arXiv:1510.06245

# An improved analytic EMRI waveform model

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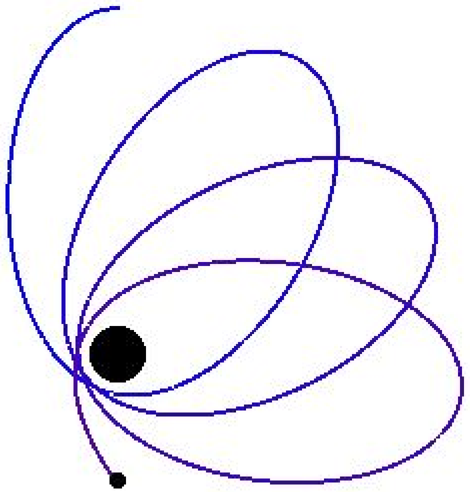
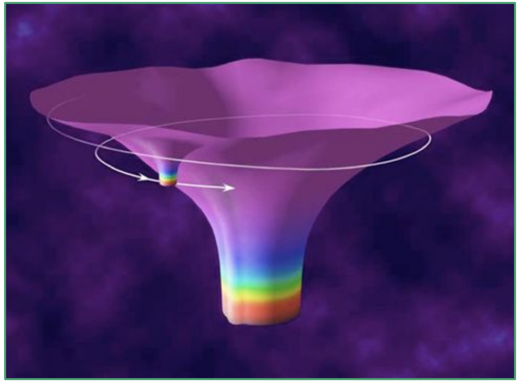
# Outline

- Extreme-mass-ratio inspirals (EMRIs)
  - Detection and parameter estimation
  - Waveform inventory
- The fast and the fiducial
  - Numerical kludge (NK)
  - Analytic kludge (AK)
- Augmenting the AK model
  - Map to Kerr fundamental frequencies
  - Phase-space trajectory corrections
  - Benchmarking



# Extreme-mass-ratio inspirals

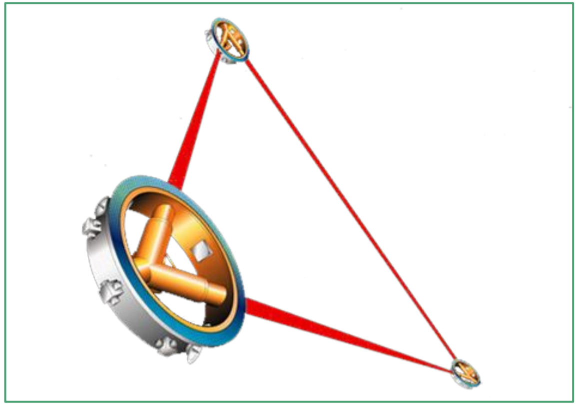
- Capture of stellar-mass compact object by supermassive black hole
- Adiabatic orbital decay due to radiation reaction
  - Inspiralling: Orbital separation decreases
  - Circularisation: Orbital eccentricity decreases
- Pronounced relativistic effects in last few years
  - Periapsis precession
  - Lense-Thirring precession
- All imprinted on gravitational-wave signal



NASA

# Gravitational waves from EMRIs

- Source for millihertz space-based detectors
- Inspiral in $10^{-4}$-$10^{-1}$ Hz band before plunge
- Long duration in band: Months to years
- Complex waveforms: $10^5$ cycles
- Can retain significant eccentricity in band
- Signal-to-noise ratio dominated by inspiral



