Antonino Marcianò

Fudan University

Inflation from fermionic matter interacting with a gauge field

based on

S. Alexander, A. Marciano & D. Spergel

JCAP 1304 (2013) 046, arXiv:1107.0318.

S. Alexander, D. Jyoti, A. Kosowsky & A. Marciano

JCAP 1505 (2015) 005, arXiv:1408.4118.

S. Alexander, S. Brahma, P. Dona, A. Marciano & Z. Yang
to appear.

Physical Idea

S. Alexander, AM & D. Spergel, JCAP 1304 (2013) 046

At GUT energy scales the Hypercharge gauge field (photon at low energies) can drive inflation.

The Hypercharge field is sourced by the quantum fermionic charge density.

The interaction Energy between Charge and Hypercharge field sources inflation.

Consistency and Stability

S. Alexander, D. Jyoti, A. Kosowsky & AM, JCAP 1505 (2015) 005

Consistency of the model is ensured thanks to the Stueckelberg mechanism.

There exists exactly one stable solution, and stability has been checked numerically.

Inflation arises without fine tuning, and any effective potential must be postulated.

Theoretical framework

S. Alexander, AM & D. Spergel, JCAP 1304 (2013) 046

$$S = S_D + \int_{\mathcal{M}_4} d^4x\sqrt{-g}\left[\frac{M_p^2\,R}{16\pi} - \frac{1}{2}\partial_\mu\theta\,\partial^\mu\theta + \frac{1}{2}m^2\,\theta^2 - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + \frac{\theta}{4M_*}F_{\alpha\beta}\tilde{F}^{\alpha\beta}\right]$$

$$S_D = \frac{1}{2}\int_{\mathcal{M}_4} d^4x\sqrt{-g}\,\left(-i\overline{\psi}\not{\nabla}\psi + M\overline{\psi}\psi + q\,\overline{\psi}\,\gamma^I e_I^\mu\psi\,A_\mu + c.c.\right)$$

$$\mathcal{J}^\mu = \overline{\psi}\gamma^I\,e_I^\mu\psi \longrightarrow \mathcal{J}^\mu\,A_\mu$$

Interaction Lagrangian, sourcing inflation

Energy-momentum tensor

$$T^\mu_\nu = -q\delta^\mu_\nu\,A_\rho\mathcal{J}^\rho - qA_{(\nu}\mathcal{J}^{\mu)} + F_\alpha{}^\mu F^\alpha{}_\nu - \frac{1}{4}\delta^\mu_\nu\,g^{\alpha\rho}g^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma}$$
$$+\frac{\imath}{4a}\left(\bar{\psi}\gamma_\nu\partial^\mu\psi + h.c.\right) - \frac{\delta^\mu_\nu}{a}\,\bar{\psi}(\imath\gamma_\rho\partial^\rho - m\,a)\psi$$

Gauge Field's Equations

Gauge potential $A_0^{\rm tot} = \boxed{A_0} + \boxed{\delta A_0}$

Vector field $\vec{A}_{\rm tot} = \boxed{\vec{A}} + \boxed{\delta \vec{A}}$

Inhomogenous perturbations

Ford (abelian), Zhang & Parker (YMC),
Mukhanov (stochastic fields),
Sheikh-Jabbari (Non-abelian gauge),
Adshead (Chromo Natural), etc.,.

Homogenous background

Ansatz

$$a = a_0 \exp H_0 t$$

$$\ddot{A}_0(t) = a^4 \mathcal{J}_0$$

$$\ddot{\vec{A}}(t) = \cancel{a^4 \vec{\mathcal{J}}}$$

vanishing

Gauge Field's Solutions

Homogeneous background components

Isotropic components $$A_0 = \bar{A}_0 a$$

Anisotropic components $$\vec{A} = \vec{c} + \vec{c}\,'/a$$

Inhomogeneous perturbation components

Perturbations

$$\delta A_0(k) = \delta A_0 \, \exp(\imath k_0 t)$$

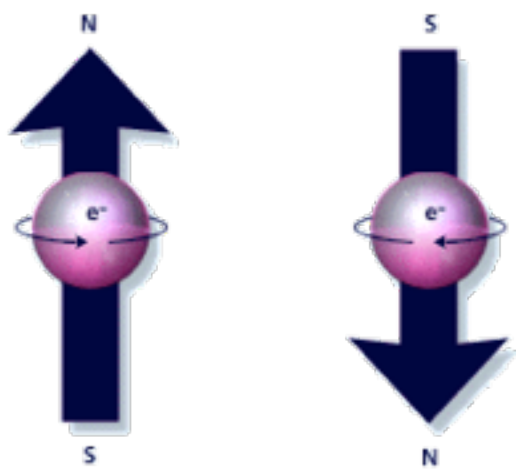
$$\delta \vec{A}(k) = \delta \vec{A}_0 \, \cos(\gamma_k t) + \delta \vec{A}_0' \, \sin(\gamma_k t)$$

