Light bending in superrenormalizable higher-derivative gravity

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Based in collaboration with Antonio Accioly and Ilya L. Shapiro:

*Accioly, Giacchini & Shapiro (2016) arXiv:*1604.07348

Accioly et al., PRD ***91*** *(2015) arXiv:*1506.00270

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Contents:

1. Superrenormalizable higher-derivative gravity (HDG)
2. Gravitational “seesaw”?
3. Light bending in semiclassical HDG
4. Conclusions

Conventions: $\hbar = c = 1$; Minkowski metric: $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$.

“On the gravitational seesaw and light bending”
Accioly, Giacchini & Shapiro, 2016, arXiv:1604.07348

1. Superrenormalizable higher-derivative gravity

- General Relativity (GR): Einstein, 1915
- Solar eclipse, 1919: light bending
- Higher-derivative gravity (HDG):

$$\mathcal{L} = \sqrt{-g}\left[\frac{2R}{\kappa^2} + \frac{\alpha}{2}R^2 + \frac{\beta}{2}R_{\mu\nu}^2\right]$$

$\kappa^2 = 32\pi G$

  - Renormalizable but non-unitary

    [Stelle, *PRD*, 1977]

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

New York Times, 11/19/1919

- Superrenormalizable HDG

[Asorey, López & Shapiro, *Int. Jour. Mod. Phys. A*, 1997]

$$\begin{aligned} S &= \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda) + \int d^4x\sqrt{-g}\Big\{c_1 R^2_{\mu\nu\alpha\beta} + c_2 R^2_{\mu\nu} + c_3 R^2 \\ &+ d_1 R_{\mu\nu\alpha\beta}\Box R^{\mu\nu\alpha\beta} + d_2 R_{\mu\nu}\Box R^{\mu\nu} + d_3 R\Box R + d_4 R^3 + d_5 R R^{\mu\nu} R_{\mu\nu} + \dots \\ &+ f_1 R_{\mu\nu\alpha\beta}\Box^k R^{\mu\nu\alpha\beta} + f_2 R_{\mu\nu}\Box^k R^{\mu\nu} + f_3 R\Box^k R + \dots + f_{\dots} R^{k+2}_{\dots}\Big\}, \end{aligned}$$

- If *k* is odd ≥ 1 it is possible to have all massive poles complex, and S-matrix becomes unitary in the Lee-Wick sense. [Modesto, Shapiro, *PLB* 2016]

- However, even with real massive poles, ghost instabilities may be put under control by other mechanisms close to the Planck scale.

[Salles & Shapiro, *PRD* 2014; Shapiro, Pelinson & Salles, *MPLA* 29, 2014]

- We shall assume that higher derivatives exist, and look for its consequences even at the low-energy realm.

- For small curvatures we may write

$$S = S_{grav} + \int d^4x\sqrt{-g}\,\mathcal{L}_m\,,$$

$$S_{grav} = \int d^4x\sqrt{-g}\left\{\frac{2}{\kappa^2}R + \frac{\alpha}{2}R^2 + \frac{\beta}{2}R_{\mu\nu}^2 + \frac{A}{2}R\Box R + \frac{B}{2}R_{\mu\nu}\Box R^{\mu\nu}\right\}$$

- String theory prescribes $A^{-1},\ B^{-1},\ \kappa^{-2} \sim M_P^2$

- Would it be possible to have a seesaw-like mechanism in this theory, so that huge-mass parameters κ^2, A^{-1}, B^{-1} combine into a small physical mass?

- What are observable consequences of a small-mass excitation?

- How does light deflect in presence of massive complex poles in the propagator?