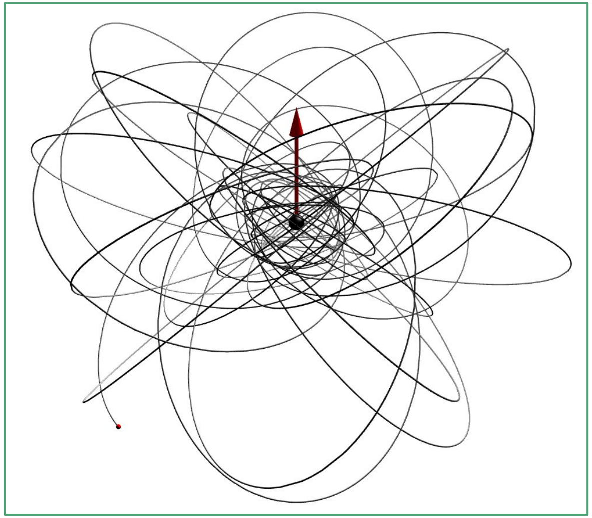
NASA/JPL-Caltech

# EMRI detection and parameter estimation

- Large parameter space
  - Dimensionality: 14
  - Fisher-Rao volume: $10^{30}$ templates needed to cover space
- Hierarchical search
  - Short-duration templates for detection (identification of candidates and rough localisation)
    - Must use **computationally efficient** waveform models
  - Long-duration templates for parameter estimation (precise localisation)
    - Must use **accurate** waveform models
  - Desirable to improve speed/accuracy trade-off for both detection and measurement templates

# EMRI waveform inventory

- Extreme mass ratio and number of orbits is challenging for numerical relativity; black hole perturbation theory is current state of the art
- Self-force waveforms
  - Dissipative and conservative effects beyond first order
- Teukolsky waveforms
  - Dissipative first-order effects only
- Kludge waveforms
  - Numerical kludge
  - Analytic kludge
  - Post-Newtonian (PN) expansion of Teukolsky solutions



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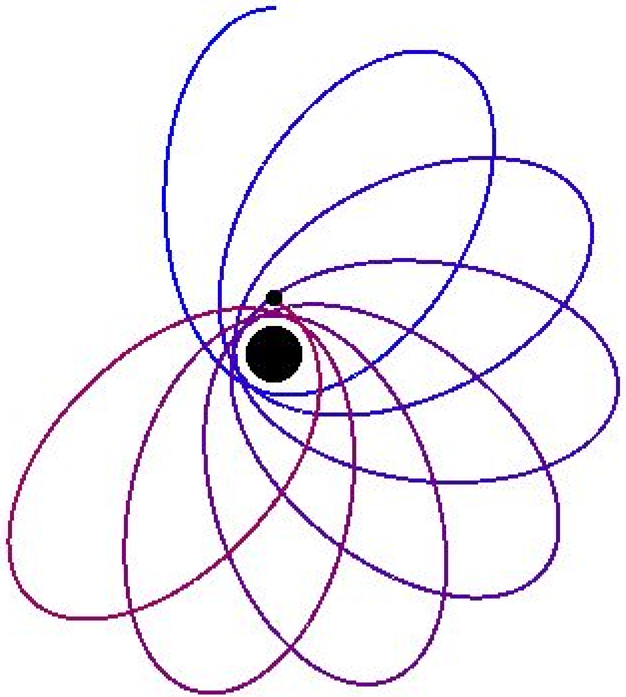
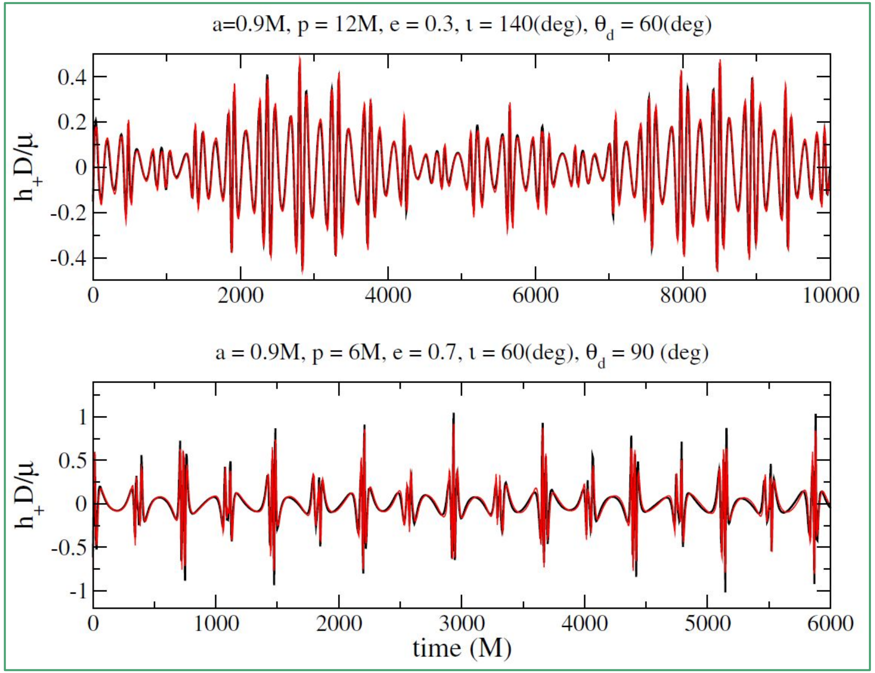


