Double layers in quadratic theories of gravity

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Outline

1. **Motivation and setting**
2. **Concentrated sources in gravity: General Relativity**
3. **Concentrated sources in gravity: Quadratic gravity**

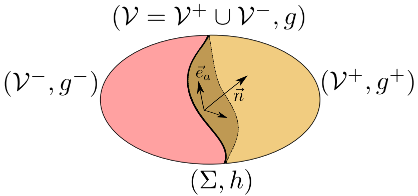
Outline

Motivation

- We want to represent very concentrated sources of matter/ energy in gravity theories.
- These model thin shells of matter, braneworlds, impulsive waves, ...
- To treat properly these objects, one has to resort to theory of distributions:
  - The sums of distributions, the derivative of a distribution, and the tensor product of a tensor field with a tensor distribution are well defined.
  - In general, the product of distributions is not well defined.
- We will make use of a class of metrics that generate a distributional curvature tensor, so that the field equations make sense.
  - Smooth metrics, except on a localized hypersurface where they are only continuous.

Israel (1966), Taub (1980), Clarke and Dray (1987), Mars and Senovilla (1993)

Setting



- The (timelike) hypersurface Σ splits $\mathcal{V}$ into $\mathcal{V}^\pm$.
- $\vec{n}$: (spacelike) unit normal vector to Σ. ($\boldsymbol{n}$: normal one form)
  $\vec{e}_a$: basis of vectors tangent to Σ. ($\boldsymbol{\omega}^a$: dual basis)
- In $\mathcal{V}^\pm$, the metric g agrees with $g^\pm$ and it is smooth.
- The metric g is only continuous across Σ. The induced metric is $h_{\alpha\beta} = g_{\alpha\beta}|_\Sigma - n_\alpha n_\beta$, and $\overline{\nabla}$ its associated covariant derivative.
- The jump/discontinuity of f is denoted by

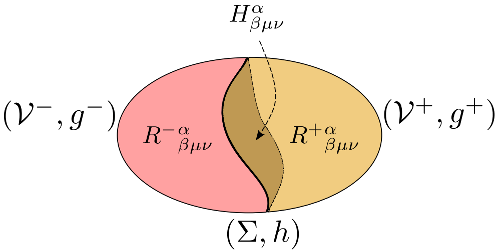
$$\forall q \in \Sigma, \qquad [f](q) \equiv \lim_{\substack{x \to q \\ \mathcal{V}^+}} f^+(x) - \lim_{\substack{x \to q \\ \mathcal{V}^-}} f^-(x) \ .$$

- We define $f|_\Sigma := (1/2)(\lim_{\substack{x \to q \\ \mathcal{V}^+}} f^+(x) + \lim_{\substack{x \to q \\ \mathcal{V}^-}} f^-(x))$.

Concentrated sources in gravity: geometry

- Metric: $g = g^+\theta + g^-(1-\theta), \quad \underline{g} = g^+\,\underline{\theta} + (\underline{1} - \underline{\theta})\,g^-.$
- Riemann tensor distribution

$$\underline{R}^{\alpha}_{\beta\mu\nu} \quad = \quad R^{+\,\alpha}_{\ \ \beta\mu\nu}\,\underline{\theta} + R^{-\,\alpha}_{\ \ \beta\mu\nu}\,(\underline{1} - \underline{\theta}) + H^{\alpha}_{\beta\mu\nu}\,\delta^{\Sigma},$$



Second fundamental form: $\kappa^{\pm}_{ab} := e^{\alpha}_{a} e^{\beta}_{b} \nabla^{\pm}_{\alpha} n_{\beta}, \quad \kappa^{\pm}_{\alpha\beta} = \omega^{a}_{\alpha}\omega^{b}_{\beta}\kappa^{\pm}_{ab}$

