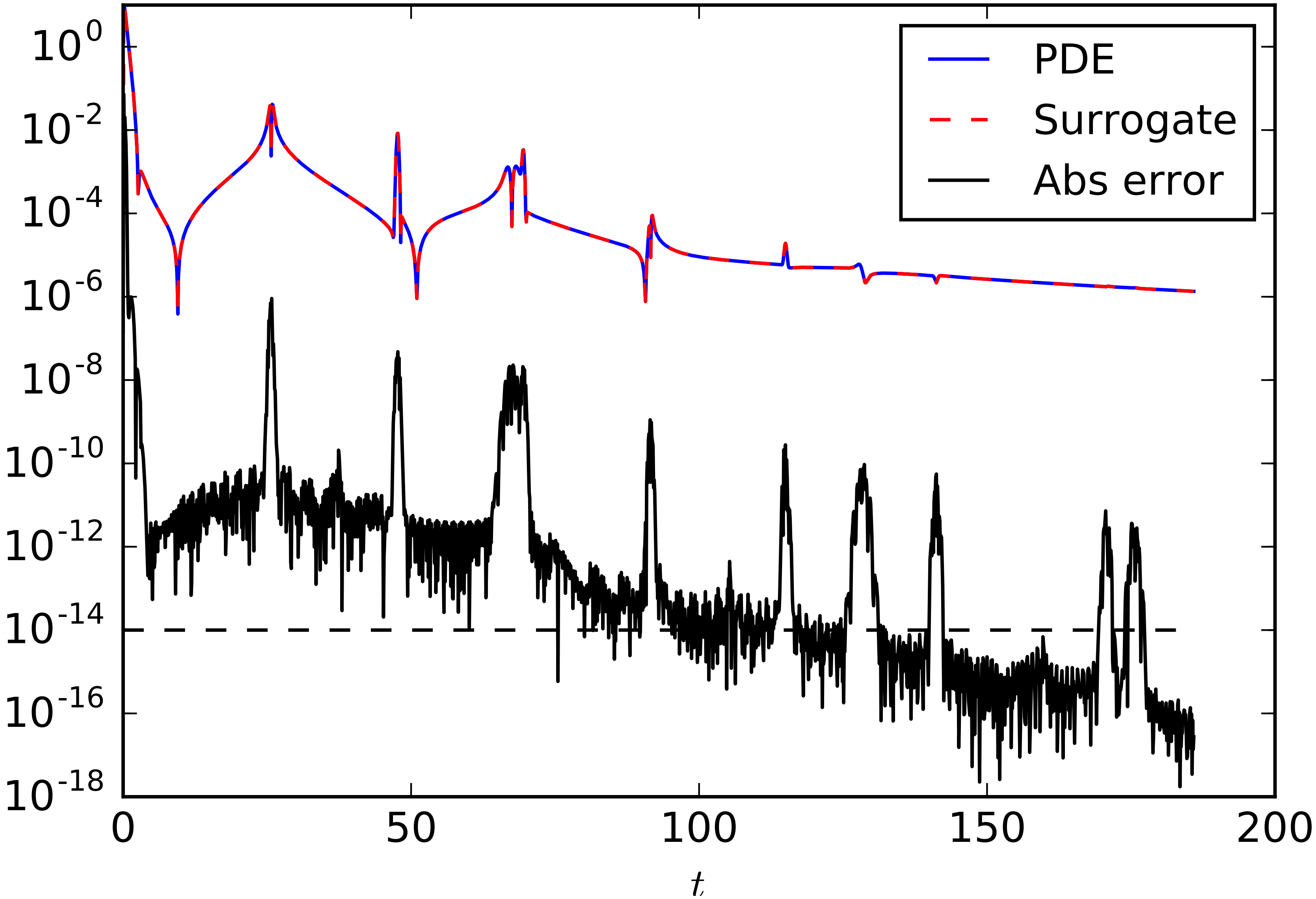

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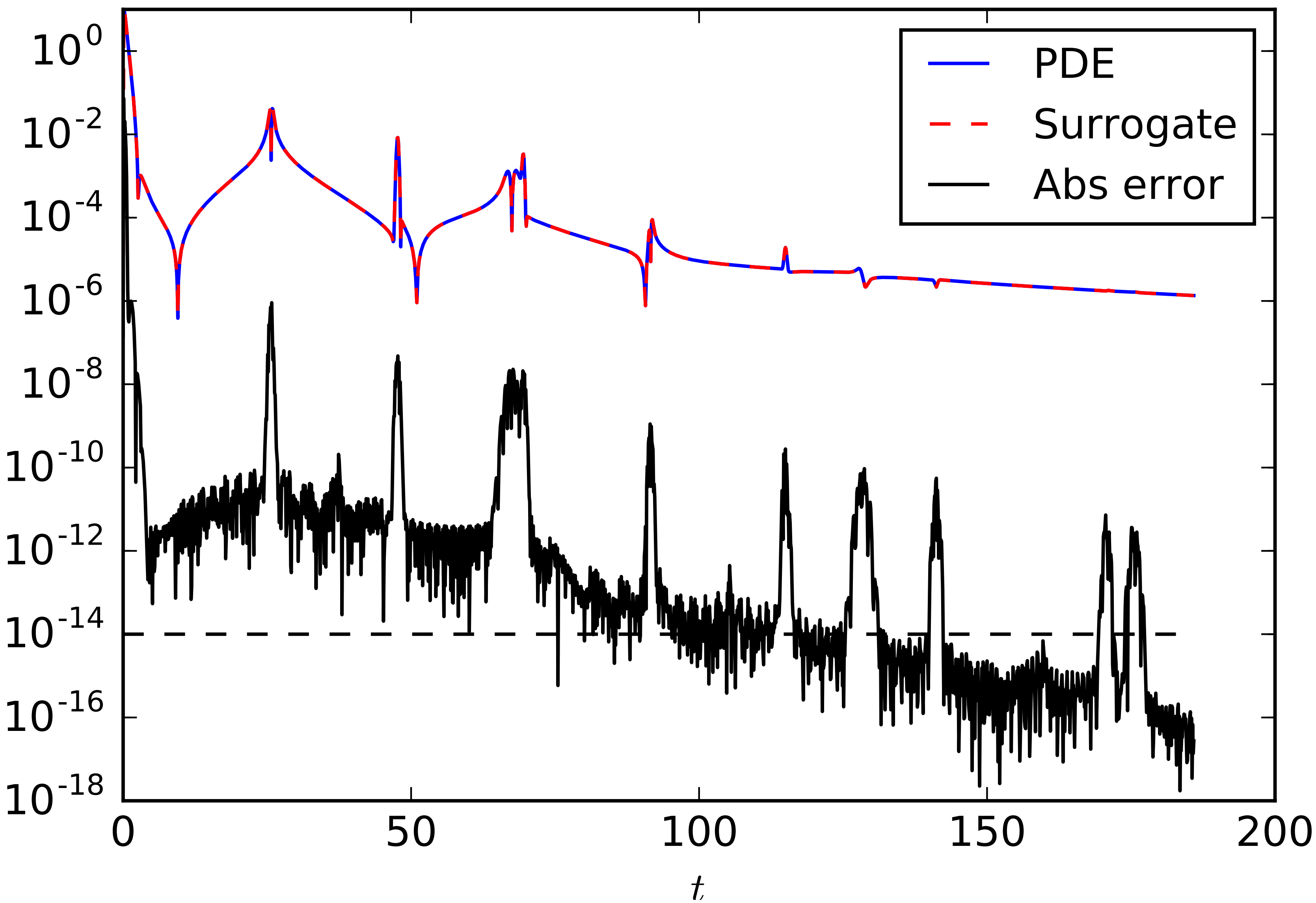