# The fiducial waveform: Numerical kludge

- Orbital evolution in curved space; waveform generation in flat space
  - Babak, Fang, Gair, Glampedakis & Hughes (2007)
- Trajectory built out of **Kerr geodesics**
  - Phase space: Evolve orbital constants with Teukolsky-fitted PN equations
    - Introduces radiation reaction
  - Configuration space: Integrate Kerr geodesic equations along phase-space trajectory
    - Yields relativistic precession for free
- Waveform generation: Quadrupole-octupole formula
  - Curved-space coordinates of compact object associated artificially with flat-space coordinates

# The fiducial waveform: Numerical kludge

- Good agreement with Teukolsky waveforms in strong-field regime
- Matches of over 97%; accurate enough to serve as detection templates
- But still rather unwieldy for online analysis



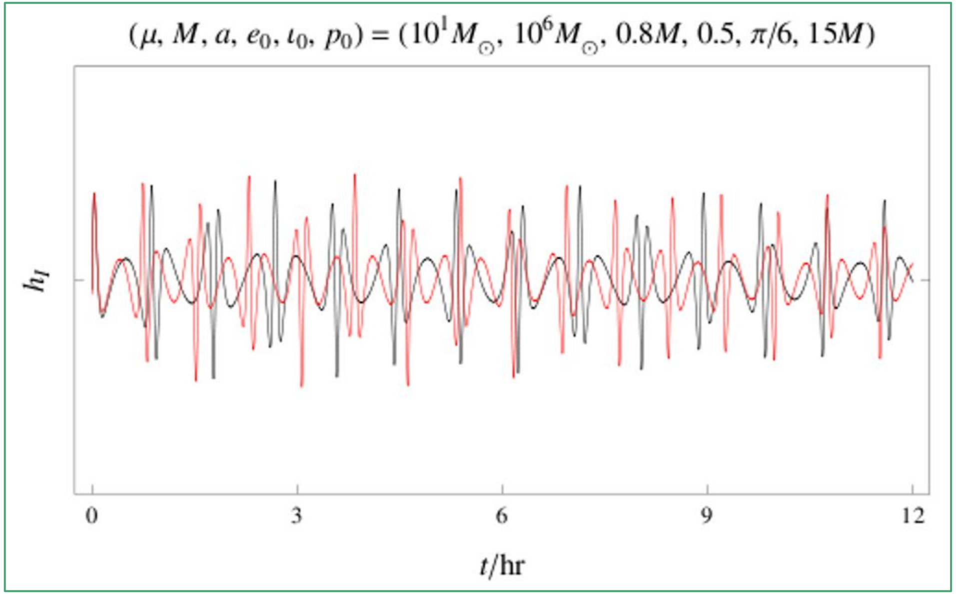

Babak et al. (2007)

