Viewing the shadow of black hole through a magnetised plasma

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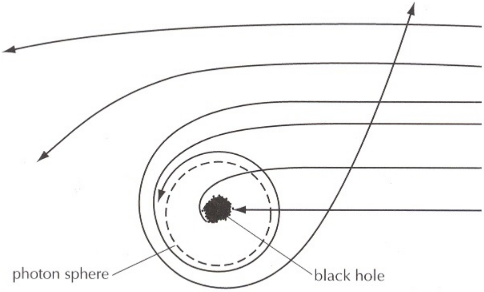
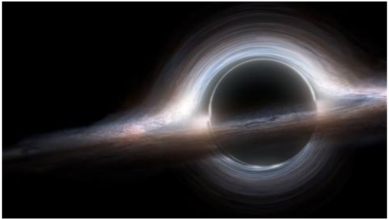
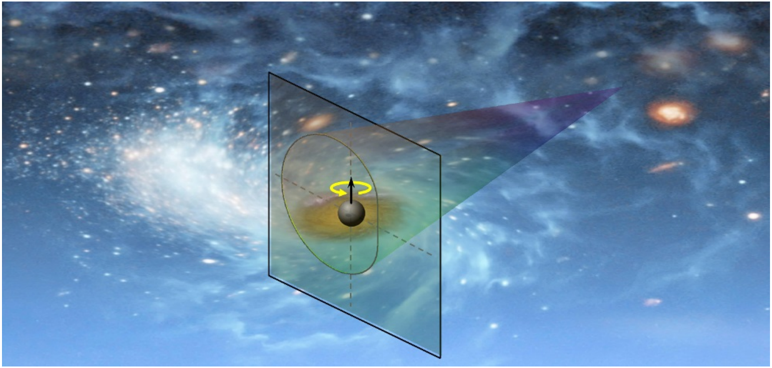
Lensing by black holes



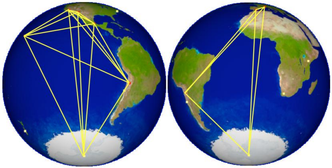


- Rays passing very close to a black hole experience large deflections
- Rays even closer fall into the photon sphere and remain trapped
- Photon sphere has a radius of $r_{ph} = \frac{3GM}{c^2}$
- Observer at r_0 sees a dark disk a.k.a. shadow

Shadows test strong field gravity



- An observer will (**ideally!**) see the shadow as a circle (Schwarzschild BH's) or an asymmetric prolate spheroid (Kerr BH's)
- **Significant** deviations from these shapes violate the no hair theorem, which implies that GR is falsified!



A planet-sized radio telescope combining existing and future arrays (VLBI). First detection in 2017?

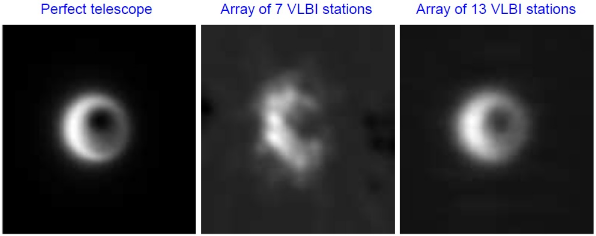


Figure: V.L. Fish, S.S. Doeleman, IAU Symposium 261, 1304 (2009)

Simulations of Sag A* demonstrate the improvement of image resolution as number of telescopes increases.

Photon orbits in Schwarzschild

- Consider photons travelling past a Schwarzschild black hole i.e. $ds^2 = -\left(1 - 2M/r\right) dt^2 - \left(1 - 2M/r\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$
- Photons travel on null geodesics of spacetime with 4–momentum p^μ
- From the Lagrangian $\mathcal{L} = g_{\mu\nu} p^\mu p^\nu = 0$ the equations of motion are

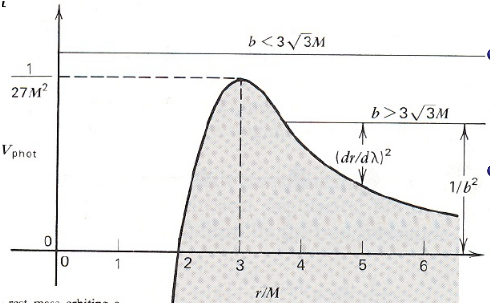
$$\frac{dt}{d\lambda} = \frac{1}{b\left(1 - 2M/r\right)} \tag{1}$$

$$\frac{d\phi}{d\lambda} = \frac{1}{r^2} \tag{2}$$

$$\left(\frac{dr}{d\lambda}\right)^2 = \frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2M}{r}\right) \tag{3}$$

where we've defined the impact parameter $b \equiv \frac{L}{E}$ in terms of the photon energy E and angular momentum L.

