Max-Planck-Institut für Gravitationsphysik

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Catch a Wave From Space

Transient Continuous Gravitational Waves (tCW): Sources and Searches

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System

- **Transient gravitational wave emission (tCW)** requires **transient non-axisymmetry** *aka* transient quadrupole moment in isolated compact objects (*read* **neutron stars**).

- Such transient quadrupole moment could be driven by transient **differential rotation** of **neutron star's** crust with respect to its core i.e. superfluid bulk matter. **Glitches** are considered to be a common source for such differential rotation.

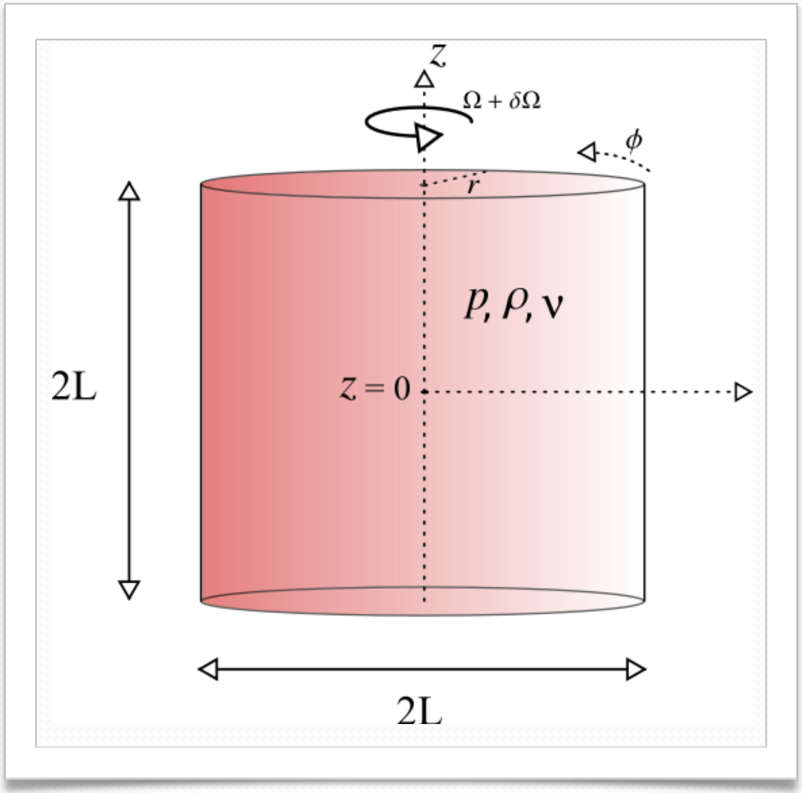
- Previous works have explored **tCW emission during the post-glitch relaxation phase** in simplified systems by Abney & Epstein (1996), Melatos & Van Eysden (2008, 2010). **Equation of state** plays a crucial role in characterising tCW emission.

- We extend the previous works to include more general set of equations of state, besides generalising other assumptions.

- Unlike GW emission from Compact Binary Coalescence **(CBC)**, **tCW** signals have more concentrated frequency content *aka* nearly monochromatic decaying oscillations.

Singh A, arXiv:**1605.08420**, 2016

Star* in a Monty Python Universe



*this assumption is reasonable! - spherical geometry results in average over latitudes (of spherical harmonics) and doesn't affect the order of magnitude of **emitted signal amplitude**, or their **decay time-scales** [Melatos & Van Eysden (2013)]

Singh A, arXiv:**1605.08420**, 2016

More Assumptions

- Our hypothetical star is **equivalent to a sphere of radius L.**

- The **adiabatic sound speed** v_c (defined via the equation of state) varies along the z-direction: equivalent to radial variation in a sphere.

- The **equilibrium sound speed** v_{eq} (pre-glitch) also varies only in z-direction.

- Ignore the effects of magnetic field on our system and consider a purely hydrodynamic system. This is also reasonable! - magnetic field is limited to the crust, due to strong screening at the crust (possibly screened by polar alignment of the macroscopic domain vortices).

Singh A, arXiv:**1605.08420**, 2016

Main Equations

- Navier-stokes equation: Navier-stokes equation for a comp- -ressible fluid in a rotating frame with viscosity ν,

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} + 2\vec{\Omega} \times \vec{v} = -\frac{1}{\rho}\nabla p + \nu\, \nabla^2 \vec{v} + \frac{\nu}{3}\nabla(\nabla \cdot \vec{v}) + \nabla[\vec{\Omega} \times (\vec{\Omega} \times r)] + \vec{g}$$

considering a simplified gravitational field: $\vec{g} = -\frac{z}{|z|} g \hat{z}$

- Continuity equation: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$

- Energy equation*: $\left\{\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right\}\rho = \left\{\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right\}\frac{p}{v_c^2}$

*Alternative differential form of equation of state.