PDE	~380s
Surrogate	~25s
Speed-up*	~15x

* *PDE in C++*
Surrogate in Python

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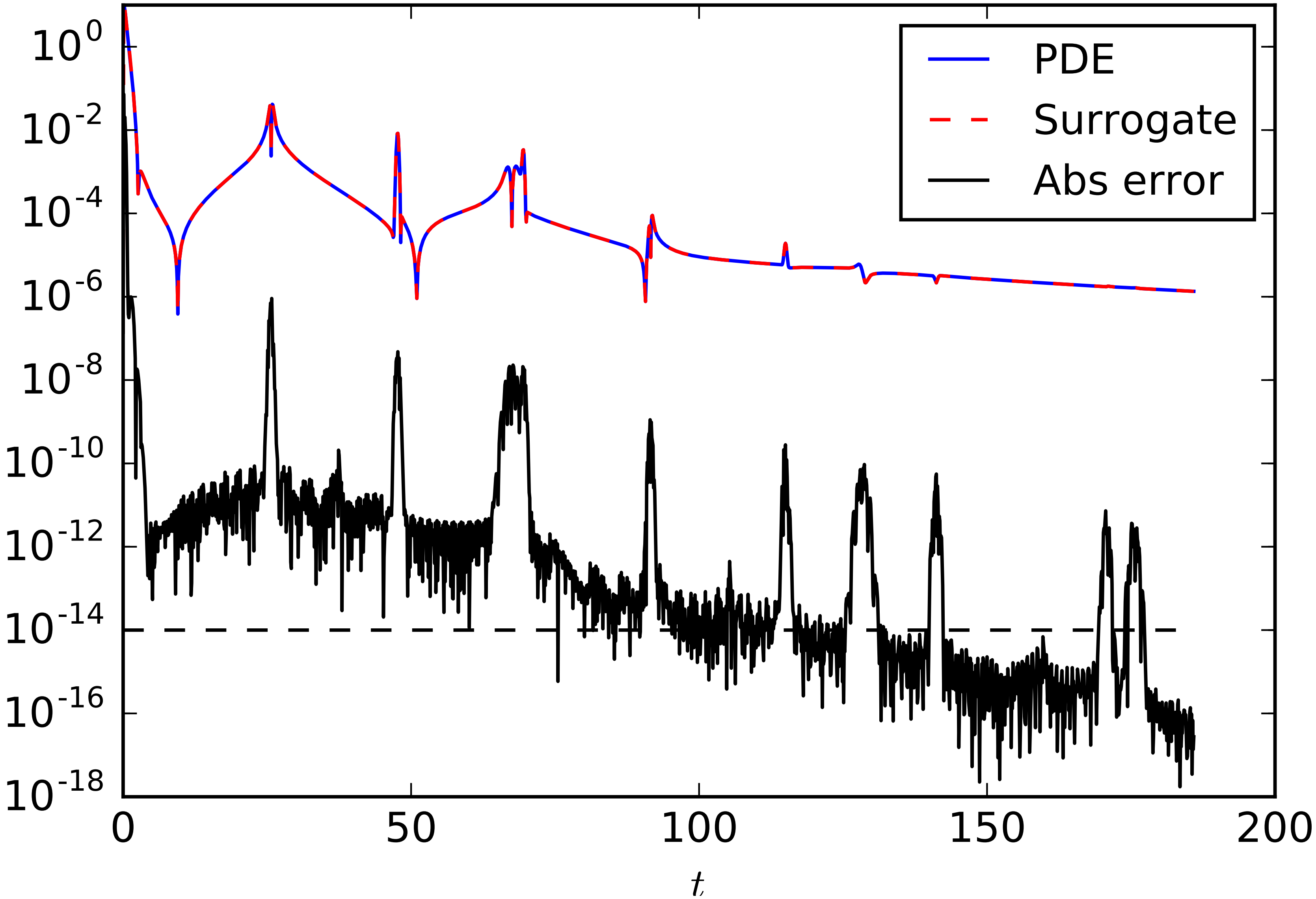

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Physical memory: 30GB reduced to 2GB