# The fast waveform: Analytic kludge

- Both orbital evolution and waveform generation in flat space
  - Barack & Cutler (2004)
- Trajectory built out of **Keplerian ellipses**
  - Phase space: Evolve orbital constants with PN equations
    - Introduces radiation reaction
  - Configuration space: Evolve ellipse orientation with PN equations
    - Introduces relativistic precession
- Waveform generation: Quadrupole formula
  - Peters-Mathews approximation: Decomposition into harmonics of Keplerian orbital frequency

# The fast waveform: Analytic kludge

- Poor agreement with NK waveforms even at early inspiral
- One full cycle out of phase within **hours**
- But an order of magnitude faster to generate




Chua & Gair (2015)

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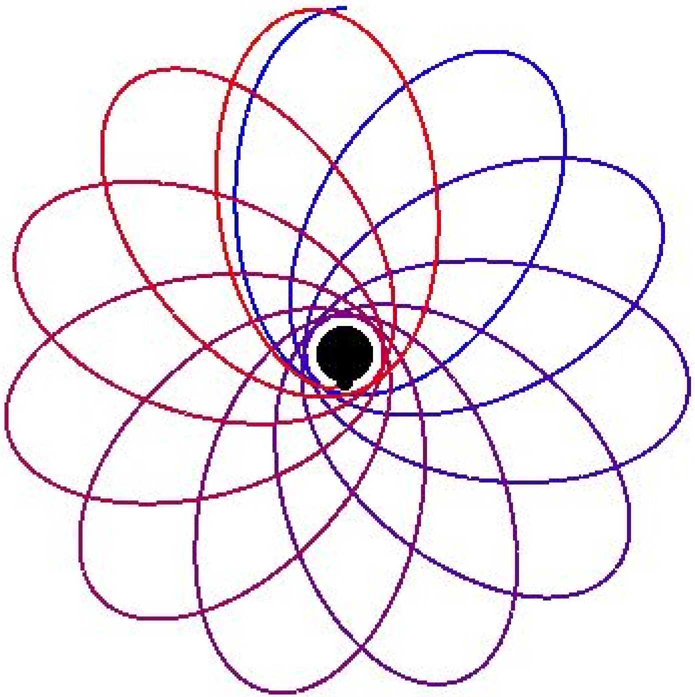


# Kerr fundamental frequencies

- Kerr geodesic equations parametrised by Mino time $\lambda$

$$\frac{dr}{d\lambda} = \sqrt{R(r)}, \quad \frac{d\theta}{d\lambda} = \sqrt{\Theta(\theta)}, \quad \frac{d\phi}{d\lambda} = \Phi(r,\theta), \quad \frac{dt}{d\lambda} = T(r,\theta)$$

- Manifestly periodic in $r$ and $\theta$

$$\Omega_r \propto \left( \int \frac{dr}{\sqrt{R(r)}} \right)^{-1}, \quad \Omega_\theta \propto \left( \int \frac{d\theta}{\sqrt{\Theta(\theta)}} \right)^{-1}$$

- Periodic on average in $\varphi$ and $t$

$$\Omega_\phi \propto \left\langle \iint d\lambda \, d\lambda' \, \Phi(r(\lambda), \theta(\lambda')) \right\rangle, \quad \Omega_t \propto \left\langle \iint d\lambda \, d\lambda' \, T(r(\lambda), \theta(\lambda')) \right\rangle$$

# Frequency map

- Consider a point ($e$,$\iota$,$p$) on phase-space trajectory of an EMRI
  - In NK model this is a Kerr geodesic; in AK model this is a rotating ellipse
- Match Keplerian, periapsis and Lense-Thirring frequencies for AK ellipse to correct combinations of fundamental frequencies for NK geodesic
- Induces a 3-D map in AK parameter space; we choose to map ($M$,$a$,$p$)

