The incredible properties of Epsilon Near Zero materials:

Analogue cosmological quantum particle creation

Angus Prain

Heriot-Watt University, Edinburgh, **Scotland**

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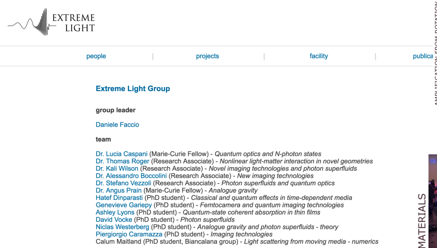
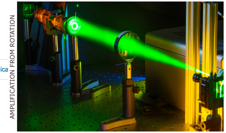
Columbia University, New York

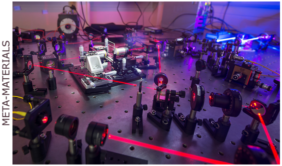
The lab and group

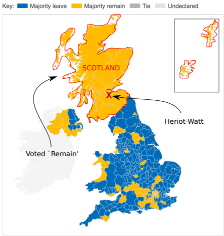
Extreme light group[1] at Heriot-Watt: Non-linear optics group interested in a variety of analogue gravity experiments

- 1D black holes in optical fibres – dispersive Hawking mechanism (**long story**)
- Cosmological expansion in ‘epsilon near zero’ materials – dynamical particle creation (**this talk**)
- Rotating black holes in ‘photon fluids’ – Zel’dovich effect, superradiance (**the future**)









[1] http://extremelight.eps.hw.ac.uk/

This talk in one slide

- Homogeneous time dependent environments spontaneously produce quanta from the vacuum e.g. cosmological $a(t)$.

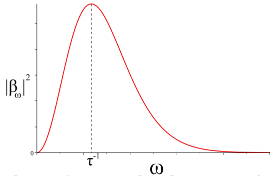
- There is an **analogy** between gravity and dielectrics for electromagnetic wave propagation:

$$ds^2 = -dt^2 + h_{ij}(t,x)dx^i dx^j, \longleftrightarrow D^i = \epsilon^{ij}(t,x)E_j$$
$$\mu_{ij},\, \epsilon^{ij} = \sqrt{\mathbf{g}}\, h^{ij}$$

- The quanta that are most abundantly produced have a frequency

$$\omega_{\text{peak}} = \max \frac{\dot{a}}{a}$$

and the number spectrum $k^2|\beta_k|^2$ usually looks like this:



- **However**, in optics 'normal' laboratory conditions are **adiabatic** and **perturbative** so we're in the exponential tail and the peak is small

- **New Physics**: There is an **exciting** and **revolutionary** new kind of material which beats both the adiabatic and perturbative limits: Epsilon Near Zero (ENZ) materials (I called these thin film metamaterials in original talk title but they're not really metamaterials. . .)

Moral of the story: Cosmological particle creation in the lab just got a massive boost

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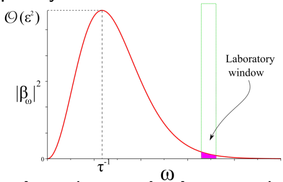

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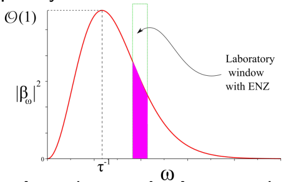

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What analogy? Why **perturbative** and why **adiabatic**?

Assuming a homogeneous variation of permittivity is possible (subtlties) the wave equations for the electric field are

$$\nabla^2 E = \frac{\partial}{\partial t} a(t) \left(\frac{\partial}{\partial t} a(t)\, E \right) \qquad \text{– cosmology}$$

$$\nabla^2 E = \frac{\partial}{\partial t} \mu(t) \left(\frac{\partial}{\partial t} \epsilon(t)\, E \right) \qquad \text{– optics}$$

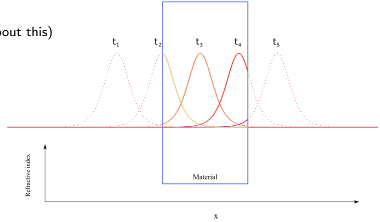
- Its an analogy at the level of the wave equation, not just geometric optics...
- We're not talking about an analogy with a scalar field, we're talking about electromagnetism...