Surrogate self-force evaluation

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\begin{aligned}
&G_{\rm ret}(z^{\mu}, z^{\mu'}) \\
&\approx \theta(\tau - \tau_{\rm ql})\mathrm{Pade}[V_{\rm ql}(z^{\mu}, z^{\mu'})] \\
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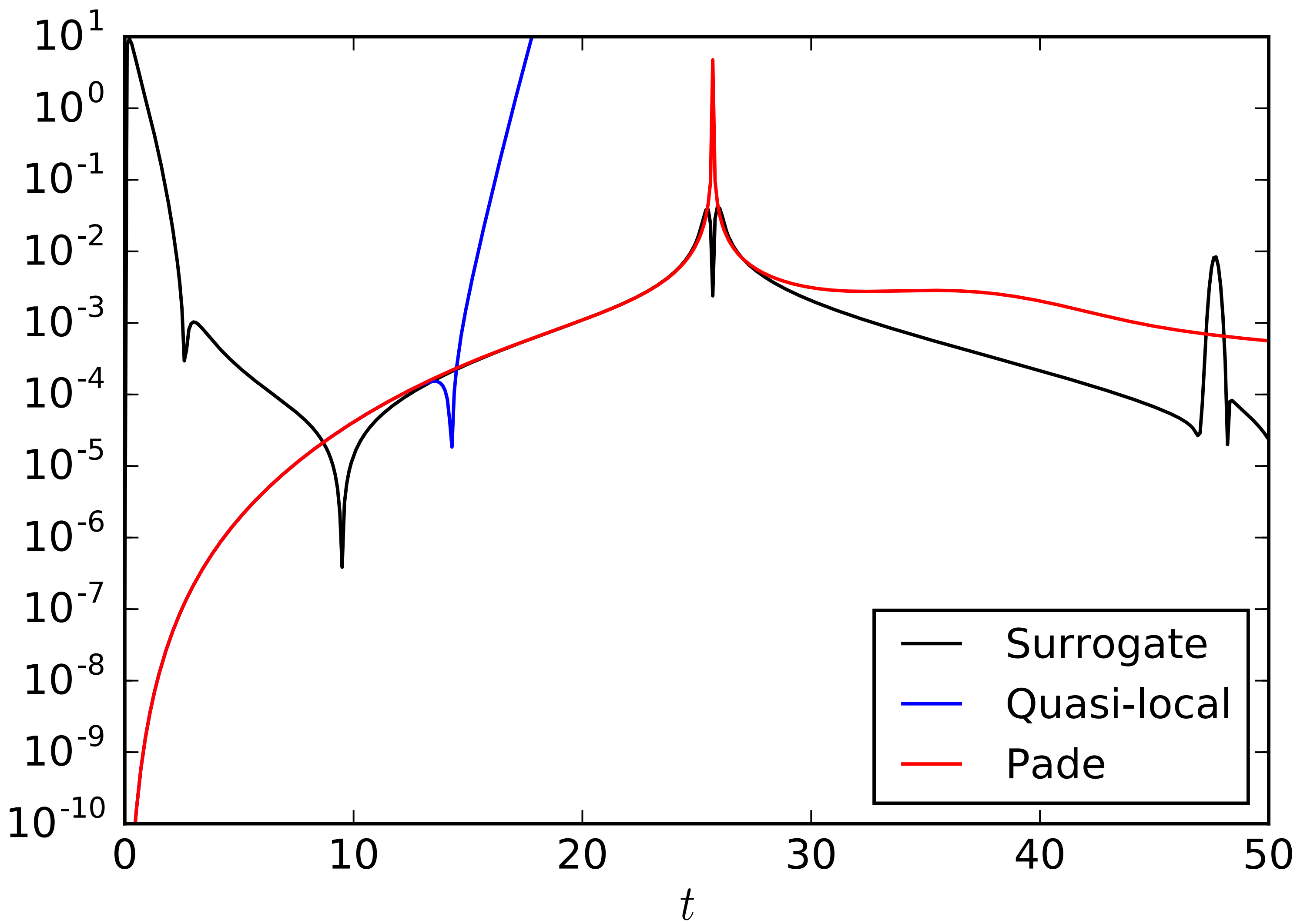
- Method of matched expansions
  Anderson & Wiseman (05); Casals et al (13)
- Quasi-local expansions
  Ottewill & Wardell (08); Wardell's thesis
- Pade approximants *Casals et al (09)*