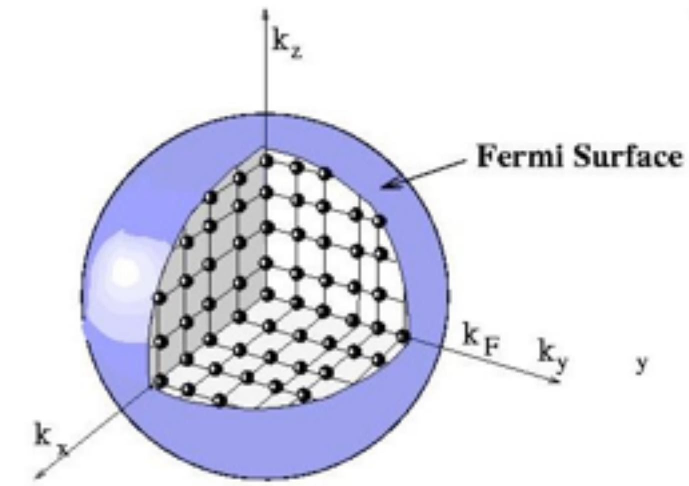
$$\gamma_k^2 = k(\dot{\theta}/M_* + k)$$

Fermionic Current

Fermionic current

$$\boxed{J^I = \overline{\psi}\gamma^I\psi}$$





VEV on homogenous FLRW background

$$\langle 0|J^I|0\rangle \simeq \lim_{y\to x} S^{ab}(x,\,y)\gamma^I_{ba}$$

$$S^{ab}(x,y) \simeq \frac{H^2}{16\pi^2}\left[\frac{m^2}{H}\left(\frac{1+\gamma^0}{2}\right)^{ab} - \frac{m^2}{H}\left(\frac{1-\gamma^0}{2}\right)^{ab}\right] + O\left(\frac{m^2}{H^2}\right)$$

$$J^0 \simeq m^2 H + O\left(\frac{m^3}{H^3}\right)$$

$$\mathcal{J}^0 = e^0_I J^I = J^0/a(t) \qquad J^i = 0$$

Gravitational Field

Einstein Equation for Scale Factor

$$3\frac{\dot{a}^2}{a^2} = \frac{8\pi G_N}{a^4}\underbrace{\cancel{(\vec{E}^2 + \vec{B}^2)}}_{\text{negligible}} + \underbrace{\boxed{8\pi G_N A_0 \mathcal{J}^0}}_{\text{constant}} \longleftarrow \left\{ \begin{array}{l} A_0 \simeq a \\ \mathcal{J}^0 \simeq \dfrac{1}{a} \end{array} \right.$$

Anisotropic background components redshift away in the Einstein EOM

Asymptotic de Sitter phase

$$\boxed{a = a_0 \, \exp H_0 t}$$

Stuekelberg mechanism

S. Alexander, D. Jyoti, A. Kosowsky & AM, JCAP 1505 (2015) 005

Scalar field as a DOF of a massive gauge field

$$S = \int_{\mathcal{M}_4} d^4x\sqrt{|g|}\left[\frac{M_p^2 R}{8\pi} - \frac{1}{4}G_{\alpha\beta}G^{\alpha\beta} - \frac{1}{2}m^2 C_\mu C^\mu + C_\mu \mathcal{J}^\mu + \mathcal{L}_{\rm fer}\right]$$

$$C_\mu = A_\mu - \frac{1}{m}\partial_\mu\theta\,, \quad G_{\mu\nu} = \partial_{[\mu}C_{\nu]} = \partial_{[\mu}A_{\nu]} = F_{\mu\nu}$$

$$\mathcal{L}_{\rm fer} = -\imath\overline{\psi}\nabla\!\!\!\!/\,\psi + c.c. + M\overline{\psi}\psi\,, \quad \mathcal{J}^\mu = q\,\bar{\psi}\gamma^\mu\psi$$

invariant under

$$\boxed{\begin{aligned} A_\mu &\to A'_\mu = A_\mu + \partial_\mu\Lambda \\ \theta &\to \theta' = \theta + m\,\Lambda \end{aligned}}$$

Dynamics

$$\ddot{A}_0 + 3H\dot{A}_0 + 3\dot{H}A_0 + m^2 A_0 + J/(1-\epsilon) = 0$$

$$\theta = \frac{1}{m}\nabla_\mu A^\mu = \frac{1}{m}\nabla_0 A^0 = \frac{1}{m}\left(\dot{A}^0 + 3\bar{H}A^0\right) = \frac{3\bar{H}J}{m^3}$$

