# Scalar field critical collapse in 2+1

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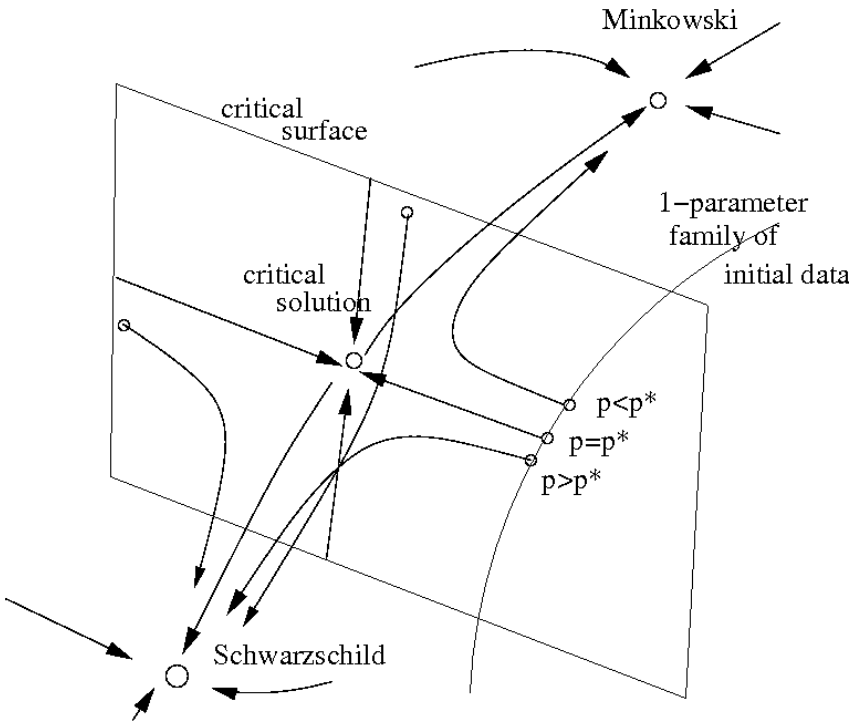
- Fine-tune to threshold of collapse
- (Approximately) scale-invariant physics
  - Arbitrarily small black hole mass
  - Universality and critical exponents
- Critical solution: self-similar with exactly one growing mode



- Continuous self-similarity in adapted coordinates $x^\mu = (\tau, x^i)$:

$$g_{\mu\nu}(\tau, x^i) = e^{-2\tau}\tilde{g}_{\mu\nu}(x^i)$$

- $\tau$ indicates scale:

$$\text{any length} \sim e^{-\tau}, \qquad Ricc \sim e^{2\tau}, \qquad M \sim e^{-(d-2)\tau}$$

- but $\tau$ can also be time

- Near the critical solution $\phi_*$:

$$\begin{aligned}\phi(x,\tau) &\simeq \phi_*(x) + (p-p_*)\, e^{\lambda_0 \tau}\, \phi_0(x) + \text{decaying modes} \\ &\simeq \phi_*(x) \pm \phi_0(x) \quad \text{when AH forms/dispersion starts}\end{aligned}$$