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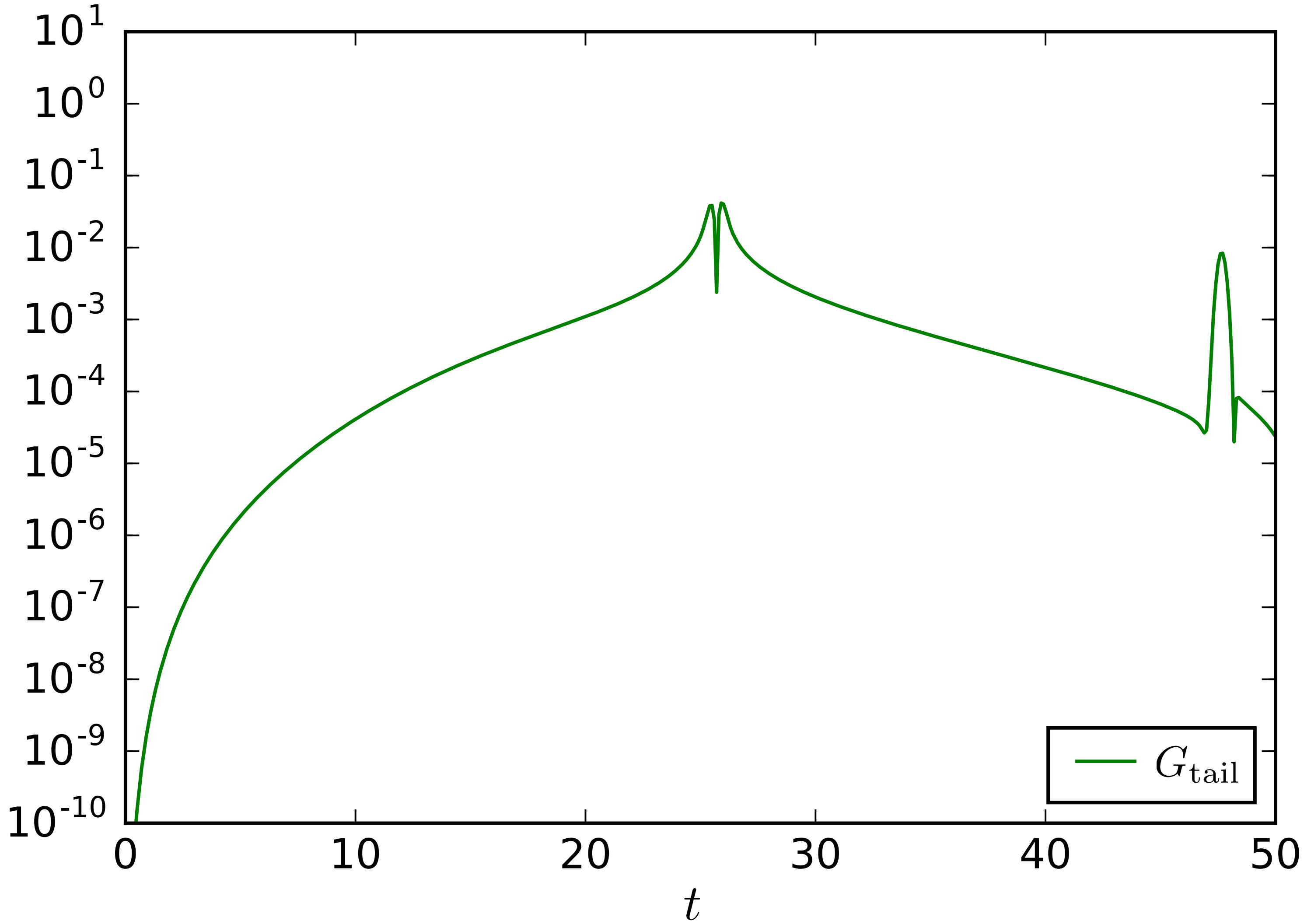

