"**Gravitational**" scalar-tensor theory

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introduction

Einstein's General Relativity

- "**The classical theory of fields**" by Landau & Lifshitz,

    GR is a **unique theory of gravity** provided

    - composed only by metric and its derivatives

    - 4D

    - covariant theory

    - EOM is (at most) 2nd order

- have to abandon one of assumptions above (or more ?)

    metric?   **> 4D?**   **covariance?**   **2nd order EOM?**

f (R) theory

- The action is given by a **non-linear** function of **R**

- **EOM is 4th order** because $R \supset \ddot{g} + \dot{g}^2$

$$\int \mathrm{d}^4 x \sqrt{-g}\, f(R) \sim \int \mathrm{d}^4 x\, f(\ddot{g}) \;\rightarrow\; \dot{f}(\ddot{g}) \supset \ddddot{g} \quad : \text{EOM}$$

- Under a **Weyl (conformal) transformation**,

$$g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$$
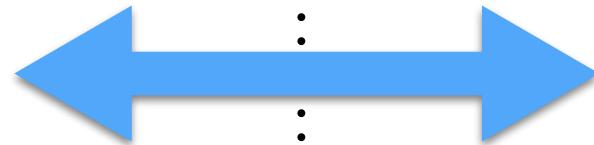
f (R) theory **=** Einstein + **canonical scalar**

➡ change of gravity law = introduction of matter !!

correspondence

metric + φ (scalar-tensor) | metric (purely gravitational)
exotic matter ? | modified gravity ?

canonical scalar ⟷ f (R) theory

Horndeski's theory

- the most general S-T theory **with 2nd order EOM for g & φ** :

canonical :
$$\mathcal{L} = X - V(\phi) = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)$$

k-essence :
$$\mathcal{L}_2 = K(\phi, X)$$

$$X = -(\nabla\phi)^2/2$$

KGB :
$$\mathcal{L}_3 = G(\phi, X)\,\Box\phi$$

Horndeski :
$$\mathcal{L}_4 = G_4(\phi, X)\,R - \frac{\partial G_4}{\partial X}\left[(\nabla\nabla\phi)^2 - (\Box\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)\,G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{3}\frac{\partial G_5}{\partial X}\left[(\nabla\nabla\phi)^3 + \cdots\right]$$

(cf. GLPV theory)

dual description ??

metric + φ (scalar-tensor) exotic matter ?	metric (purely gravitational) modified gravity ?
<u>canonical scalar</u> ⟷	<u>f (R) theory</u>
k-essence	?
KGB (⊃ Box)	??
Horndeski (⊃ Box^2)	???
Horndeski (⊃ Box^3)	????

⚠ a special Horndeski ↔ f (Gauss-Bonnet) ⚠

Kobayashi et.al. (2011)

question

What if we introduce derivatives of R ?   cf. f(R)

$$f\left(R, (\nabla R)^2, \Box R, \cdots\right)$$

What is the corresponding S-T theory, if it exists ??

model

model

- The action is given by **R** and **derivatives of R** :

$$f\left(R, (\nabla R)^2, \Box R, \cdots\right) \qquad (\nabla R)^2 = g^{\mu\nu}\nabla_\mu R\nabla_\nu R$$

c.f. **f (R, □R, … , □ⁿR)** theory   Wands (1993)

f (Riemann) theory   Deruelle et.al. (2009)

- We call this model "**gravitational** scalar-tensor theory" since the theory is constructed only in terms of gravitational language, namely metric & its derivatives.

Ostrogradski's ghost ??

- **non-degenerate** Lagrangian with **higher derivatives**

$$L = L(q\,,\dot{q}\,,\ddot{q}) \qquad (\mathrm{d}^2 L/\mathrm{d}\ddot{q}^2 \neq 0)$$

$$H = P_1\,Q_2 + P_2\,f(Q_1\,,Q_2\,,P_2) - L(Q_1\,,Q_2\,,P_2)$$
$$(Q_1 = q\,,Q_2 = \dot{q})$$

➡ **Hamiltonian is unbounded below**

- Although $f\Big(R\,,(\nabla R)^2\,,\Box R\,,\cdots\Big) \supset f\Big(g\,,\dot{g}\,,\ddot{g}\,,\cdots\Big)$
  there is **no Ostrogradsky's ghost** in our model
  because the Lagrangian is **degenerate**.

proof of healthiness

- consider a simple model : $f\left(R, (\nabla R)^2\right)$

- rewrite **R** with **φ** & **λ** :

  $$!!\ \lambda \neq f_\phi\ !!$$

  $$f\left(R, (\nabla R)^2\right) = f\left(\phi, (\nabla \phi)^2\right) - \lambda(\phi - R)$$