Caveat: We need $\epsilon(t) \propto \mu(t) \in \mathbb{R}$ (lets talk about this)

Mechanism: Non-linear Kerr effect:

$$\delta n \propto |E|^2$$

Typically[2] $\delta n \simeq 10^{-4} - 10^{-3}$



Numbers Typical optical frequencies are of the order of fs $\sim 10^{-15}$s; solitons which induce the variations can *at very best* be of the oder of 10s to 100 fs $\sim 5 \times 10^{-14}$s so the picture is more like:

[2] Intensity can be 10^{13} W/cm^2, Kerr index is about 10^{-16}cm^2/W – this is **non-linear** optics after all...

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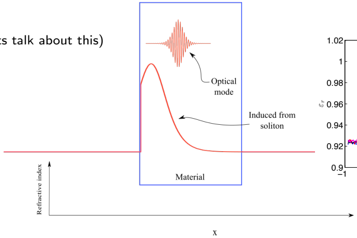

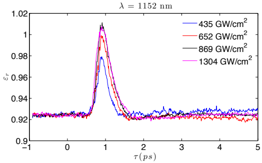

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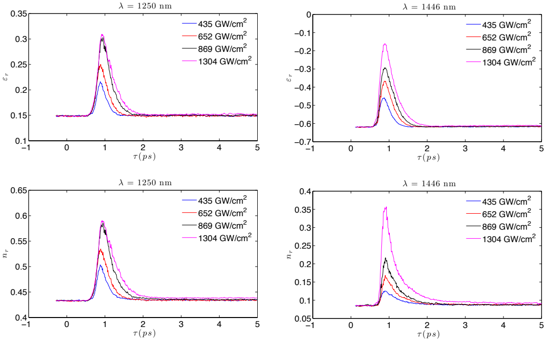
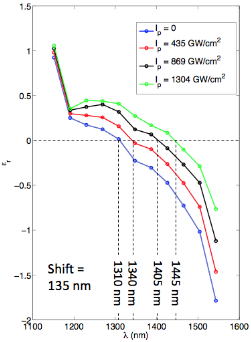
Enter ENZ materials!

The ENZ material: 'oxygen-deprived aluminium-doped zinc oxide' (AZO)

Epsilon Near Zero means that its background value can officially *be* zero or even **negative**.

The non-linear (Kerr) response is always positive.





- Huge non-linear response
- Extremely fast response
- Massive relative changes

There is always an 'imaginary part' so to get *n* and the damping you need to take the complex square root:

$$\text{Refractive index} = \mathcal{R}e\sqrt{\epsilon + i\epsilon''}$$

$$\text{Damping coefficient} \simeq \mathcal{I}m\sqrt{\epsilon - i\epsilon''} \quad (0.1)$$

A prediction: Lots more quanta produced near ENZ

Equation of motion:

$$\frac{d^2}{dt^2}E_k + \omega^2(t)E_k = 0, \qquad \omega^2(t) = \frac{c^2k^2}{\epsilon(t)}$$

How to do the QFT calculations: Options

1) Reduce problem to 'Bremmer' integral

$$\beta := \frac{1}{2}\int_{-T}^{T} \frac{\dot{\omega}}{\omega} e^{-2\imath \int^t \omega(s)ds} dt$$

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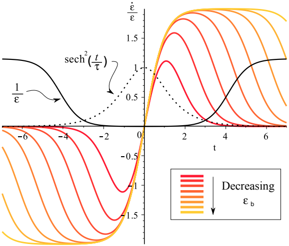

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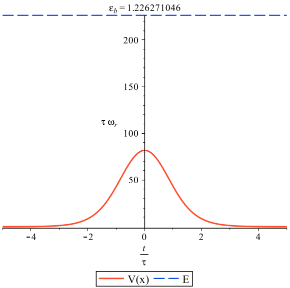

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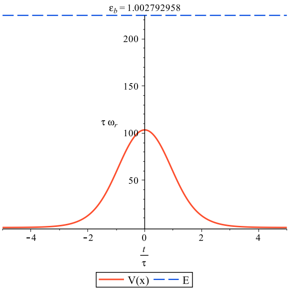

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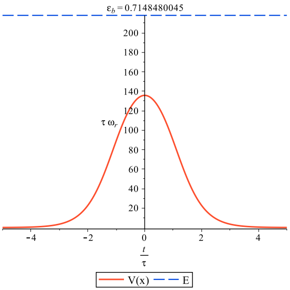

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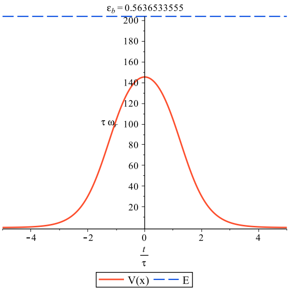