Conservation and dynamics

Conservation of the total energy momentum tensor

$$T^{\mu\nu} = G_\alpha^{\ \mu} G^{\alpha\nu} - \frac{1}{4} g^{\mu\nu} G_{\alpha\beta} G^{\alpha\beta} - \frac{1}{2} m^2 g^{\mu\nu} C_\alpha C^\alpha - g^{\mu\nu} C_\rho \mathcal{J}^\rho + C^{(\nu} \mathcal{J}^{\mu)} + {}^{(\psi)}T^{\mu\nu}$$

$${}^{(\psi)}T^{\mu\nu} = \frac{\imath}{2} \left[\bar{\psi} \Gamma^{(\mu} \nabla^{\nu)} \psi - \nabla^{(\nu} \bar{\psi} \Gamma^{\mu)} \psi \right] - g^{\mu\nu} \left[\frac{\imath}{2} \left(\bar{\psi} \Gamma^\mu \nabla_\mu \psi - \nabla^\mu \bar{\psi} \Gamma_\mu \psi \right) + M \bar{\psi} \psi \right] \qquad \Gamma^\mu = \gamma^I e_I^\mu$$

$$\nabla_\mu T^{\mu\nu} = 0 \implies \dot{\rho} + 3H(\rho + P) = 0 \qquad P = T_{ii} = -\frac{1}{2} m^2 \mathcal{C}^2 - \mathcal{C} \cdot \mathcal{J} - MJ \qquad \rho = T_{00} = \frac{1}{2} m^2 \mathcal{C}^2 + \mathcal{C} \cdot \mathcal{J} + MJ$$

Dynamical System

$$\rho = \frac{1}{2} m^2 \left(A_\mu - \frac{1}{m} \partial_\mu \theta\right)^2 - \left(A_\mu - \frac{1}{m} \partial_\mu \theta\right) J^\mu + MJ$$

$$H^2 = \frac{8\pi}{3M_p^2} \left[\frac{1}{2} m^2 A^{0 2} + A^0 \left\{ \frac{J}{(1-\epsilon)} - m\dot{\theta} \right\} + \frac{1}{2} \dot{\theta}^2 + MJ \right]$$

$$\frac{J}{Mm^2} \gg 1$$

$$A^0 = \frac{J}{m^2}$$

$$\bar{H}^2 \simeq \frac{4\pi J^2}{M_p^2 m^2}$$

Attractor solution

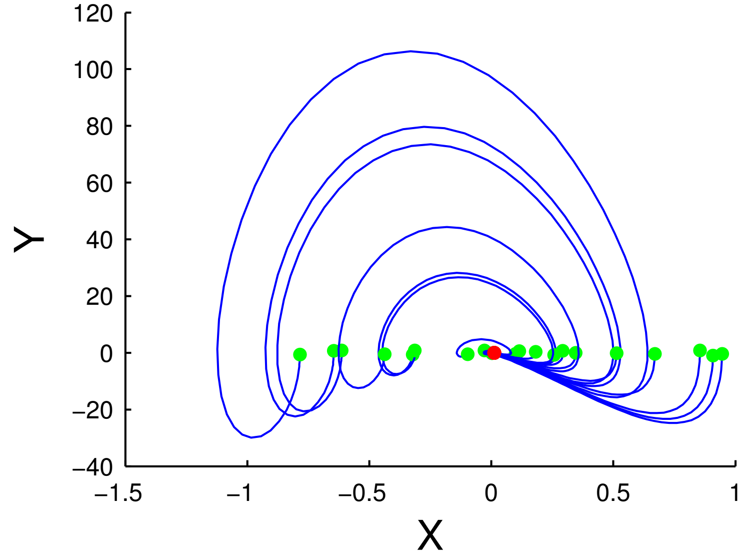
S. Alexander, D. Jyoti, A. Kosowsky & A.M. , JCAP 1505 (2015) 005

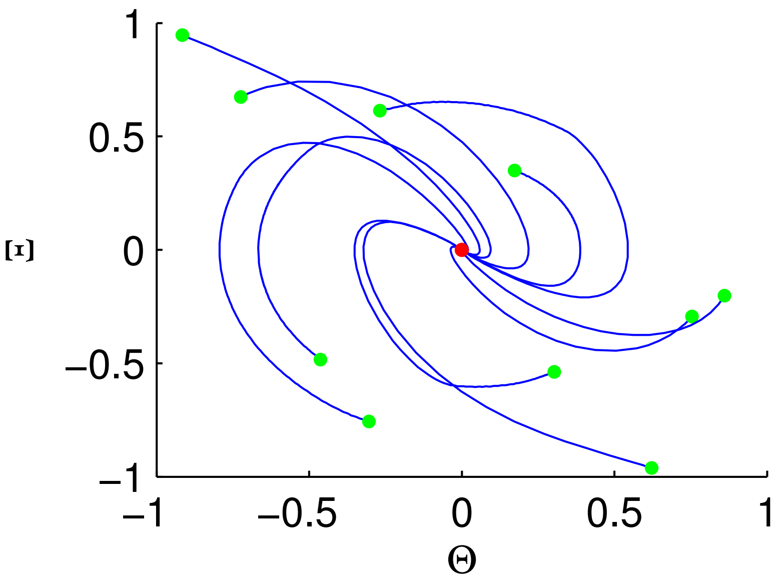
$$\tilde{t} \equiv t\,m, \qquad X \equiv \frac{m^2}{J} A^0, \qquad \Theta \equiv \frac{m^2}{J}\theta, \qquad \mathcal{H} \equiv H/m$$