- This happens at some $\tau_*(p)$ defined by

$$(p-p_*)\, e^{\lambda_0 \tau_*} = 1$$

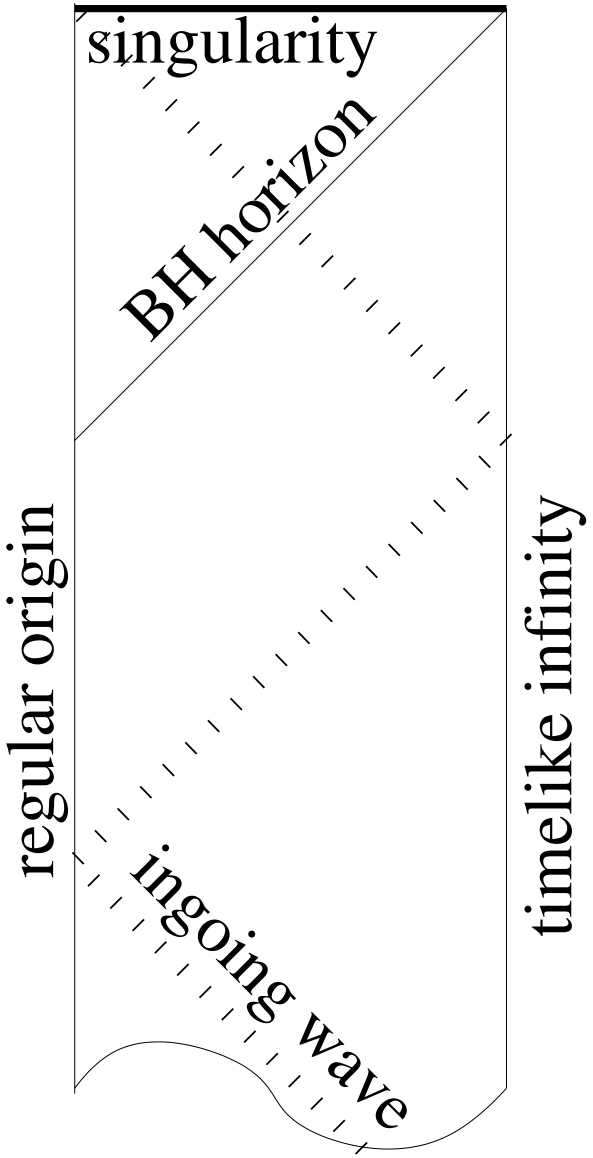
- Hence for $p > p_*$

$$M_{\rm BH}(p) \sim e^{-\tau_*} \sim (p-p_*)^{\frac{1}{\lambda_0}}$$

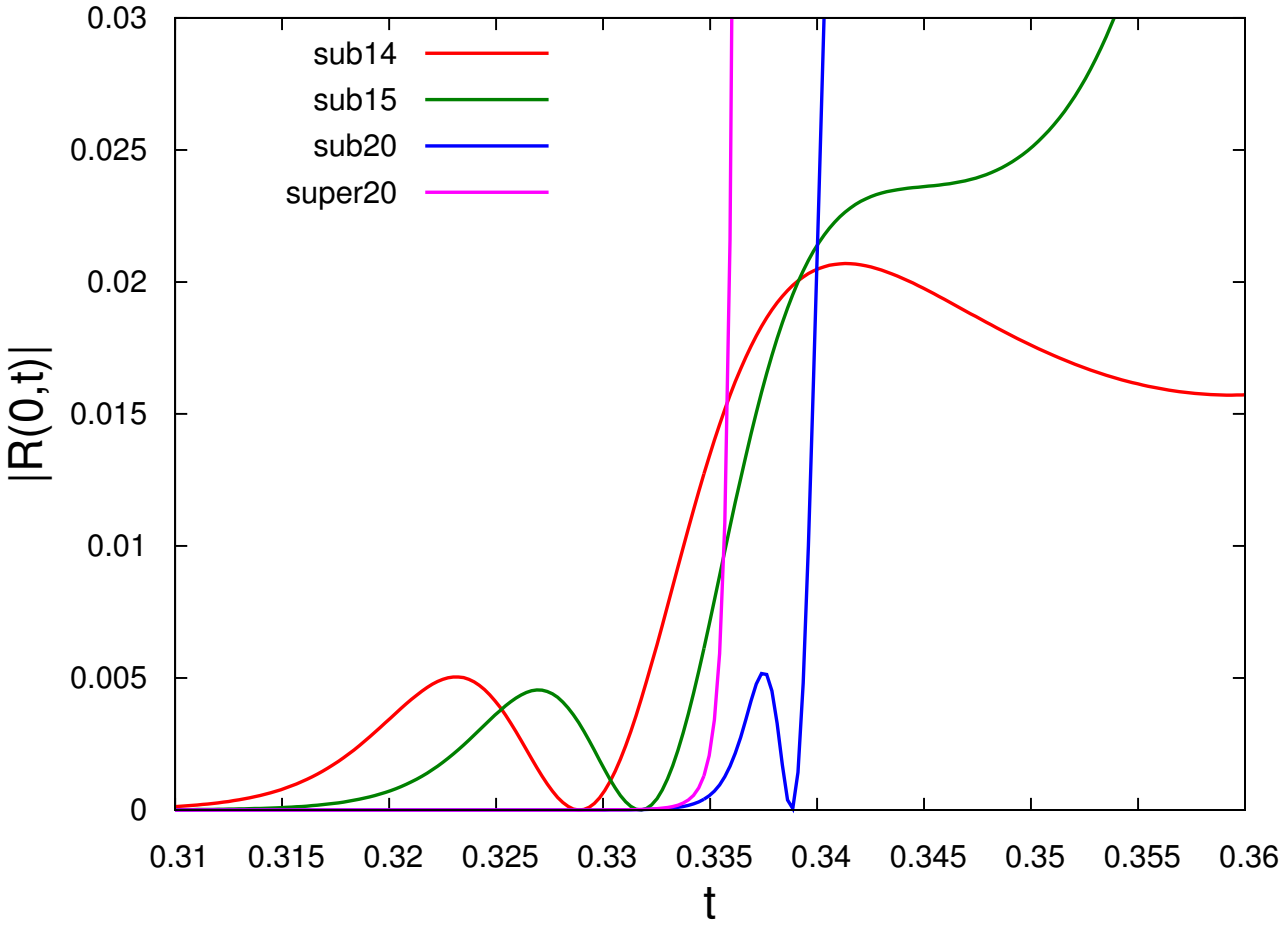
and for $p < p_*$

$$Ricc_{\max}(p) \sim e^{2\tau_*} \sim (p_*-p)^{-\frac{2}{\lambda_0}}$$

- No gravitational waves
- No collapse without $\Lambda < 0$ (but then no exact scale invariance)
- $M$ is dimensionless
- Mass gap: vacuum adS has $M = -1$, black holes have $M > 0$
- Timelike infinity (scalar field has reflecting boundary condition)