- conformal tr. :

  $$\frac{1}{2}\widetilde{R} + f\left(\phi, 2\lambda(\widetilde{\nabla}\phi)^2\right) - \frac{1}{2}(\widetilde{\nabla}\chi)^2$$

  $$g^{\mu\nu} = 2\lambda\,\widetilde{g}^{\mu\nu}$$

  $$\lambda \equiv e^{\sqrt{2/3}\,\chi}$$

☞ # of d.o.f.s : **2 (GW) + 2 (scalar)** ($\neq$ **2 + 1** in f(R))

☞ sub class of bi-scalar k-essence ⊃ **healthy** domain

some remarks

- $(\Box R)^2$ is **not allowed** in general -> **Ostrogradski's ghost**

  among models with the form of $\ f\left(R,(\nabla R)^2,\Box R\right)$

  $$\blacktriangleright \quad K\left(R,(\nabla R)^2\right) + G\left(R,(\nabla R)^2\right) \times \Box R$$

- $(\Box R)^2$ **can be introduced** "with specific combinations"

  $$Q\left(R,(\nabla R)^2\right)R + Q_X\left(R,(\nabla R)^2\right)\left[(\Box R)^2 - (\nabla_\mu\nabla_\nu R)^2\right]$$

- Way to construct a healthy gravitational S-T theory :

  $$\mathcal{L}^{\mathrm{Horn}}(\phi,g_{\mu\nu}) \quad \rightarrow \quad \mathcal{L}^{\mathrm{Horn}}(R,g_{\mu\nu})$$

dual description ??

metric + φ (scalar-tensor) exotic matter ?	metric (purely gravitational) modified gravity ?
canonical scalar ⟷	f (R) theory
k-essence	?
KGB (⊃ Box)	??
Horndeski (⊃ Box^2)	???
Horndeski (⊃ Box^3)	????

to be continued ...

summary

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- We have considered a theory of gravity in which the action is given by R and derivatives of R.

- Despite the higher derivative nature of action, the theory is healthy (if f is properly chosen)
  = no ghost & no Ostrogradsky's instabilities

- # of d.o.f.s = 2 (GW) + 2 (scalar)
  $\Leftrightarrow$ 2 scalars-tensor theory $\supset$ bi-scalar k-essence

- Way to construct a healthy gravitational S-T theory :
$$\mathcal{L}^{\mathrm{Horn}}(\phi\,,g_{\mu\nu}) \quad \rightarrow \quad \mathcal{L}^{\mathrm{Horn}}(R\,,g_{\mu\nu})$$

Thank you very much for your attention !!

ありがとう (Arigatou)

how to construct GST

Horndeski (# : **2** + **1**)	Gravitational S-T (# : **2** + **2**)
k-essence : $K\big(\phi,(\nabla\phi)^2\big)$	$K\big(R,(\nabla R)^2\big)$
KGB : $G(\phi,X)\,\Box\phi$	$G(R,X_R)\,\Box R$
Horn. (L4) : $G_4\,R - G_{4,X}\big[(\nabla\nabla\phi)^2 - (\Box\phi)^2\big]$	$G_4\,R - G_{4,X}\big[(\nabla\nabla R)^2 - (\Box R)^2\big]$
Horn. (L5) : $G_5\,G_{\mu\nu}\nabla^\mu\nabla^\nu\phi + \cdots$	$G_5\cdots + \cdots$

how to construct GST

| Horndeski ($\textcolor{blue}{\mathbf{2}}$ + $\textcolor{red}{\mathbf{1}}$) | gravitational S-T ($\textcolor{blue}{\mathbf{2}}$ + $\textcolor{red}{\mathbf{2}}$) |

Horn : $\quad \mathcal{L}_5 = G_5(\phi, X)\, G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \dfrac{1}{3}\dfrac{\partial G_5}{\partial X}\Big[(\nabla\nabla\phi)^3 + \cdots\Big]$

$$\mathcal{L}_5 \supset f\big(R, (\nabla R)^2\big)\Big[\nabla^\alpha(\nabla_\alpha\Box - \Box\nabla_\alpha)R - \frac{1}{2}(\nabla R)^2 - \frac{1}{2}R\,\Box R\Big]$$

$$+ \frac{1}{3}\frac{\partial f}{\partial(\nabla R)^2}\big(R, (\nabla R)^2\big)\Big[(\Box R)^3 - 3(\Box R)(\nabla_\mu\nabla_\nu R)^2 + 2(\nabla_\mu\nabla_\nu R)^3\Big]$$

$\textcolor{green}{\textbf{identity}}$: $\quad G_{\mu\nu}\nabla^\mu\nabla^\nu\psi = \nabla(\nabla\Box - \Box\nabla)\psi - \dfrac{1}{2}\nabla R\,\nabla\psi - \dfrac{1}{2}R\,\Box\psi$