$$X'' + 3\mathcal{H}X' + (3\dot{\mathcal{H}} + 1)X = 1/\left(1 + \dot{\mathcal{H}}/\mathcal{H}^2\right)$$

$$\mathcal{H}^2 \simeq \alpha^2 \left[\left(X - \dot{\Theta}\right)^2 + 2X/\left(1 + \dot{\mathcal{H}}/\mathcal{H}^2\right)\right]$$

$$\alpha \equiv \sqrt{\frac{4\pi}{3}} \left(\frac{J}{m^2 M_p}\right)$$





$$X' \equiv Y,$$

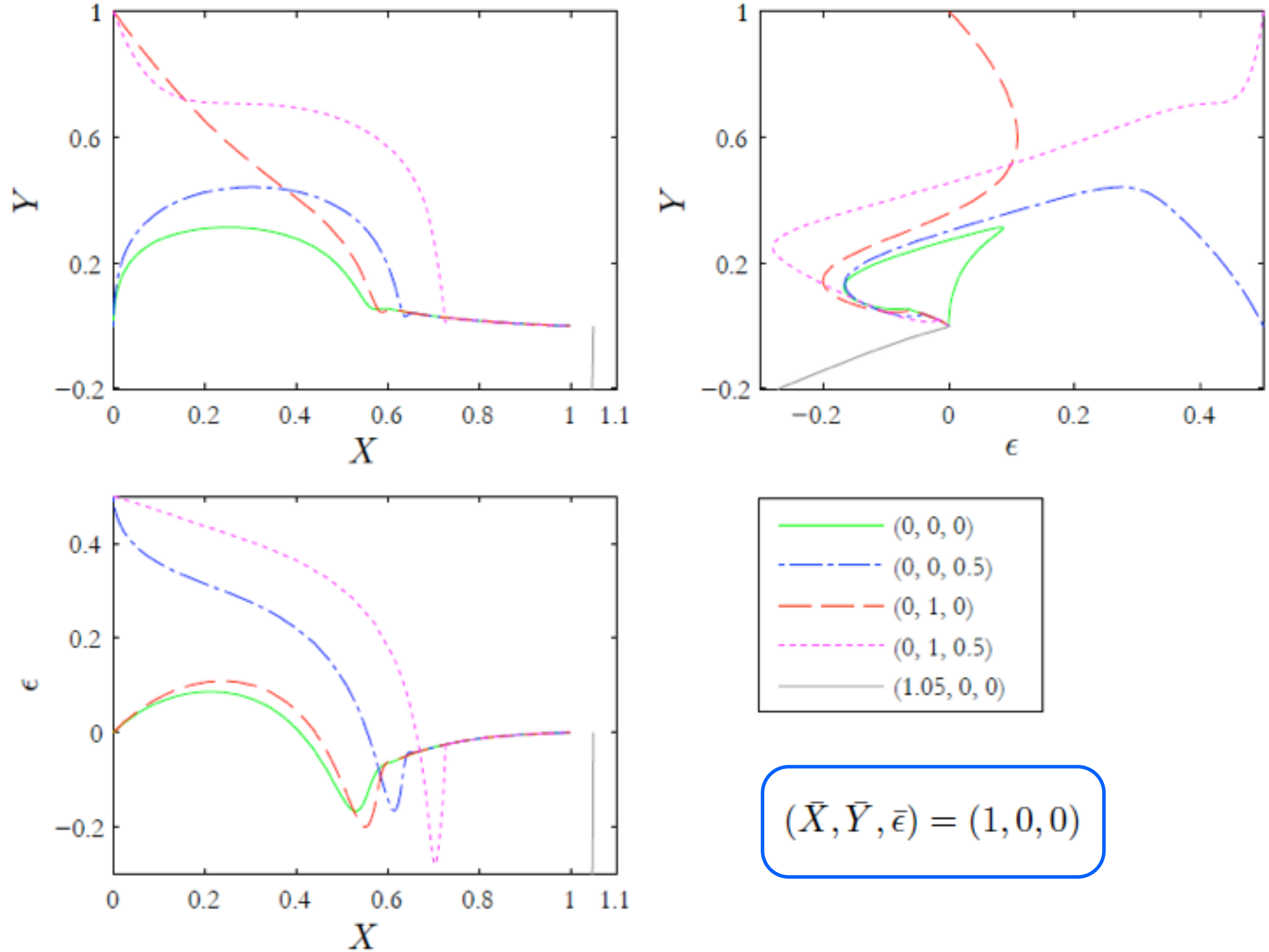
$$Y' = (1-\epsilon)^{-1} - X - 3\mathcal{H}Y + 3\mathcal{H}^2\epsilon X,$$