Singh A, arXiv:**1605.08420**, 2016

Reduce

- Dimensionless parameters: η, the Froude number **F**, the Scaled Compressibility **K**, the Ekman Number **E**, and the Rossby number ϵ.

$$\mathrm{E} = \frac{\nu}{\mathrm{L}^2\Omega}, \quad \eta = \frac{v_c^2}{c^2}, \quad \mathrm{K} = g\frac{\mathrm{L}}{c^2}, \quad \mathrm{F} = \Omega^2\frac{\mathrm{L}}{g}, \quad \epsilon = \frac{\delta\Omega}{\Omega}$$

- Induce perturbations: Induce perturbations from Glitch,

$$\rho \rightarrow \rho + \epsilon\delta\rho, \; p \rightarrow p + \epsilon\delta p, \vec{v} \rightarrow \delta\vec{v}.$$

- Introduce Brunt-Väisälä frequency:

$$\mathrm{N}^2 = \frac{\mathrm{K}}{\mathrm{F}}\left[\frac{v_c^2}{v_{\mathrm{eq}}^2} - 1\right] - \frac{\partial_z \eta}{\mathrm{F}}$$

Singh A, arXiv:**1605.08420**, 2016

GW Emission via *Mass Quadrupole*

- **Gravitational Wave Emission:** From mass-quadrupole,

$$h_{+}^{\mathbb{M}^{\mathsf{P}}}(t) = h_{\mathrm{o}}^{\mathbb{M}} \sum_{\gamma=1}^{\infty} \kappa_{2\gamma} \left[-4\omega_{2\gamma} \mathsf{E}^{\frac{1}{2}} \,\mathsf{Sin}(2\Omega t) + (4 - \mathsf{E}\omega_{2\gamma}^{2}) \mathsf{Cos}(2\Omega t) \right] e^{-\mathsf{E}^{\frac{1}{2}} \omega_{2\gamma} \Omega t}$$

where,

$$h_{\mathrm{o}}^{\mathbb{M}} = \pi \rho_{\mathrm{o}} \Omega^{4} \mathrm{L}^{6} \epsilon \frac{\mathrm{G}}{c^{4} d_{\mathsf{source}}\, g}$$

Also, calculate $h_{\times}^{\mathbb{M}^{\mathsf{P}}}(t)$, $h_{+}^{\mathbb{M}^{\mathsf{E}}}(t)$ and $h_{\times}^{\mathbb{M}^{\mathsf{E}}}(t)$. The associated decay time-scale is given by

$$t_{\nu\gamma} = \mathsf{E}^{-\frac{1}{2}} \Omega^{-1} \omega_{\nu\gamma}^{-1}$$

- **Resonances:** Resonances for mass-quadrupole*,

$$\omega_{\mathrm{R}}^{2} = 4\Omega^{2} + t_{2\gamma}^{-2}$$

*not all modes and/or polarisations; some of them emit at Ω, instead of 2Ω.

Singh A, arXiv:**1605.08420**, 2016

GW Emission via *Current Quadrupole*

- **Gravitational Wave Emission:** From current-quadrupole,

$$h_+^{\mathbb{C}^P}(t) = h_o^{\mathbb{C}} \sum_{\gamma=1}^{\infty} V_{2\gamma}\Big[-4t_{2\gamma}^{-1}\Omega^{-1}\mathrm{Cos}(2\Omega t) - (4 - t_{2\gamma}^{-2}\Omega^{-2})\mathrm{Sin}(2\Omega t)\Big]e^{-t_{2\gamma}^{-1}t}$$

where,

$$h_o^{\mathbb{C}} = 2\pi G \frac{\rho_o L^6 \epsilon\, \Omega^3}{3c^5 d_{\mathrm{source}}}$$

Also, calculate $h_\times^{\mathbb{C}^P}(t)$, $h_+^{\mathbb{C}^E}(t)$ and $h_\times^{\mathbb{C}^E}(t)$.