Introducing the effective potential, $V_{eff} \equiv \frac{1}{r^2}\left(1 - \frac{2M}{r}\right)$ we characterise photon trajectories via $\left(\frac{dr}{d\lambda}\right)^2 = \frac{1}{b^2} - V_{eff}$

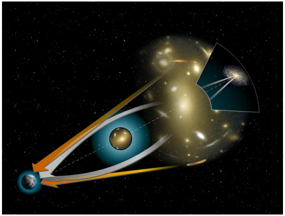


- Capture and scattering orbits separated by critical impact parameter $b_c = 3\sqrt{3}M$
- Photon sphere occurs at $r_{ph} = 3M$

In any static, spherically symmetric spacetime

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + D(r)d\Omega^2 \tag{4}$$

an observer at r_0 infers the angular size of a shadow, α_s
(From Perlick, Tsupko and Bisnovatyi-Kogan, PRD 2015)

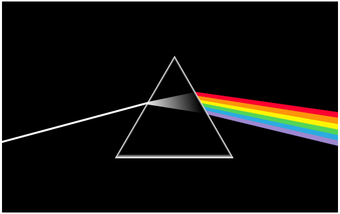


- General deflected rays obey $\cot^2\alpha = \frac{B(r)}{D(r)}\frac{dr}{d\phi}|_{r=r_0}$
- As $R \to r_{ph}$ we approach the shadow, $\alpha \to \alpha_{sh}$
- An observer at r_0 measures the angular size of a Schwarzschild shadow,

$$\sin^2\alpha_{sh} = \frac{27M^2(1-2M/r_0)}{r_0^2} \tag{5}$$

Galaxies are hazy

- We don't observe in a vacuum. Sag A* is engulfed by magnetic fields and a plasma i.e. ionised gases
- Plasmas are dispersive i.e. light waves of different frequencies (or wavelengths) travel at different speeds (Waters *et al* 1973)



- Shadow shape and size – observed through interstellar medium – is frequency–dependent!

Rays in plasmas in curved spacetimes

- Plasmas are characterised by their plasma frequency

$$\omega_p^2(r) = \frac{4\pi e^2}{m} N(r) \tag{6}$$

where e, m, N are the electron charge, mass and number density.

- The refractive index of a plasma is $n^2 = 1 - \frac{\omega_p^2}{\omega^2}$
- The Hamiltonian for light rays in a plasma is $H = \frac{1}{2}\left(g^{\mu\nu} p_\mu p_\nu + \omega_p^2\right)$ See Synge (1966)
- The dynamics follow from

$$\dot{p}_\mu = -\frac{\partial H}{\partial q^\mu} \tag{7}$$

$$\dot{q}_\mu = \frac{\partial H}{\partial p_\mu}. \tag{8}$$

Shadows in plasmas

Lensing by black holes is now frequency–dependent! (Perlick, Tsupko and Bisnovatyi-Kogan, PRD 2015)

In vacuo:

- $r_{ph} = 3M$
- $\sin^2 \alpha_{sh} = \frac{27M^2(1-2M/r_0)}{r_0^2}$

In a plasma:

- Photon sphere at roots of $0 = \frac{r(r-3M)}{(r-2M)^2} - \frac{\omega_p^2}{E^2} - \frac{\omega_p \omega_p'}{E^2} r$
- $\sin^2 \alpha_{sh} = \frac{r_{ph}^2}{r_0^2} \frac{\left(r_{ph}/(r_{ph}-2M) - \omega_p^2(r_{ph})/E^2\right)}{\left(r_0/(r_0-2M) - \omega_p^2(r_0)/E^2\right)}$

Require knowledge of the number density; typically "plasmas shrink shadows"

Waves in magnetised plasma

- Simplest, flat spacetime case is complicated!
- $\mathbf{B} = \text{const}\hat{\mathbf{n}}$ $p_0, \rho_0 = \text{const.}$ $\mathbf{u}_0 = 0$
- waves propagate at angle ψ w.r.t $\mathbf{B}$ i.e. $\mathbf{k} \cdot \mathbf{B} = kB\cos\psi$
- fluid displacements in the $(\mathbf{k}, \mathbf{B})$-plane are fast and slow MHD modes

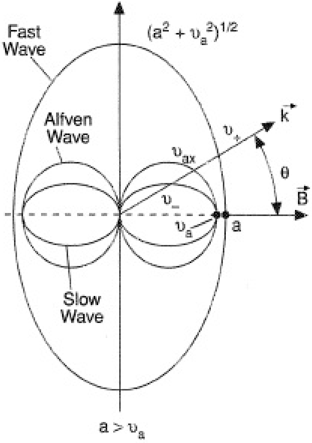
$$\frac{\omega^2}{k^2} = \frac{1}{2}\left(v_A^2 + a_s^2\right) \pm \frac{1}{2}\left[\left(v_A^2 + a_s^2\right)^2 - 4v_A^2 a_s^2 \cos^2\psi\right]^{1/2} \tag{9}$$

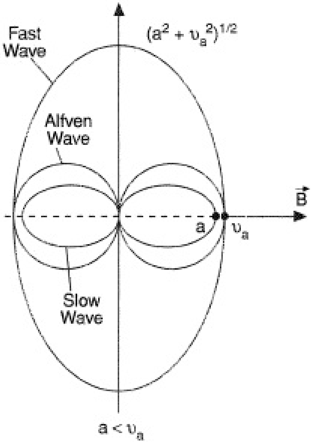
- fluid displacements orthogonal to $(\mathbf{k}, \mathbf{B})$-plane are Alfven modes

$$\frac{\omega^2}{k^2} = v_A^2 \cos^2\psi \tag{10}$$

MHD wave geometry

Wave propagation in magnetised plasma is dispersive and anistropic





Outlook

- More realistic **B** eg. toroidal or poloidal fields
- Accurate models of plasmas surrounding SMBH's
- Arbitrary wave propagation angle, ψ
- Faraday rotation
- Rigorous derivation of Hamiltonian (Breuer and Ehlers, 1980)
- Kerr spacetime
- Unmodeled, known physics or modeled, unknown physics?