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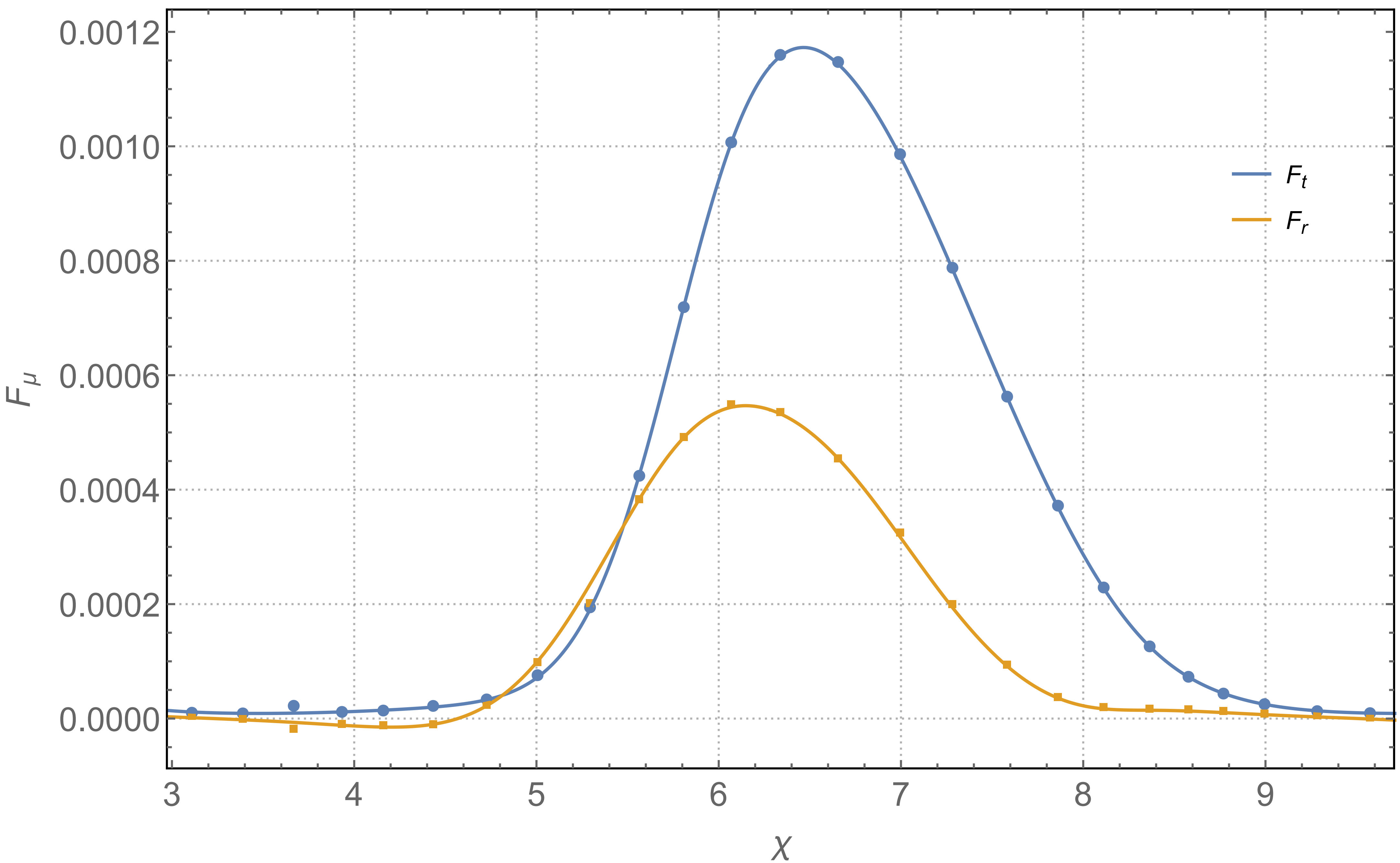