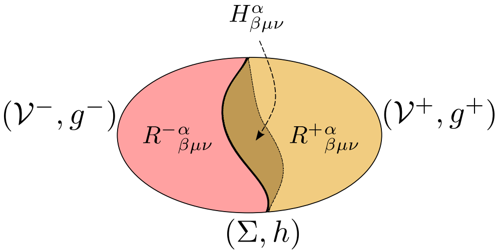
Singular part of the Riemann tensor distribution

$$H_{\alpha\beta\mu\nu} = n_{\alpha}([\kappa_{\beta\mu}]\,n_{\nu} - [\kappa_{\beta\nu}]\,n_{\mu}) + n_{\beta}([\kappa_{\alpha\nu}]\,n_{\mu} - [\kappa_{\alpha\mu}]\,n_{\nu})$$

Concentrated sources in gravity: geometry

- Metric: $g = g^+\theta + g^-(1-\theta)$, $\quad \underline{g} = g^+\,\underline{\theta} + (\underline{1} - \underline{\theta})\,g^-$.
- Riemann tensor distribution

$$\underline{R}^{\alpha}_{\beta\mu\nu} \quad = \quad {R^+}^{\alpha}_{\ \beta\mu\nu}\,\underline{\theta} + {R^-}^{\alpha}_{\ \beta\mu\nu}\,(\underline{1} - \underline{\theta}) + H^{\alpha}_{\beta\mu\nu}\,\delta^{\Sigma},$$



Second fundamental form: $\kappa^{\pm}_{ab} := e^{\alpha}_{a} e^{\beta}_{b} \nabla^{\pm}_{\alpha} n_{\beta}$, $\quad \kappa^{\pm}_{\alpha\beta} = \omega^{a}_{\alpha}\omega^{b}_{\beta}\kappa^{\pm}_{ab}$

At any point $x \in \Sigma$ where the hypersurface is non-null, the neccessary and sufficient condition for $H_{\alpha\beta\mu\nu}$ to vanish is that $\kappa_{\alpha\beta}$ is continuous across the hypersurface.

Concentrated sources in gravity: geometry

From contractions of the Riemann tensor distribution

- Ricci tensor distribution

$$\underline{R}_{\beta\nu} = R^{+}{}_{\beta\nu}\,\underline{\theta} + R^{-}{}_{\beta\nu}\,(\underline{1} - \underline{\theta}) + H_{\beta\nu}\,\delta^{\Sigma},$$

  with singular part $H_{\beta\nu} \equiv H^{\rho}{}_{\beta\rho\nu} = -[\kappa_{\beta\nu}] - [\kappa^{\alpha}_{\alpha}]n_{\beta}n_{\nu}$.

- Ricci scalar distribution

$$\underline{R} = R^{+}\,\underline{\theta} + R^{-}\,(\underline{1} - \underline{\theta}) + H\,\delta^{\Sigma},$$

  with singular part $H \equiv H^{\beta}{}_{\beta} = -2[\kappa^{\beta}_{\beta}]$.

$$\boxed{H = 0 \Leftrightarrow [\kappa^{\alpha}_{\alpha}] = 0.}$$

- Einstein tensor distribution

$$\underline{G}_{\beta\nu} = G^{+}{}_{\beta\nu}\,\underline{\theta} + G^{-}{}_{\beta\nu}\,(\underline{1} - \underline{\theta}) + \mathcal{G}_{\beta\nu}\,\delta^{\Sigma},$$

  with singular part $\mathcal{G}_{\beta\nu} \equiv H_{\beta\nu} - \frac{1}{2}g_{\beta\nu}H = -[\kappa_{\beta\nu}] - [\kappa^{\alpha}_{\alpha}]n_{\beta}n_{\nu}$.

Concentrated sources in gravity: geometry

Some properties of the singular part of the Einstein tensor $\mathcal{G}_{\alpha\beta}$:

$$\mathcal{G}_{\beta\nu} = -[\kappa_{\beta\nu}] + [\kappa^{\alpha}_{\alpha}]h_{\beta\nu},$$

$$n^{\beta}\mathcal{G}_{\beta\nu} = 0.$$

The Bianchi identities hold in the distributional sense *Mars, Senovilla (1993)*

$$\nabla^{\alpha}(\underline{G}_{\alpha\beta}) = 0.$$

This implies

$$\kappa^{\Sigma}_{\rho\sigma}\mathcal{G}^{\rho\sigma} = n^{\beta}n^{\mu}\left[G_{\beta\mu}\right], \quad \text{(normal)}$$

$$\overline{\nabla}^{\beta}\mathcal{G}_{\beta\mu} = -n^{\rho}h^{\sigma}{}_{\mu}\left[G_{\rho\sigma}\right]. \quad \text{(tangent)}$$

These properties and results are independent of the field equations, and therefore valid in GR, F(R), quadratic gravity,...