$$\epsilon' = (1-\epsilon)^2 \left[Y + \frac{3XY + \alpha^{-2}\epsilon\mathcal{H}^3}{X - (1-\epsilon)^{-1}}\right],$$

$$\frac{\mathcal{H}^2}{\alpha^2} = 3X^2 + \left(X - \frac{1}{1-\epsilon}\right)^2$$

Phase space trajectories

S. Alexander, D. Jyoti, A. Kosowsky & A.M. , JCAP 1505 (2015) 005



$$(\bar{X}, \bar{Y}, \bar{\epsilon}) = (1, 0, 0)$$

Stability landscape

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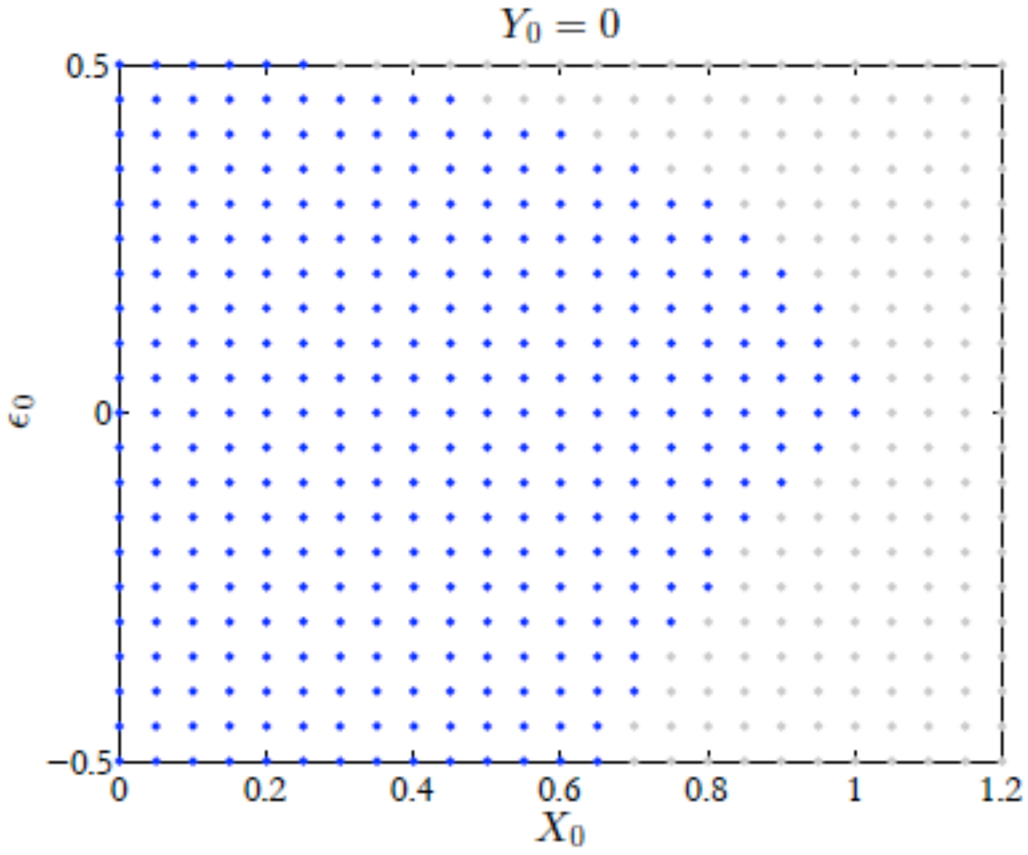
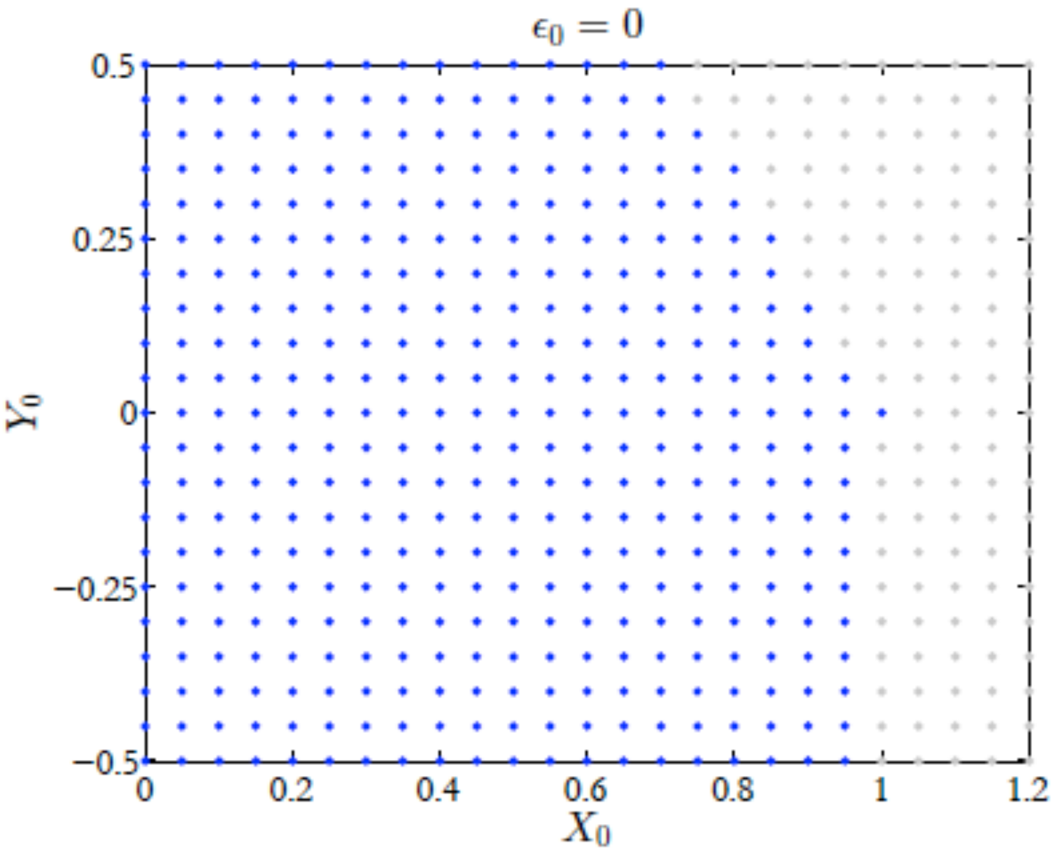
Initial conditions ending up at the inflationary fixed point