2. Gravitational seesaw

- Solution of linearized field equations sourced by a point-like mass M in rest:

$$h_{00} = \frac{M\kappa}{16\pi}\Big(-\frac{1}{r}+\frac{4}{3}F_2-\frac{1}{3}F_0\Big),$$
$$h_{11} = h_{22} = h_{33} = \frac{M\kappa}{16\pi}\Big(-\frac{1}{r}+\frac{2}{3}F_2+\frac{1}{3}F_0\Big)$$

with $$F_k = \frac{m_{k+}^2}{m_{k+}^2-m_{k-}^2}\frac{e^{-m_{k-}r}}{r}+\frac{m_{k-}^2}{m_{k-}^2-m_{k+}^2}\frac{e^{-m_{k+}r}}{r}$$

k = 0,2 labels the spin of the particles whose masses are

$$m_{2\pm}^2 = \frac{\beta \pm \sqrt{\beta^2+\frac{16}{\kappa^2}B}}{2B}, \quad m_{0\pm}^2 = \frac{\sigma_1 \pm \sqrt{\sigma_1^2-\frac{8\sigma_2}{\kappa^2}}}{2\sigma_2}$$

$$\sigma_1 = 3\alpha+\beta, \; \sigma_2 = 3A+B$$

2. Gravitational seesaw

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$$\sigma_1 = 3\alpha+\beta,\ \sigma_2 = 3A+B$$

- If $16|B| \ll \kappa^2\beta^2$ then $m_{2+}^2 \approx \frac{4}{\kappa^2|\beta|} \ll m_{2-}^2 \approx \frac{\beta}{B}$

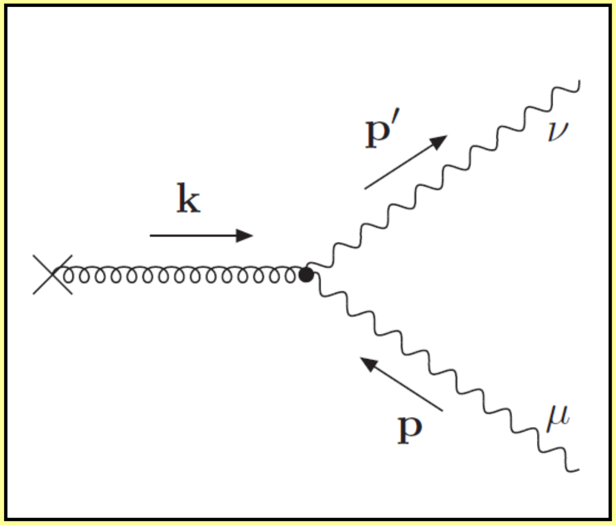
- In order to have m_{2-} on the order of Planck mass:
  - i. $\beta \sim 1$ and $|B| \sim M_P^{-2}$
  - ii. $\beta \gg 1$ and $|B| \gg M_P^{-2}$ (on account of reducing m_{2+})

Only "weak" seesaw is possible: making a large mass even larger by increasing the dimensionless parameter β.

- This result can be generalized to actions with arbitrary order on the operator $\Box$

Small physical mass only with small massive parameters on the action. Lighter mass cannot be reduced by tuning the coefficients of the sixth- (or more-) derivative terms.

3. Light bending in semiclassical SHDG



- Gravitational field: external and classical
- Vertex function:

$$\begin{aligned} V_{\mu\nu}(p,p') &= \frac{\kappa}{2} h^{\lambda\rho}_{\text{ext}}(\mathbf{k}) \big[-\eta_{\mu\nu}\eta_{\lambda\rho}\, p\cdot p' + \eta_{\lambda\rho} p'_\mu p_\nu \\ &+ 2\left(\eta_{\mu\nu} p_\lambda p'_\rho - \eta_{\nu\rho} p_\lambda p'_\mu - \eta_{\mu\lambda} p_\nu p'_\rho + \eta_{\mu\lambda}\eta_{\nu\rho}\, p\cdot p'\right) \big] \end{aligned}$$

where $h^{\lambda\rho}_{\text{ext}}(\mathbf{k}) = \int d^3\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} h^{\lambda\rho}_{\text{ext}}(\mathbf{r})$