Applications

- Higher-order self-force and radiation *CRG (12a, 12b)*

$$ma^\mu \supset P^{\mu\nu}\left(\lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau'\,\nabla_\nu G_{\rm ret}(z^\mu, z^{\mu'})\right)\left(\lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau''\,G_{\rm ret}(z^\mu, z^{\mu''})\right),$$

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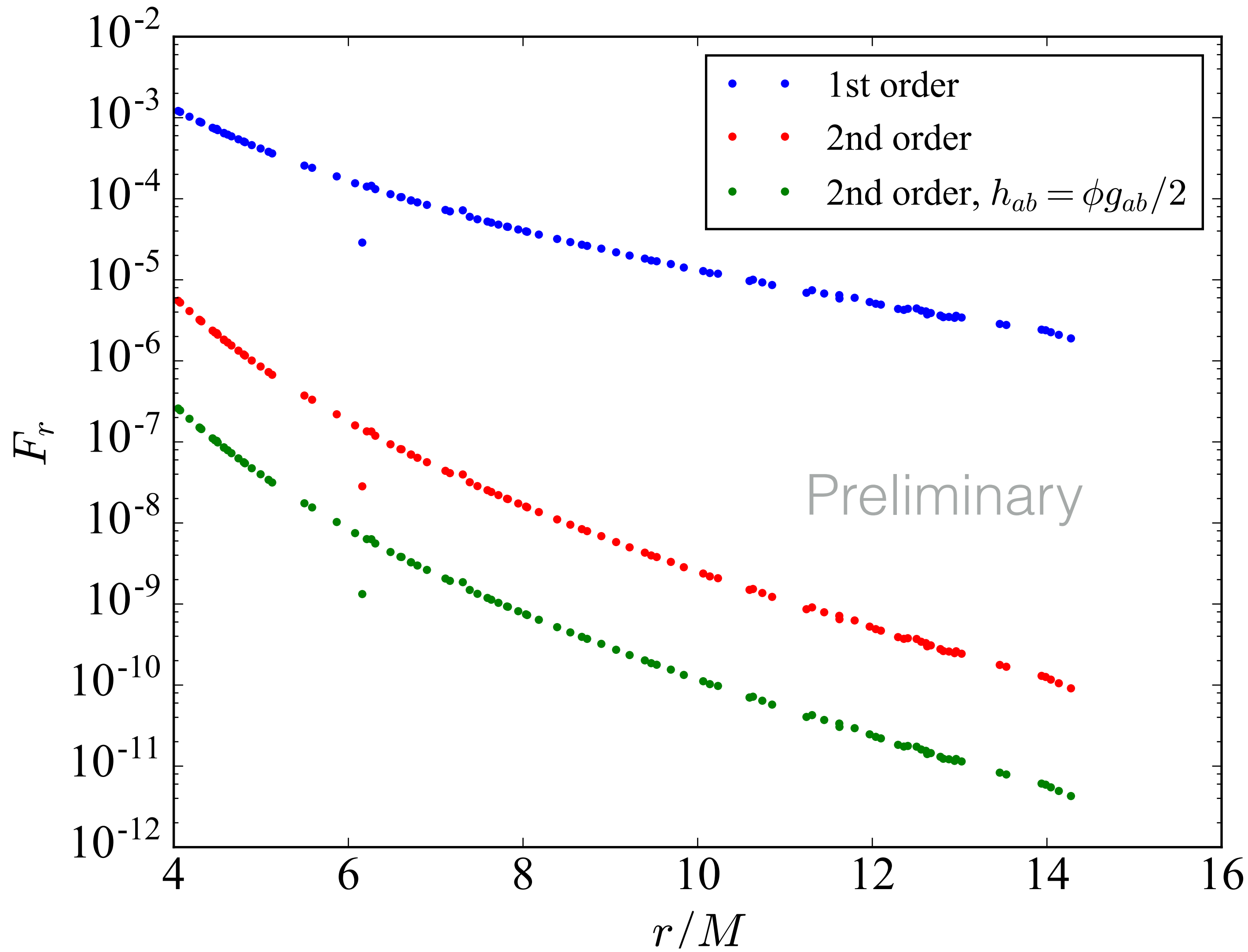