Outline

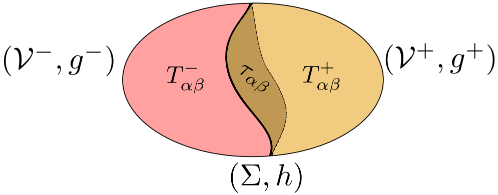
General Relativity

Field equations in **General Relativity** in the distributional sense

$$\underline{G}_{\alpha\beta} = 8\pi \underline{T}_{\alpha\beta}.$$

Hence the structure of the energy momentum tensor distribution must be

$$\underline{T}_{\alpha\beta} = T^{+}_{\alpha\beta}\underline{\theta} + T^{-}_{\alpha\beta}(\underline{1} - \underline{\theta}) + \tau_{\alpha\beta}\,\delta^{\Sigma}.$$



Singular part of the Energy Momentum tensor distribution

1. $n^{\alpha}\tau_{\alpha\beta} = 0$
2. Israel equations

$$h^{\sigma}{}_{\mu} n^{\rho}\,[T_{\rho\sigma}] + \overline{\nabla}^{\beta}\tau_{\beta\mu} = 0,$$
$$n^{\sigma} n^{\rho}\,[T_{\rho\sigma}] - \kappa^{\Sigma}_{\rho\sigma}\tau^{\rho\sigma} = 0.$$

General Relativity

Proper matching

Energy momentum tensor without singular terms

$$\tau_{\alpha\beta} = 0 \Leftrightarrow [\kappa_{ab}] = 0.$$

- Removes the singular part of the curvature tensor distribution.
- Restricts the possible jumps, encoded in $B_{\alpha\beta} = B_{\beta\alpha}$, with $n^\alpha B_{\alpha\beta} = 0$

$$\begin{aligned}
&[R_{\alpha\beta\mu\nu}] = B_{\beta\nu} n_\alpha n_\mu + B_{\alpha\mu} n_\beta n_\nu - B_{\alpha\nu} n_\beta n_\mu - B_{\beta\mu} n_\alpha n_\nu, \\
&[R_{\alpha\beta}] = B^\rho_\rho n_\alpha n_\beta + B_{\alpha\beta}, \\
&[R] = 2B^\alpha_\alpha, \\
&[G_{\alpha\beta}] = B_{\alpha\beta} - B^\rho_\rho h_{\alpha\beta} \Rightarrow \quad n^\alpha [G_{\alpha\beta}] = 0.
\end{aligned}$$

In particular:

Israel conditions

$$n^\alpha [T_{\alpha\beta}] = 0.$$

Outline

Quadratic gravity

Theories of gravity arising from the Lagrangian density ($k := 8\pi G/c^2$):

$$\mathcal{L} = \frac{1}{2k}\left(R - 2\Lambda + a_1 R^2 + a_2 R_{\mu\nu}R^{\mu\nu} + a_3 R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right) + \mathcal{L}_{matter}.$$

Field equations contain higher order derivatives:

$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} + G^q_{\alpha\beta} = kT_{\alpha\beta},$$

where $G^q_{\alpha\beta}$ encodes the part that comes from the quadratic terms:

$$\begin{aligned} G^q_{\alpha\beta} \quad = \quad & 2\Big\{a_1 RR_{\alpha\beta} - 2a_3 R_{\alpha\mu}R^\mu_\beta + a_3 R_{\alpha\rho\mu\nu}R_\beta{}^{\rho\mu\nu} + (a_2 + 2a_3)R_{\alpha\mu\beta\nu}R^{\mu\nu} \\ & - \left(a_1 + \frac{1}{2}a_2 + a_3\right)\nabla_\alpha\nabla_\beta R + \left(\frac{1}{2}a_2 + 2a_3\right)\Box R_{\alpha\beta}\Big\} \\ & - \frac{1}{2}g_{\alpha\beta}\left\{(a_1 R^2 + a_2 R_{\mu\nu}R^{\mu\nu} + a_3 R_{\rho\gamma\mu\nu}R^{\rho\gamma\mu\nu}) - (4a_1 + a_2)\Box R\right\}, \end{aligned}$$

with $\Box := g^{\mu\nu}\nabla_\mu\nabla_\nu$. Terms in blue involve products and those in red derivatives of distributions.