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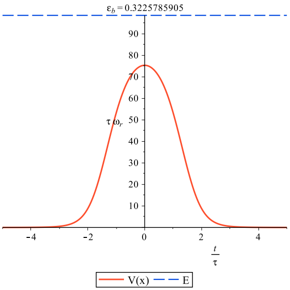

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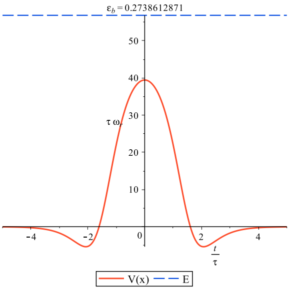

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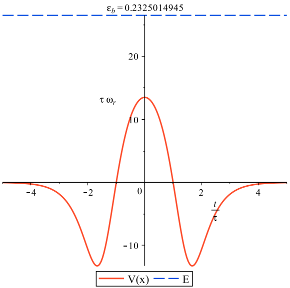

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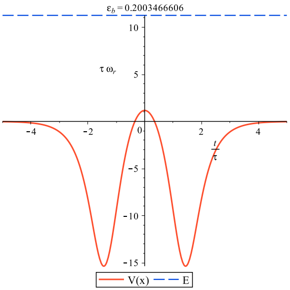

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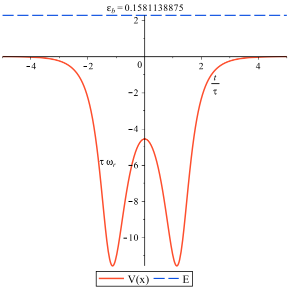

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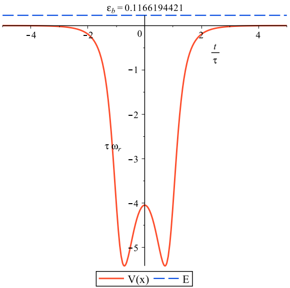

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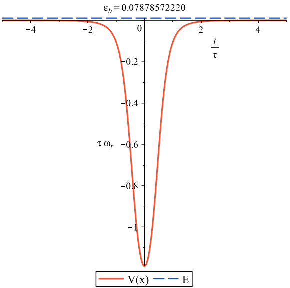

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1) Reduce problem to 'Bremmer' integral

$$\beta := \frac{1}{2}\int_{-T}^{T} \frac{\dot{\omega}}{\omega} e^{-2\imath \int^t \omega(s)ds} dt$$

2) Re-cast as QM scattering problem, use exact solutions from the 1950s:

$$\frac{d^2}{dx^2}\psi + \frac{2m}{\hbar^2}(E - V(x))\psi = 0, \quad |\beta|^2 = \frac{1}{T} - 1$$

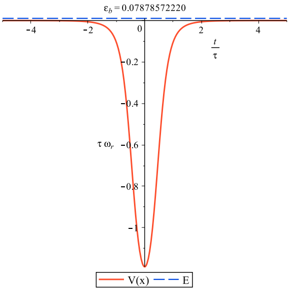
Predicted number of photons:

$$k^2|\beta_k|^2 \times \text{Vol} = 10^{-\text{stupid}} \qquad \text{– not at ENZ wavelength}$$

$$= \mathcal{O}(1)! \qquad \text{– at ENZ wavelength}$$

Next step: Actually do the experiment.

A prediction: Lots more quanta produced near ENZ



Equation of motion:

$$\frac{d^2}{dt^2}E_k + \omega^2(t)E_k = 0, \qquad \omega^2(t) = \frac{c^2k^2}{\epsilon(t)}$$

How to do the QFT calculations: Options

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Next step: Actually do the experiment.

Take home Messages

- Non-linear optics is a great place to do analogues – high precision, very active field, technologically advanced
- Scotland voted to remain – and remain eligible for MC fellowships / ERC grants
- The 'cosmological' quantum effects just got way more observable thanks to a revolutionary new kind of material
- We'll be doing the experiments soon. . .

Thank you

Epilogue - loose ends

- Co-moving black holes induced by high intensity laser pulse and the non-linear Kerr effect in media.
- The cosmology is in the orthogonal subspace
- Dispersion is accounted for by working at a single wavelength which is conserved
- The optics people are interested in the ‘redshift’ which in this case would be a ‘cosmological redshift’