$$(M, a, p) \mapsto (\tilde{M}, \tilde{a}, \tilde{p})$$

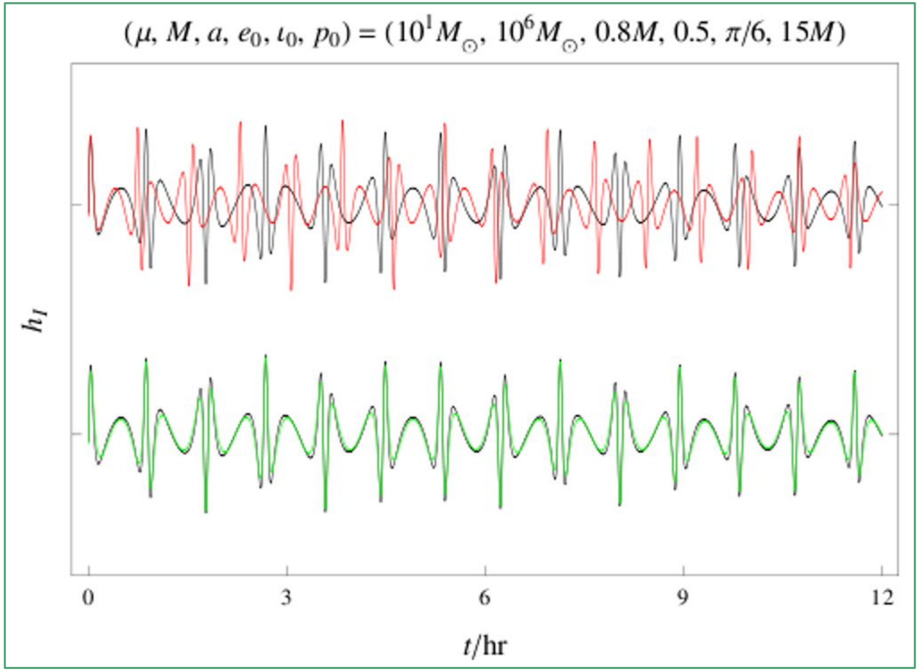
$$\omega_{\rm Kep}\big|_{(\tilde{M},\tilde{a},\tilde{p})} = \Omega_r/\Omega_t\big|_{(M,a,p)}$$

$$\omega_{\rm peri}\big|_{(\tilde{M},\tilde{a},\tilde{p})} = (\Omega_\phi - \Omega_r)/\Omega_t\big|_{(M,a,p)}$$

$$\omega_{\rm LT}\big|_{(\tilde{M},\tilde{a},\tilde{p})} = (\Omega_\phi - \Omega_\theta)/\Omega_t\big|_{(M,a,p)}$$

# Frequency map

- Provides an instantaneous correction of AK waveforms at any point in phase space
- AK waveforms with initial map show good agreement with NK waveforms at early inspiral
- Dephasing occurs over **days** rather than hours