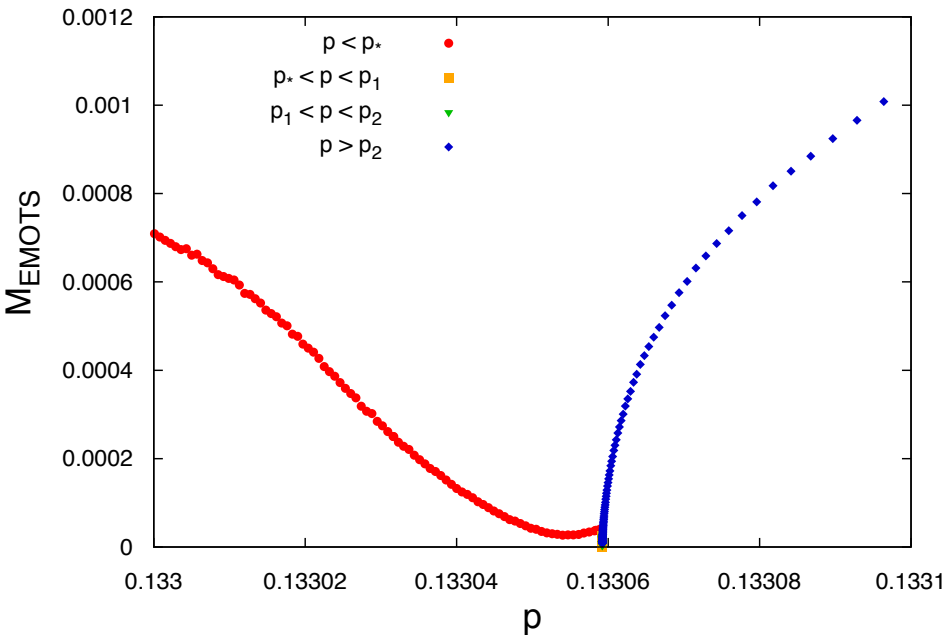


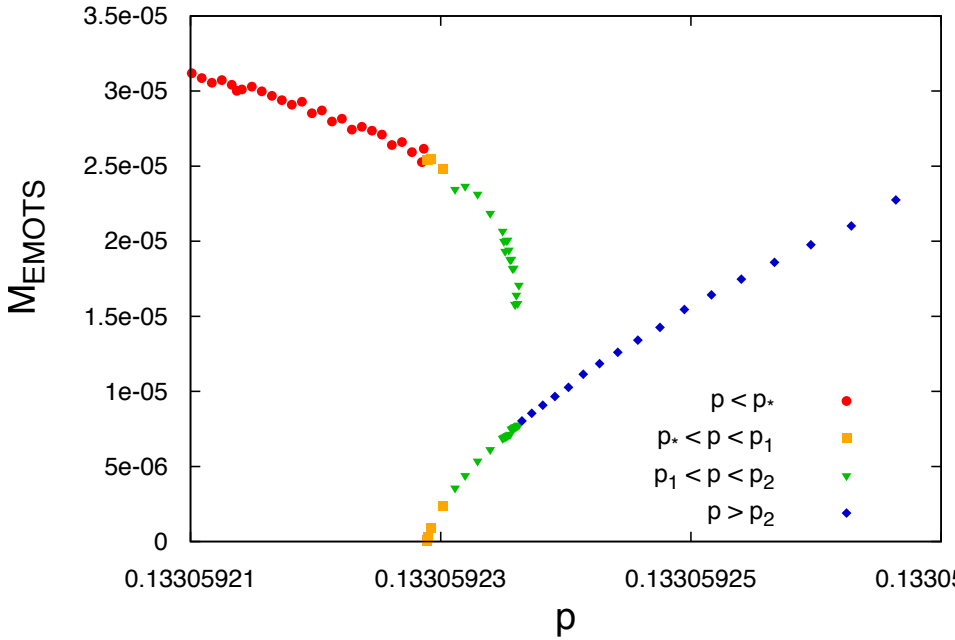
Definition of $p > p_*$: immediate blowup of Ricci at the origin.