Quadratic gravity

The derivatives in G^q act linearly in R and $R_{\alpha\beta}$

$$\nabla_\alpha \nabla_\beta R, \quad \Box R_{\alpha\beta}, \quad g_{\alpha\beta} \Box R,$$

but these imply derivatives of singular distributions: "$\delta^{\Sigma\prime}$".

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but these imply derivatives of singular distributions: "$\delta^{\Sigma\prime}$".

The following terms in G^q involve products of singular distributions (e.g. $\delta^\Sigma \delta^\Sigma$)

$a_1 R R_{\alpha\beta}, \quad a_1 R^2,$

$a_3 R_{\alpha\mu} R^\mu_\beta, \; a_3 R_{\alpha\rho\mu\nu} R_\beta{}^{\rho\mu\nu}, \; (a_2 + 2a_3) R_{\alpha\mu\beta\nu} R^{\mu\nu}, \; a_2 R_{\mu\nu} R^{\mu\nu}, \; a_3 R_{\rho\gamma\mu\nu} R^{\rho\gamma\mu\nu}.$

Product of (singular) distributions is not well defined.

Quadratic gravity

The following terms in G^q involve products of singular distributions (e.g. $\delta^\Sigma\delta^\Sigma$)

$$a_1 R R_{\alpha\beta}, \quad a_1 R^2,$$

$$a_3 R_{\alpha\mu} R^\mu_\beta,\ a_3 R_{\alpha\rho\mu\nu} R_\beta{}^{\rho\mu\nu},\ (a_2+2a_3) R_{\alpha\mu\beta\nu} R^{\mu\nu},\ a_2 R_{\mu\nu} R^{\mu\nu},\ a_3 R_{\rho\gamma\mu\nu} R^{\rho\gamma\mu\nu}.$$

We distinguish two cases depending on the theory of gravity:

1. $a_2 = a_3 = 0$: This corresponds to $f(R) = R + 2\Lambda + a_1 R^2$.

   $$\boxed{[\kappa^\alpha_\alpha] = 0.}$$

   Distributional scalar curvature without singular part, and with $[R] \neq 0$ in general: $\underline{R} = R^+\underline{\theta} + R^-(\underline{1} - \underline{\theta})$.

2. A generic case with a_2 or $a_3 \neq 0$

   $$\boxed{[\kappa_{\alpha\beta}] = 0.}$$

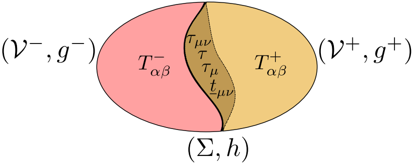
   Distributional curvature without singular part, and with $[R_{\alpha\beta\mu\nu}] \neq 0$ in general: $\underline{R}_{\alpha\beta\mu\nu} = R^+_{\alpha\beta\mu\nu}\underline{\theta} + R^-_{\alpha\beta\mu\nu}(\underline{1} - \underline{\theta})$.

Quadratic gravity

Energy momentum tensor distribution

The curvature distributions and the field equations for quadratic gravity lead to

$$\underline{T}_{\mu\nu} = T^{+}_{\mu\nu}\underline{\theta} + T^{-}_{\mu\nu}(\underline{1} - \underline{\theta}) + \tau_{\mu\nu}\delta^{\Sigma} + (2\tau_{(\mu}n_{\nu)} + \tau n_{\mu}n_{\nu})\delta^{\Sigma} + \underline{t}_{\mu\nu}.$$



- $T^{+}_{\mu\nu}$ and $T^{-}_{\mu\nu}$ are the EM tensors in $\mathcal{V}^+$ and $\mathcal{V}^-$.
- $\tau_{\mu\nu}$ is the EM tensor on Σ.