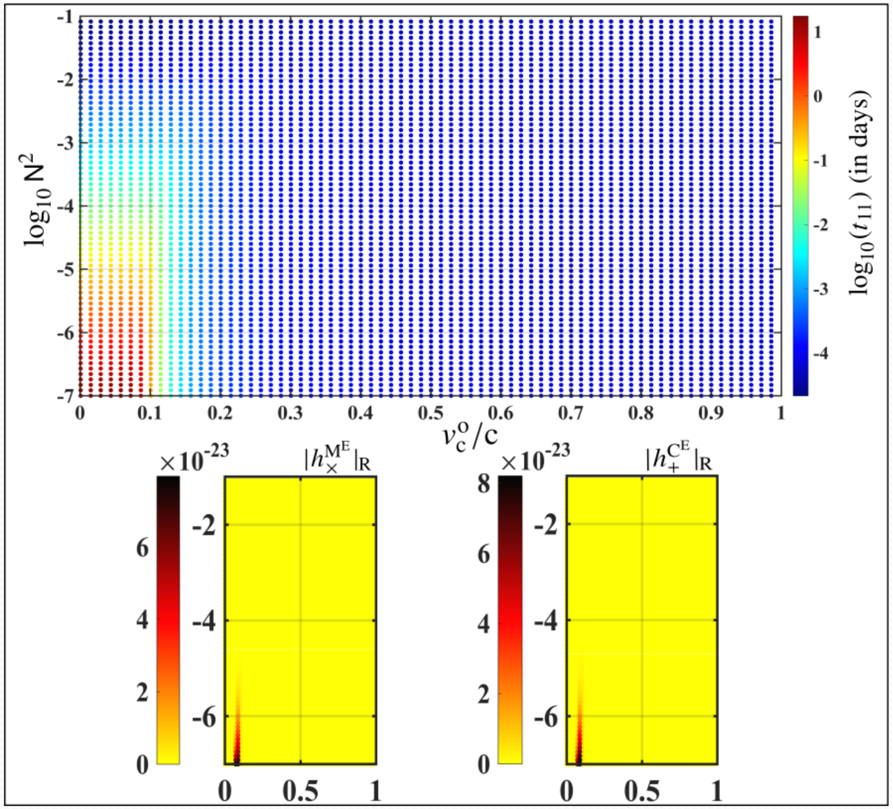
- **Other:** Resonance and decay time-scales have same expressions.
- **Other Others:**

$$\kappa_{\nu\gamma} = 2\omega_{\nu\gamma}^{-1}A_{\nu\gamma}\Bigg[\int_0^1 dr\, r^3 J_\nu(\lambda_{\nu\gamma}r)\int_0^1 dz\, \partial_z[-Z_{\nu\gamma}(z)\rho_e(z)] + K\int_0^1 dr\, r^4\partial_r[J_\nu(\lambda_{\nu\gamma}r)]\times \int_0^L dz\left[1+\frac{K}{K_s(z)}\right]Z_{\nu\gamma}(z)\rho_e(z) + \frac{\Omega^2 L^2}{c^2}\int_0^1 dr\, r^5 J_\nu(\lambda_{\nu\gamma}r)\times \int_0^1 dz\,\Big[\partial_z[-Z_{\nu\gamma}(z)\rho_e(z)] + K Z_{\nu\gamma}(z)\rho_e(z)\Big]\Bigg]$$