Phenomenology

S. Alexander, S. Brahma, P. Dona, A.M. & Z. Yang, to appear soon

Power spectrum of scalar perturbations from gauge fields

Scalar field: tilt

$$\mathcal{L}_{\text{YM}} + \frac{\theta}{4M_*} F_{\alpha\beta}\tilde{F}^{\alpha\beta} + \mathcal{L}_\theta$$

Gauge fields contribution

Chern-Simons

$$\mathrm{Tr}[\widetilde{\delta A_i}(\vec{k})] \simeq \frac{1}{k^{\frac{3}{2}}}$$

$$\mathcal{P}_A(k) \equiv \frac{k^3}{2\pi^2} |\mathrm{Tr}[\widetilde{\delta A_i}(\vec{k})]|^2 \simeq \text{const}$$

Chern-Simons term

Baryogensesis in the first version of the model

End Inflation (force term for the scalar field EOM)

Conformally invariant: consistent with Power spectrum invariance

In progress…

Phenomenological analysis: constraining the parameters

Reheating mechanism from fermion bilinears?

Non-gaussianeities and cross-correlation functions

Stay tuned!

Inflation and The Standard Model

- Guth originally proposed inflation by using ideas from:

  (1) Condensed Matter Physics: (Spontaneous symmetry breaking).

  (2) Particle Physics: SU(5) GUT.

- This idea did not work (too much fine-tuning in parameters of theory).