- Unpolarized cross-section for small angles:

$$\frac{d\sigma}{d\Omega} = 16 G^2 M^2 \left[\frac{1}{\theta^2} + \frac{E^2}{m^2_{2-} - m^2_{2+}} \left(\frac{m^2_{2+}}{E^2\theta^2 + m^2_{2-}} - \frac{m^2_{2-}}{E^2\theta^2 + m^2_{2+}} \right) \right]^2$$

- ➢ Scalar modes do not affect light propagation
- ➢ Dispersive propagation of photons

1st scenario: real poles

- Unpolarized cross-section for small angles:

$$\frac{d\sigma}{d\Omega} = 16G^2M^2\left[\frac{1}{\theta^2} + \frac{E^2}{m_{2-}^2 - m_{2+}^2}\left(\frac{m_{2+}^2}{E^2\theta^2 + m_{2-}^2} - \frac{m_{2-}^2}{E^2\theta^2 + m_{2+}^2}\right)\right]^2$$

- Since $m_{2-} > m_{2+}$, light deflects *less* than in GR
- More energetic photons undergo less deflection
  - $E \gg m_{2-}$: no deflection at all
  - $E \ll m_{2+}$: deflects like in GR
  - Nontrivial scattering for intermediate energies
    - Seesaw (huge β): $m_{2-} \gg m_{2+} \sim E$

- If $m_{2-} \gg m_{2+} \sim E$

$$\frac{1}{\theta_{GR}^2} = \frac{1}{\theta^2} + \frac{1}{\theta^2 + \frac{m_{2+}^2}{E^2}} + \frac{2E^2}{m_{2+}^2} \ln\left(\frac{\theta^2}{\theta^2 + \frac{m_{2+}^2}{E^2}}\right)$$

  - Fix E and solve for $\theta = \theta(m_{2+})$
  - *Fix θ and solve for E/m_{2+}*

For a photon grazing the Sun

- If $m_{2-} \gg m_{2+} \sim E$

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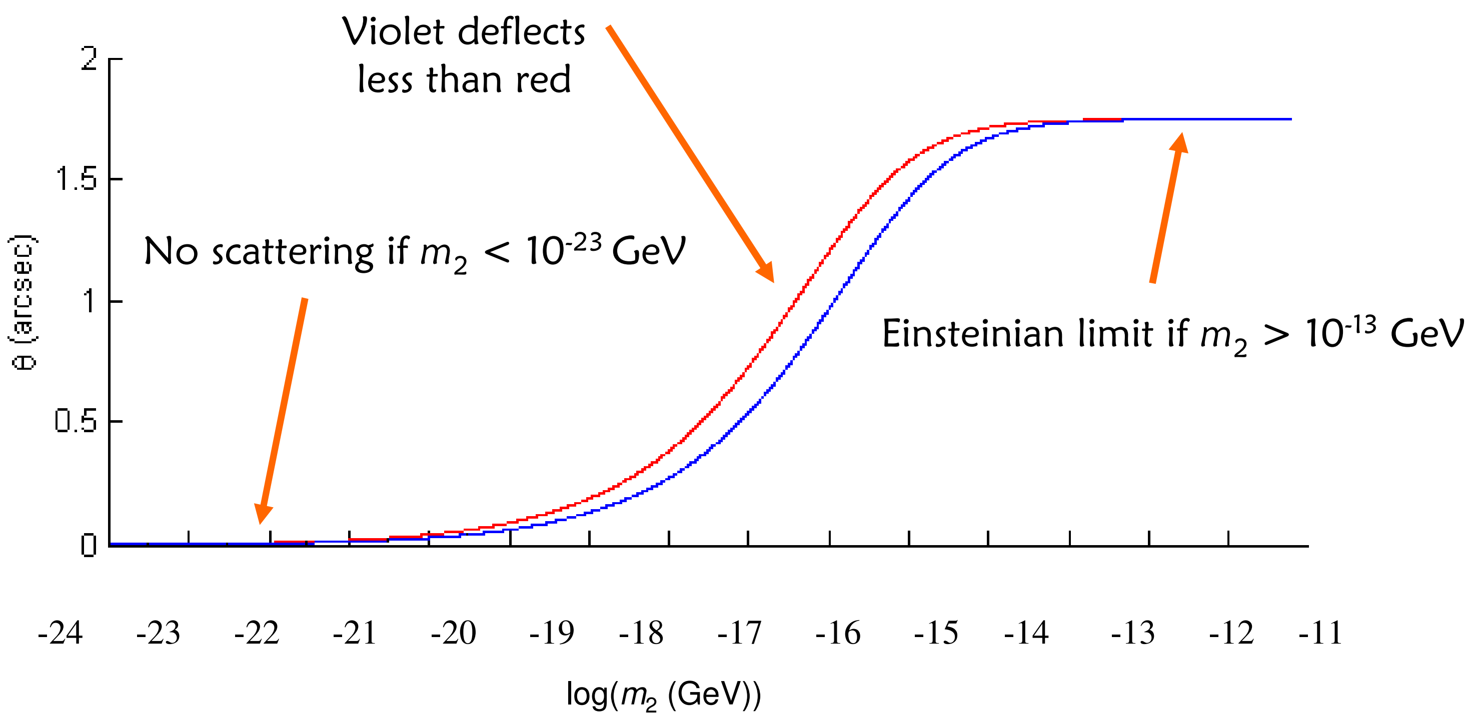