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- Self-consistent evolution

$$\begin{aligned}\frac{d^2 z^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dz^\alpha}{d\tau}\frac{dz^\beta}{d\tau} \approx q^2 P^{\mu\nu}\int_{-\infty}^{\tau} d\tau' \Big\{ &\theta(\tau' - \tau_{\rm ql}){\rm Pade}\big[\nabla_\nu V_{\rm ql}(z^\mu, z^{\mu'})\big] \\ &+ \theta(\tau_{\rm ql} - \tau')\theta(\tau' - \tau_{\rm br})\nabla_\nu G_{\rm surr}(z^\mu, z^{\mu'}) \\ &+ \theta(\tau_{\rm br} - \tau')G_{\rm br}(z^\mu, z^{\mu'})\Big\} + \text{local terms}\end{aligned}$$

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- Higher-order, self-consistent evolutions

- Self-consistent field/waveform and at higher orders

$$\phi(x^\alpha) = q\int_{-\infty}^{\tau_{\rm ret}(x)} d\tau' \Big\{\theta(\tau'-\tau_{\rm br})G_{\rm surr}(x, z^{\mu'}) + +\theta(\tau_{\rm br}-\tau')G_{\rm br}(x, z^{\mu'})\Big\}$$

- Comparing errors in osculating orbits and self-consistent evolutions (via two derivatives of the Green's function) *Pound (unpublished)*

$$F^{\mu}_{\mathrm{hist}}(\tau) = q^2 P^{\mu\nu} \lim_{\epsilon\to 0^+} \int_{-\infty}^{\tau-\epsilon} d\tau' \, \nabla_\nu G_{\mathrm{ret}}(z^\mu, z^{\mu'})$$

- Studying and visualizing basic wave propagation in black hole spacetimes

- Many similar applications in gravity plus others (e.g., NS-BH inspirals)

Summary & Outlook

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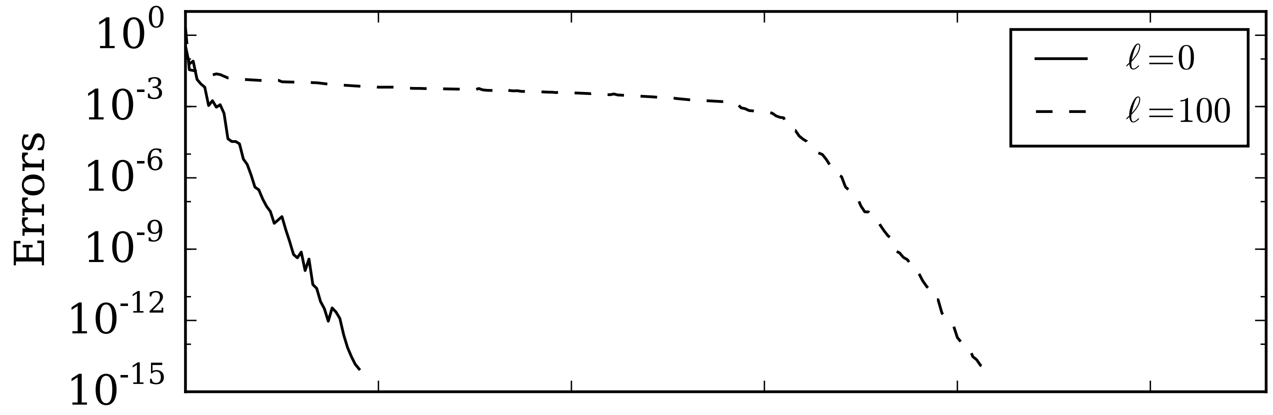
- Extending to Kerr spacetime is straightforward but may involve (much?) larger data sets because of extra parameters and reduced symmetry

- How to compute Green's function for gravitational perturbations?
  - Lorenz gauge has unstable non-radiative modes…
  - Accuracy and speed of "metric" reconstruction from curvature scalars?