Open Questions for Inflation

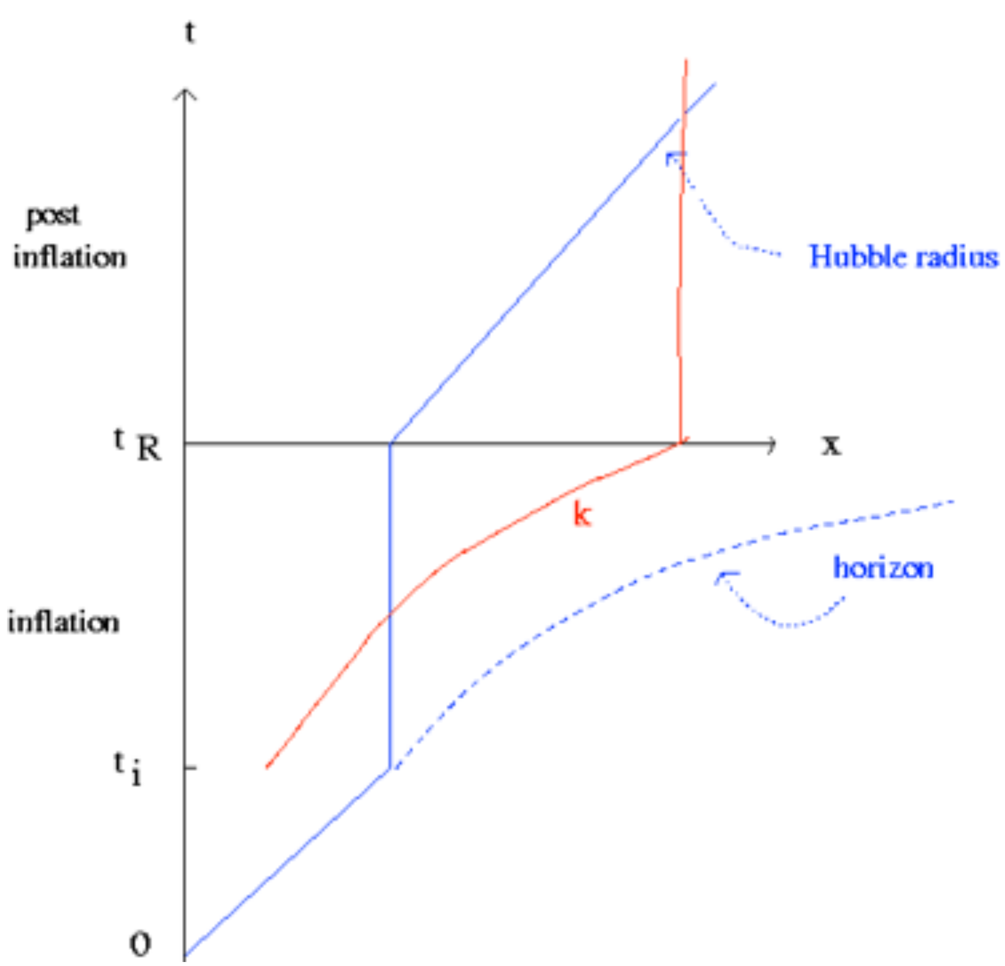
- What is the identity of the inflaton field?
- What happens "at or before" the Big-Bang singularity?

Structure formation: heuristics

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]$$

$$\phi(t, \vec{x}) = \phi_{\vec{k}}(t)\, e^{\imath \vec{k}\cdot\vec{x}}, \qquad \frac{d^2\phi_{\vec{k}}}{dt^2} + 3H\,\frac{d\phi_{\vec{k}}}{dt} + \frac{k^2}{a^2}\phi_{\vec{k}} = 0$$

Free massless scalar field



$$\frac{a}{k} \ll \frac{1}{H}$$ **harmonic oscillator behavior**

$$\frac{a}{k} \gg \frac{1}{H}$$ **overdamped oscillator**

Inflation: success and limits

Deficiencies of Inflation

Cosmological singularity: not a theory of very early Universe

High level of arbitrariness in the mechanism involving scalar field

Trans$-$Planckian problem for cosmological perturbations

Criteria to bear in mind

Horizon $\gg$ Hubble radius

Fluctuations mode have $\lambda \gg H^{-1}$for a long period (squeezing)