All maxima and minima and their proper time locations scale

## EMOTS mass versus $p$



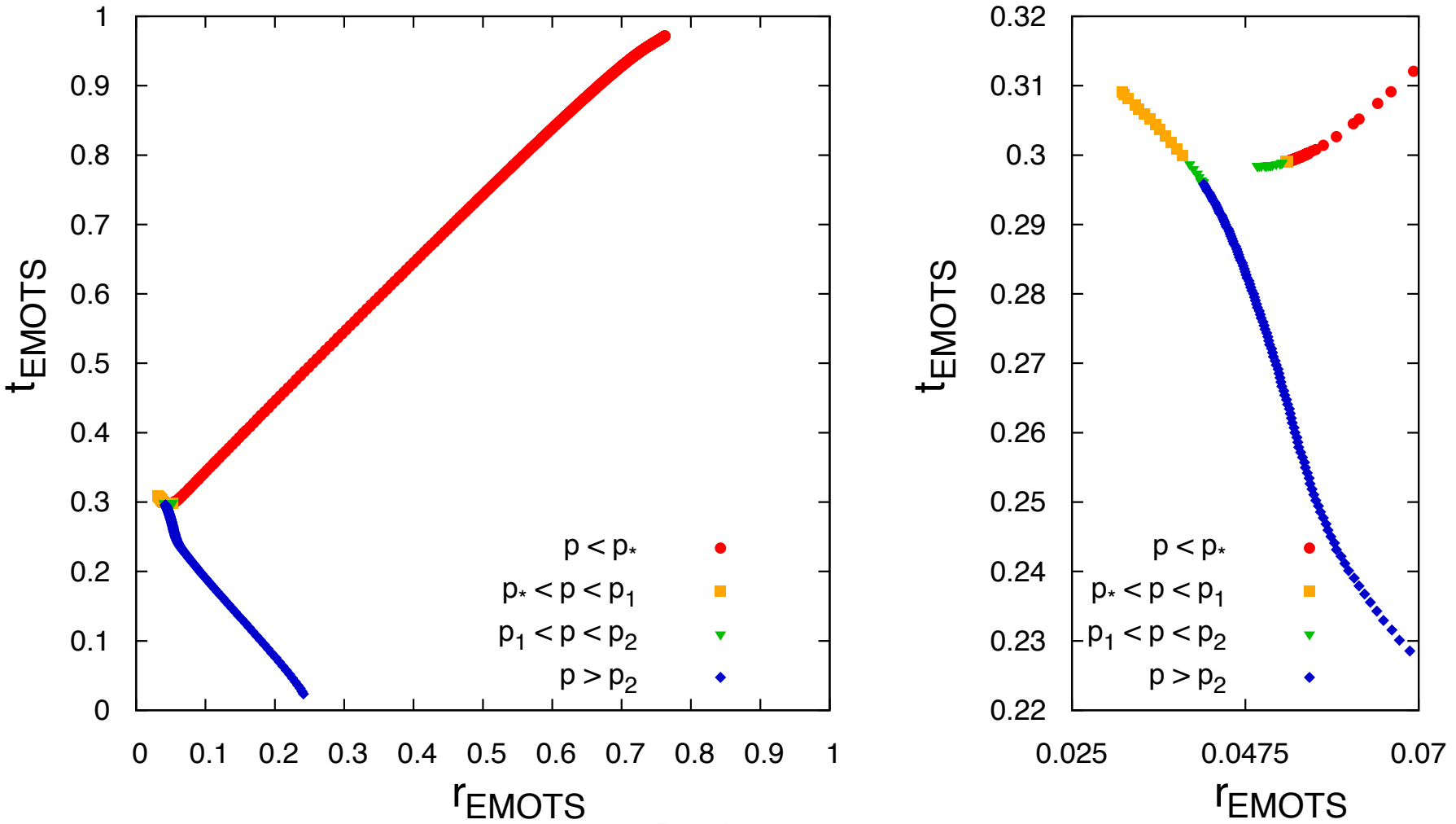

Right plot is a closeup.

Right curve in both plots is

$$M \sim (p - p_*)^{0.68}$$

# Earliest Marginally Outer-Trapped Surface

Light rays at $\pm 45^\circ$




Right plot is a closeup

- Approximate the critical solution by two $\Lambda = 0$ solutions:
- inside the lightcone: self-similar (Garfinkle, three growing modes, lightcone marginally trapped)
- outside the light cone: "outgoing Vaidya" (every point marginally trapped)
- ... each plus corrections in powers of $\Lambda$ (leads to untrapping)
- **Conjecture** this approximates an **analytic** solution with **one** growing mode

- Modulo this conjecture:
- Ricci scaling standard from the unstable mode ($\lambda_0 = 7/8$)

$$Ricc_{\max} \sim (p_* - p)^{16/7}$$

- Mass scaling from competion of the unstable mode (wants to trap) with first $\Lambda$ correction (wants to untrap)

$$M \sim (p - p_*)^{16/23}$$