Extra slides

Improving the surrogate building strategy

The plateau in the max projection errors often hints that a different representation of the data may generate a more compact basis



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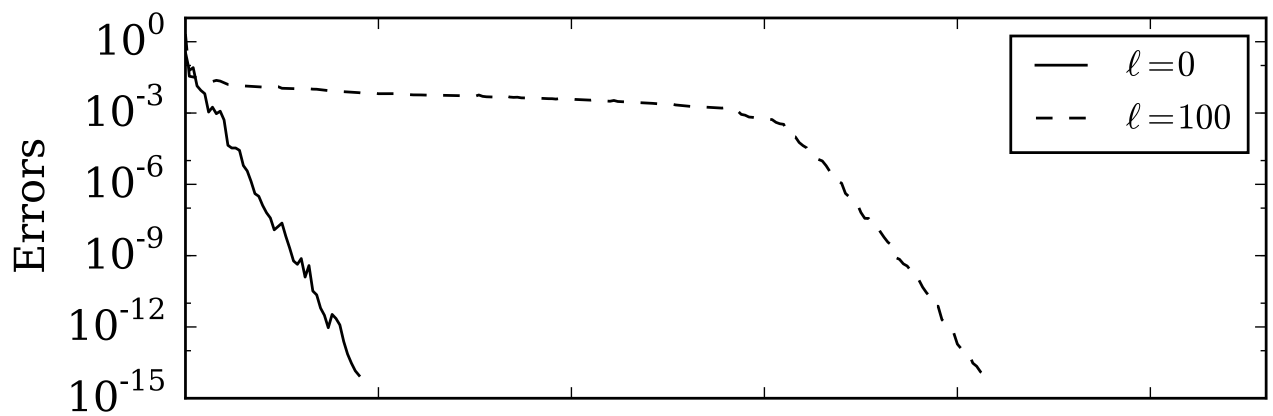

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  - "Rippling" is a problem
  - Phase at initial times is difficult to estimate
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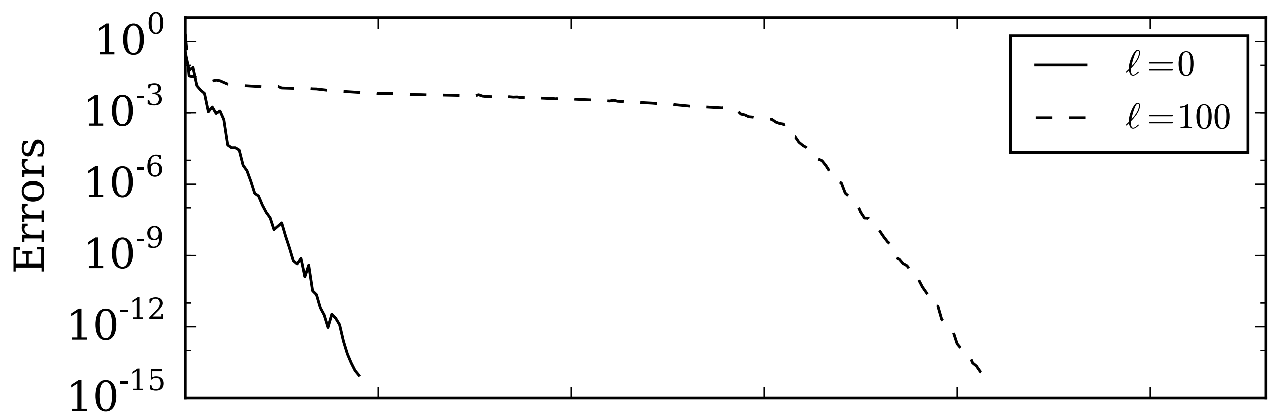

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- Some other way to represent the data?

Different and useful ways to parametrize the data?

- A more "natural" parametrization might be $\lambda = r'_*$ and regard (t, r_*) as the physical dimension

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Maybe try "invasive" approaches that project the wave equation onto the small vector space spanned by the basis