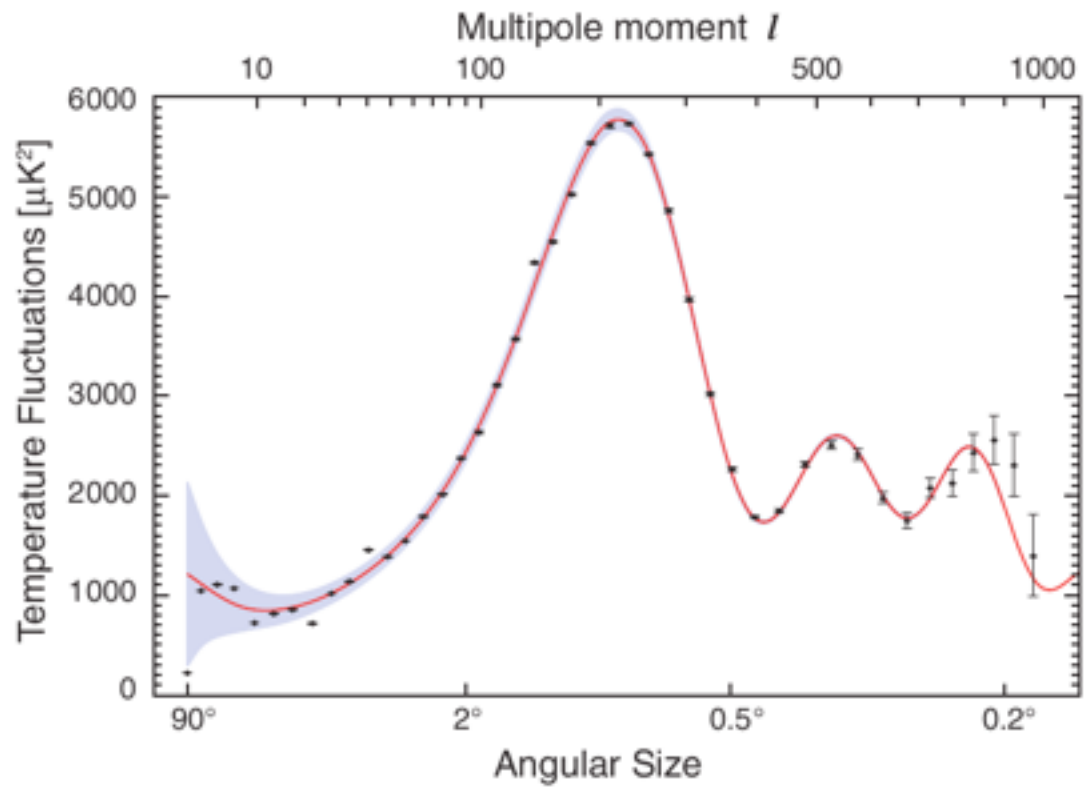
Mechanism accounting for scale$-$invariant primordial spectrum

Inflation

i) $\varrho \simeq \varrho_\phi \simeq \text{const} \to a(t) = e^{Ht}$

ii) $\varrho_K/\varrho_{\rm rad} \sim a(t)^2 \qquad \varrho_K/\varrho_\phi = 1/a(t)^2$

iii) Universe empty, then $\delta\phi$



$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \qquad \frac{\dot a^2}{a^2} \equiv H^2 = \frac{8\pi G}{3c^4}\varrho \qquad \varrho = \varrho_\phi + \varrho_{\rm rad} + \varrho_{\rm matt.} + \varrho_K$$

with a bonus!

Causal mechanism for generating primordial cosmological (Chibisov & Mukhanov 1981)

perturbations originate as quantum vacuum fluctuations!

GR21 NYC, 12th July 2016

Fermionic Current II

Feynman propagator

$$
\begin{aligned}
iS_F^{ab}(x,y) &= \langle 0|T\{\widehat{\psi}^a(x)\widehat{\overline{\psi}}^b(y)\}|0\rangle \\
&= \theta(\eta_x-\eta_y)\ \langle 0|\widehat{\psi}^a(x)\widehat{\overline{\psi}}^b(y)|0\rangle - \theta(\eta_y-\eta_x)\ \langle 0|\widehat{\overline{\psi}}^b(y)\widehat{\psi}^a(x)|0\rangle
\end{aligned}
$$

$$
\begin{aligned}
iS^{ab}(x,y) &= a(\eta_x)(i\gamma^\mu\nabla_\mu+m)\frac{H^2}{\sqrt{a(\eta_x)a(\eta_y)}}\left[iS_+(x,\,y)\frac{1+\gamma^0}{2}+iS_-(x,\,y)\frac{1-\gamma^0}{2}\right] \\
iS_\pm(x,\,y) &= \frac{\Gamma(1\mp i\frac{m}{H})\,\Gamma(2\pm i\frac{m}{H})}{(4\pi)^2\Gamma(2)}\,{}_2F_1\left(1\mp i\frac{m}{H},\,2\pm i\frac{m}{H},\,2,1-\frac{\Delta(x,y)}{4}\right)\,, \\
\Delta(x,y) &= a(x)a(y)H^2\Delta x^2\,, \qquad \Delta x^2 = (|\eta_x-\eta_y|-i\varepsilon)^2+(\vec{x}-\vec{y})\cdot(\vec{x}-\vec{y}),
\end{aligned}
$$

Fermionic current

$$
\langle 0|J^I|0\rangle \simeq\ \lim_{y\to x}\ S^{ab}(x,\,y)\gamma^I_{ba}
$$

$$
S^{ab}(x,y)\simeq\frac{H^2}{16\pi^2}\left[\frac{m^2}{H}\left(\frac{1+\gamma^0}{2}\right)^{ab}-\frac{m^2}{H}\left(\frac{1-\gamma^0}{2}\right)^{ab}\right]+O\left(\frac{m^2}{H^2}\right)
$$

$$
J^0\simeq\frac{1}{4\pi^2}\,m^2H+O\left(\frac{m^3}{H^3}\right)
$$