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- *Fix θ and solve for E/m_{2+}* – for a photon grazing the sun

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- For $\theta = \theta_{GR} - 0.1''$: $\boxed{\frac{m_{2+}^2}{E^2} = 4.30 \times 10^{-9}}$

Only one order of magnitude smaller than figures from torsion-balance experiments

- Radio (10 GHz): $m_2 > 10^{-18}$ GeV
- Visible (7.4×10^{15} Hz): $m_2 > 10^{-13}$ GeV
- Crab pulsar (10^{27} Hz = 10 TeV): $m_2 > 10^{-1}$ GeV

Planck mass $\sim 10^{19}$ GeV

2nd scenario: complex poles

$$m_{2+} = (a_2 - ib_2), \quad m_{2-} = (a_2 + ib_2)$$

- Unpolarized cross-section for small angles:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{C}} = 16G^2M^2\left[\frac{1}{\theta^2} - \frac{E^4\theta^2 + 2E^2(a_2^2 - b_2^2)}{\left(E^2\theta^2 + a_2^2 - b_2^2\right)^2 + 4a_2^2b_2^2}\right]^2$$

- If $a_2 \geq b_2$: light bend less than in GR, more energetic photons undergo less scattering.

- If $a_2 < b_2$: cross section is no longer bounded by GR's one. Stronger bending is possible, but is suppressed by the massive parameters a_2 and b_2. [Accioly, Giacchini & Shapiro, *paper in preparation*]

4. Conclusions

- Only weak seesaw is possible in HDG: to have a much lighter tensor ghost one must have small massive parameters on the action – and a huge β.
- This protects the theory from ghost instabilities by adding higher-order terms – but makes detection of higher derivatives more difficult.
- Scattering of photons is dispersive, ghosts yield repulsive forces.
- Bound on the lighter tensor excitation mass: from 4th order gravity (or "seesaw" effect): $m_2 > 10^{-13}$ GeV. [Accioly et. al, *PRD*, 2015]
- Figure comparable to torsion-balance experiments; 13 orders of magnitude greater than previous literature results. [Stelle, *Gen. Relativ. Gravit.*, 1978; Giacchini, *Proc. 14th Marcel Grossmann Meeting* (to appear, 2016)]
- More stringent bound for more energetic photons.
- Dispersive gravitational lensing can provide important information about higher derivatives in gravity.

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