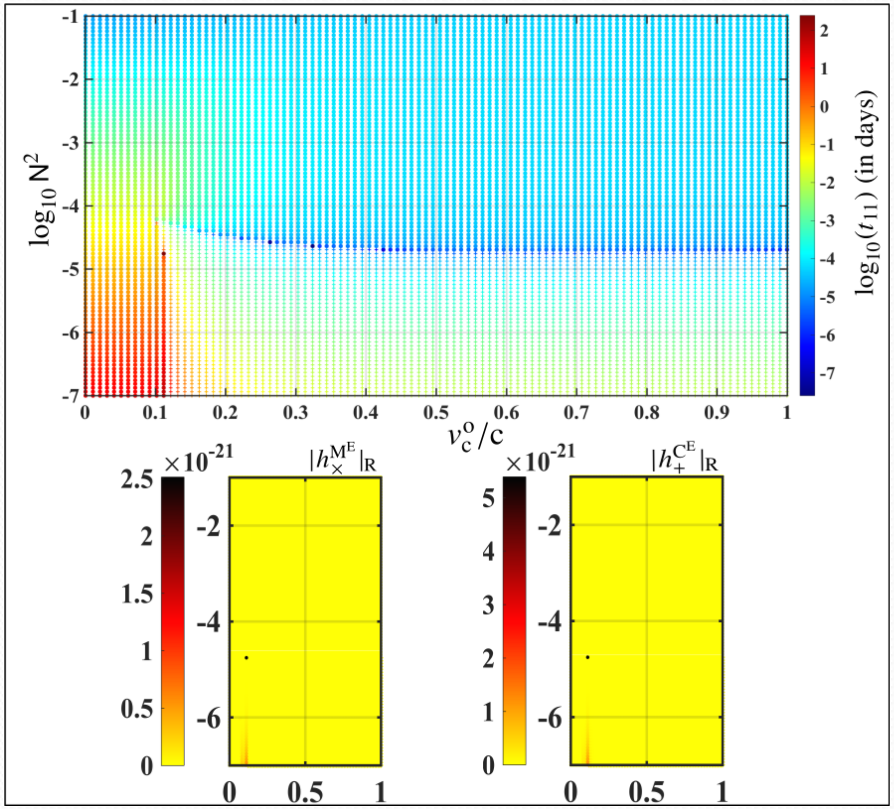
$$V_{\nu\gamma} = 2A_{\nu\gamma}\omega_{\nu\gamma}^{-1}\Bigg[\mathcal{L}_3^{(3-\nu)}\int_0^1 dr\, r^{\nu-1}[r^2\partial_r^2[J_\nu(\lambda_{\nu\gamma}r)] + r\partial_r[J_\nu(\lambda_{\nu\gamma}r)] - \nu^2 J_\nu(\lambda_{\nu\gamma}r)] - \mathcal{L}_4^{(2-\nu)}\times \int_0^1 dr\, r^{\nu+2}\partial_r[J_\nu(\lambda_{\nu\gamma}r)] + 2F\Big[\mathcal{L}_5^{(2-\nu)}\int_0^1 dr\, r^{\nu+3}J_\nu(\lambda_{\nu\gamma}r) - \mathcal{L}_4^{(3-\nu)}\int_0^1 dr\, r^{\nu+1}[r\partial_r[J_\nu(\lambda_{\nu\gamma}r)] + 2J_\nu(\lambda_{\nu\gamma}r)]\Big]\Bigg]$$

Results

- Emitted **tCW amplitude** and **time-scales** very sensitive to $\partial_z \eta$ ($\partial_z v_c$).