Appear in GR

- τ_{α} is the external flux momentum.
- τ is the external pressure.
- $\underline{t}_{\alpha\beta}$ is a double layer.

New!

Found for the 1st time in quadratic F(R) theories. *Senovilla (2013, 2014, 2015)*

Quadratic gravity

- The **energy momentum tensor on $\boldsymbol{\Sigma}$**
  - For $a_2 = a_3 = 0$

$$k\tau_{\alpha\beta} = -(1 + 2a_1 R|_\Sigma)[\kappa_{\alpha\beta}] + 2a_1(n^\rho[\nabla_\rho R]h_{\alpha\beta} - [R]\kappa_{\alpha\beta}|_\Sigma)$$

  - Otherwise

$$\begin{aligned} k\tau_{\alpha\beta} &= -(2a_1 + a_2 + 2a_3)[R]\kappa_{\alpha\beta} + \left(2a_1 + \frac{a_2}{2}\right) n^\rho[\nabla_\rho R]h_{\alpha\beta} \\ &\quad +2\left(2a_3 + \frac{a_2}{2}\right) n^\rho[\nabla_\rho R_{\mu\nu}]h^\mu_\alpha h^\nu_\beta. \end{aligned}$$

  This appears also on GR, but with a different expression.

- The **external flux momentum**

$$k\tau_\alpha = -(2a_1 + a_2 + 2a_3)\overline{\nabla}_\alpha[R] + 2\left(2a_3 + \frac{a_2}{2}\right) n^\rho[\nabla_\rho R_{\mu\nu}]n^\mu h^\nu_\alpha.$$

  Singular normal-tangent component in $\underline{T}_{\mu\nu}$, it does not exist in GR.
  It measures the energy flux/stress on Σ.

- **External pressure**

$$k\tau = (2a_1 + a_2 + 2a_3)[R]\kappa^\rho_\rho + \left(2a_3 + \frac{a_2}{2}\right)(2n^\rho[\nabla_\rho R_{\mu\nu}]n^\mu n^\nu - n^\rho[\nabla_\rho R]).$$

  This scalar does not exist in GR.
  It accounts for the (normal) tension on Σ.

Quadratic gravity

Double layer energy momentum tensor $\underline{t}_{\alpha\beta}$ with strength $\mu_{\alpha\beta}$
It acts on test tensors as

$$k\left\langle \underline{t}_{\alpha\beta}, Y^{\alpha\beta}\right\rangle = -\int_{\Sigma} k\mu_{\alpha\beta} n^{\rho}\nabla_{\rho} Y^{\alpha\beta} dv\,.$$

$$k\mu_{\alpha\beta} = (2a_1 + a_2 + 2a_3)[R]h_{\alpha\beta} + 2\left(2a_3 + \frac{a_2}{2}\right)[G_{\alpha\beta}]$$

- Absent in GR.
- The distribution $\underline{t}_{\alpha\beta}$ has support on Σ, but we have to know the extension of the test tensor $Y^{\alpha\beta}$ off Σ.
- It models a **dipole distribution** with strength $\mu_{\alpha\beta}$.
- The double layer is fundamental for the **conservation of the energy momentum tensor** distribution.
- Relation between the three new objects

$$\tau_{\alpha} = -\overline{\nabla}^{\rho}\mu_{\rho\alpha}, \quad \tau = \kappa^{\alpha\beta}|_{\Sigma}\mu_{\alpha\beta}$$

External flux momentum and external pressure do not exist without the double layer!

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- It models a **dipole distribution** with strength $\mu_{\alpha\beta}$.
  - In electrostatics, the Poisson equation for the potential Φ generated by a dipole-layer distribution with strength D localized at some surface S reads

$$\Delta\Phi = -\Delta'_S, \quad < \Delta'_S, f >= \int_S D\vec{n}\cdot\vec{\nabla} f dA.$$

- It models a **dipole distribution** with strength $\mu_{\alpha\beta}$.

Quadratic gravity

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  External flux momentum and external pressure do not exist without the double layer!