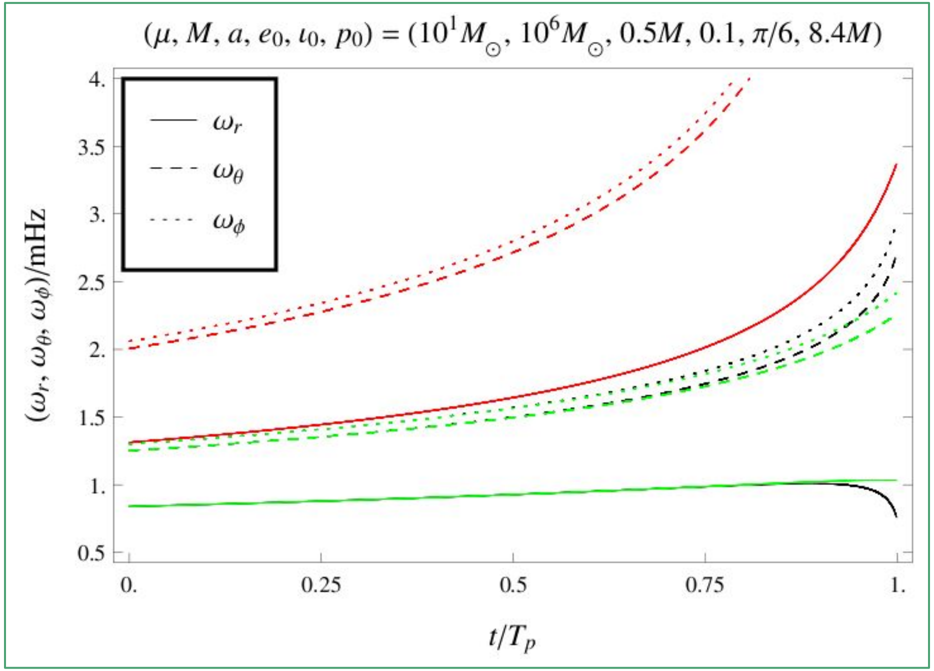

Chua & Gair (2015)

# Phase-space trajectory corrections

- Evolution of AK trajectory from initially mapped point can be improved
- Higher-order (in both mass ratio and eccentricity) PN equations for orbital constants and fundamental frequencies
    - Sago & Fujita (2015)
- Local fit to mapped trajectory of more accurate model
    - Map computed for three consecutive points on NK trajectory
    - Difference between AK and mapped NK trajectories extrapolated as quadratic
    - Unphysical black hole mass and spin are allowed to evolve in AK phase space as well

# Phase-space trajectory corrections

- Augmented AK waveforms stay in phase with NK waveforms for **months**
- Added computational cost from map-and-fit is negligible



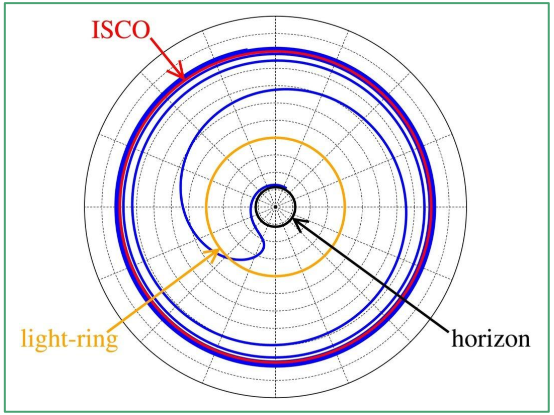

Chua & Gair (2015)

# Benchmarking the augmented AK

- Waveform accuracy
  - Matches of over 90% with NK for low-eccentricity orbits up to around $p = 6M$
  - Structure is captured less well for more eccentric orbits (without summing extra harmonics)
  - Both initial map and subsequent evolution perform less well further into strong field
- Computational cost
  - Order of magnitude faster than NK for year-long low-eccentricity waveforms
  - More speedup over NK for longer waveform durations
  - Less speedup over NK for higher eccentricities

# Work in progress

- Higher-order polynomial fit to mapped trajectory of more accurate model
  - Keep added computational cost low (under 1%)
- Improved behaviour at plunge
  - Fast internal calculation of last stable orbit
  - Smooth truncation: Sophisticated treatment not necessary
- Release version of code
  - Publicly available soon in preparation for next round of mock LISA data challenges

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# Summary

- EMRI detection requires fast waveform models that are as accurate as possible
- Fast AK model is augmented with frequency map and trajectory corrections
- Augmented AK stays in phase with more accurate waveforms for months
- Key references

    - L. Barack & C. Cutler, *LISA capture sources: Approximate waveforms, signal-to-noise ratios, and parameter estimation accuracy*, Phys. Rev. D 69:082005, 2004.
    - S. Babak et al., *"Kludge" gravitational waveforms for a test-body orbiting a Kerr black hole*, Phys Rev. D 75:024005, 2007.
    - A. J. K. Chua & J. R. Gair, *Improved analytic extreme-mass-ratio inspiral model for scoping out eLISA data analysis*, Class. Quantum Grav. 32:232002, 2015.