$\partial_z v_c = 0$

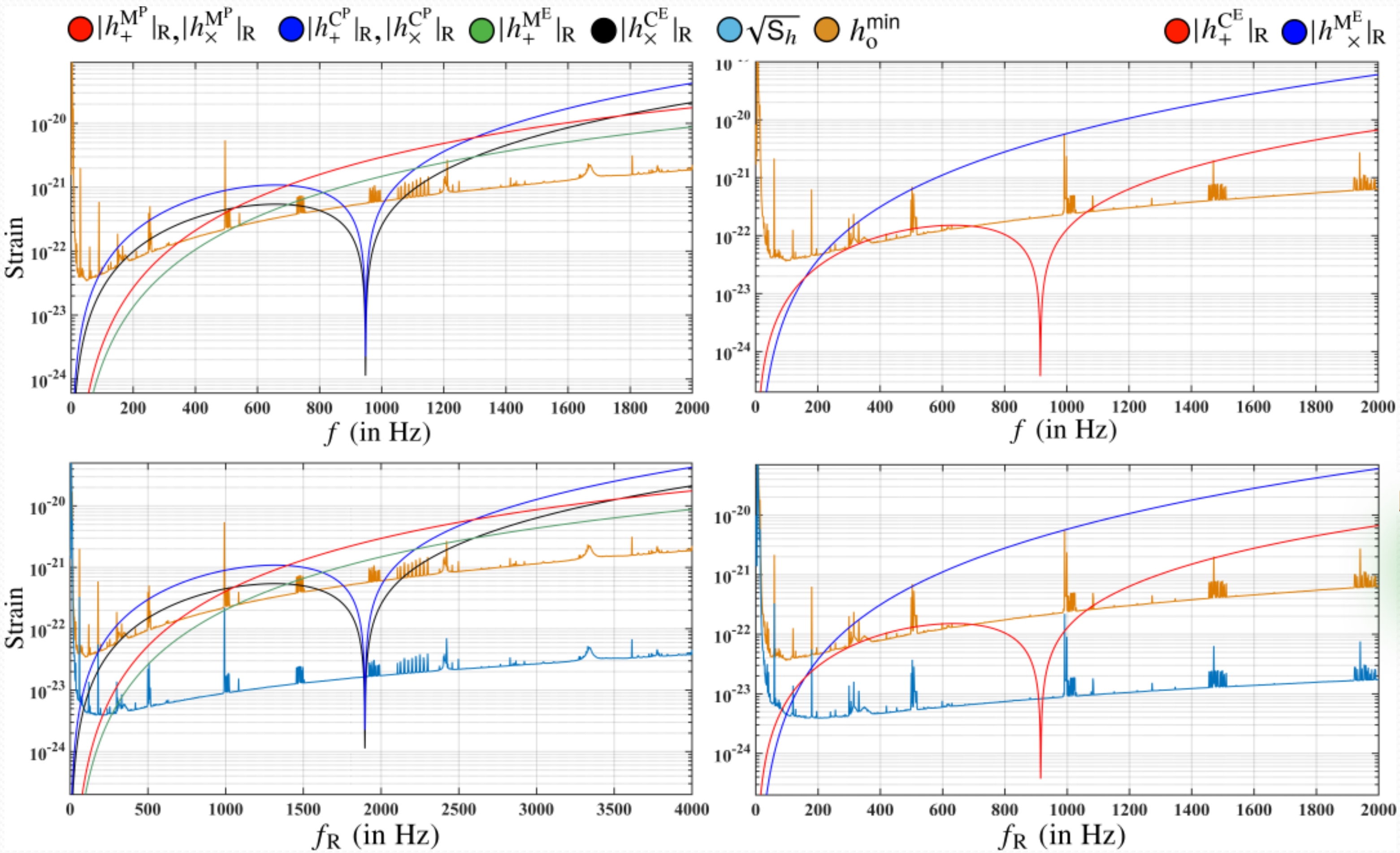


$\partial_z v_c = -10^{-4} c$

Singh A, arXiv:**1605.08420**, 2016

Results

- Compare with aLIGO sensitivity



$$v_{\mathrm{c}}^{\mathrm{o}} = 0.1c$$
$$\mathrm{N}^2 = 10^{-4}$$
$$\mathrm{E} = 10^{-10}$$
$$\epsilon = 10^{-4}$$
$$d_{\mathrm{source}} = 1\,\mathrm{kpc}$$
$$\mathrm{L} = 10^4\mathrm{m}$$
$$g = 10^{12}\mathrm{m/sec}^2$$
$$\rho_{\mathrm{o}} = 10^{17}\mathrm{Kg/m}^3$$
$$\boxed{\partial_z v_{\mathrm{c}} = 0}$$

Singh A, arXiv:**1605.08420**, 2016

Search

- First search for tCW in progress in **aLIGO (O1)** data using Einstein@Home volunteer distributed computing network. (~ 500,000 volunteers, ~ 44,000 **active volunteers**)

- **tCW** all-sky search runs in parallel with the **CW** all-sky search. (talk by Sinéad Walsh)

- Set-up: 20-100 Hz, N_{seg} = 12, T_{coh} = 210 hours. (talk by Sinéad Walsh)

- **tCW** search uses a new detection (-ranking-) statistic** $B_{StS/GLtL}$ (*line-robust signal & transient line statistic*) in combination with the traditional **$\mathcal{F}$-statistic**. (see Ref.) which is sensitive to **tCW** signals as well as **CW** signals.

** Separate implementation of $B_{tS/GLtL}$ and $B_{S/GLtL}$ in the search

Keitel D, PRD, 93:084024, 2016

Search

- The search uses semi-coherent GCT (Global Correlation Transform) method where, a) data is divided into a number of *segments* (N_{seg}), and b) the core $\mathcal{F}$-**statistic** is an **incoherent sum** over coherently calculated $\mathcal{F}$-**statistic** over a single segment.

- The new $B_{StS/GLtL}$ statistic excludes signals that a) persistently appear in only one detector across multiple segments (instrumental line) (/GL), and b) appear in only one detector within one segment (transient instrumental line) (/GLtL).

- The $B_{StS/GLtL}$ statistic is sensitive to a) persistent signals across segments in multiple detectors (S/G), and b) transient signals contained within a segment in multiple detectors (tS/G).

Keitel D, PRD, 93:084024, 2016

While we await results…

- <u>Source-modelling:</u> The **Ekman Pumping** model for **tCW** emission explored here is great for an order-of-magnitude estimate; it could be improved by

    a) including the magnetic field,

    b) possibly exploring more exotic equations of state, and

    c) solving numerically for a real spherical geometry for more accurate predictions.

- <u>Searches:</u> $B_{StS/GLtL}$ statistic doesn't perform as well when the transient signal (**tCW**) is distributed across several segments (ultra-long transients), or too short (short transients); options are:

    a) find an optimal search set-up based on priors from models, and/or

    b) derive a new detection statistic which is independent of the set-up.

    In the current **O1 search**, short transients are missable due to long segment length (T_{coh} = 210 hours).

Singh A, arXiv:**1605.08420**, 2016 | Keitel D, PRD, 93:084024, 2016

Thanks!