Quadratic gravity

Generalization of the **Israel equations** of General Relativity:

Field equations on the layer

$$
\begin{aligned}
n^\alpha h^\rho_\beta [T_{\alpha\rho}] + \overline{\nabla}^\alpha \tau_{\alpha\beta} &= -\mu_{\alpha\rho} \overline{\nabla}_\beta \kappa^{\alpha\rho}|_\Sigma + \overline{\nabla}_\rho (\mu^{\alpha\rho} \kappa_{\alpha\beta}|_\Sigma - \mu_{\alpha\beta} \kappa^{\alpha\rho}|_\Sigma), \\
n^\alpha n^\beta [T_{\alpha\beta}] - \tau_{\alpha\beta} \kappa^{\alpha\beta} &= \overline{\nabla}^\alpha \overline{\nabla}^\beta \mu_{\alpha\beta} + \mu^{\rho\nu} \left(n^\alpha n^\gamma R^\Sigma_{\alpha\rho\gamma\nu} + \kappa^\alpha_\rho |_\Sigma \kappa_{\nu\alpha}|_\Sigma \right).
\end{aligned}
$$

Three distinct type of terms:

- Jumps of the normal component of the energy momentum tensor.
- Energy momentum in the shell, including its divergence.
- Double layer strength (plus extrinsic curvature).

These equations agree with Israel equations in absence of double layers.

Quadratic gravity

Proper matchings

$$\underline{T}_{\mu\nu} = T^{+}_{\mu\nu}\underline{\theta} + T^{-}_{\mu\nu}(\underline{1} - \underline{\theta})$$

$$[h_{\alpha\beta}] = 0, \quad [\kappa_{\alpha\beta}] = 0,$$ (as in GR)

That now must be supplemented with

Case with $a_2 = a_3 = 0$

$$[R] = 0, \quad [\nabla_\rho R] = 0.$$

Generic case $^{(*)}$

$$[R_{\alpha\beta}] = 0, \quad [\nabla_\rho R_{\alpha\beta}] = 0.$$

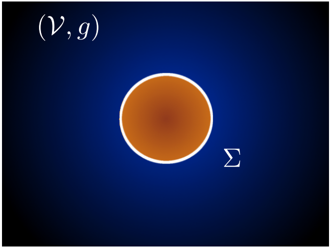
This actually implies that the full Riemann tensor and its first derivatives have no jumps across Σ:

$$[R_{\alpha\beta\lambda\mu}] = 0, \qquad [\nabla_\rho R_{\alpha\beta\lambda\mu}] = 0.$$

$^{(*)}$ $4a_3 + a_2 \neq 0$ and $4a_3 + (1+n)a_2 + 4na_1 \neq 0$

Quadratic gravity

Consider the proper matching, in GR, of a perfect fluid ball with vacuum:



- Denote by p^{GR} and ρ^{GR} the isotropic pressure and density of the fluid as computed in GR, and by u_α the unit fluid flow.
- Σ is determined by $p^{GR}|_\Sigma = 0$.
- The discontinuities of the Einstein and Ricci scalar read:

$$[G_{\alpha\beta}] = k\rho^{GR} u_\alpha u_\beta|_\Sigma, \quad [R] = k\rho^{GR}.$$

- Take this same spacetime to a quadratic theory of gravity and recall

$$k\mu_{\alpha\beta} = (2a_1 + a_2 + 2a_3)[R]h_{\alpha\beta} + (4a_3 + a_2)[G_{\alpha\beta}], \;\; \tau_\alpha = -\overline{\nabla}^\rho \mu_{\rho\alpha}, \; \tau = \kappa^{\alpha\beta}|_\Sigma \mu_{\alpha\beta}.$$

The proper matching hypersurface in GR will develop double layer and surface terms in quadratic gravity!

Quadratic gravity

Conclusions

1. We have found the generalized Israel equations for sources localized in a hypersurface Σ.
2. In general, double layers can develop in the hypersurface Σ.
   - Essential for the conservation of the energy momentum tensor distribution.
3. In absence of double layers, the generalized Israel equations are identical to the **Israel equations derived in GR**.
4. A solution **properly matched in GR** is not a solution satisfying a **proper matching in quadratic gravity**, in general.

References

Thank your for your attention!

This talk is based on the works:

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