Orbital Dynamics of Eccentric Compact Binaries

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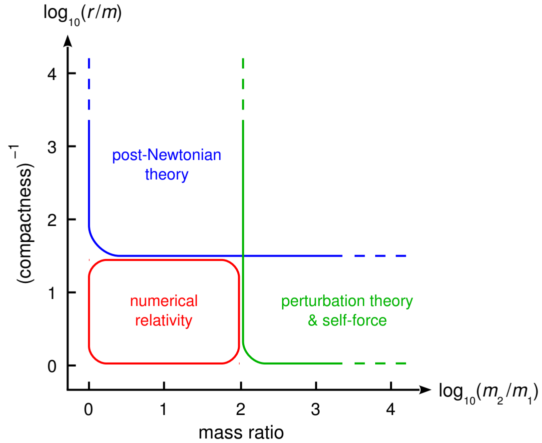
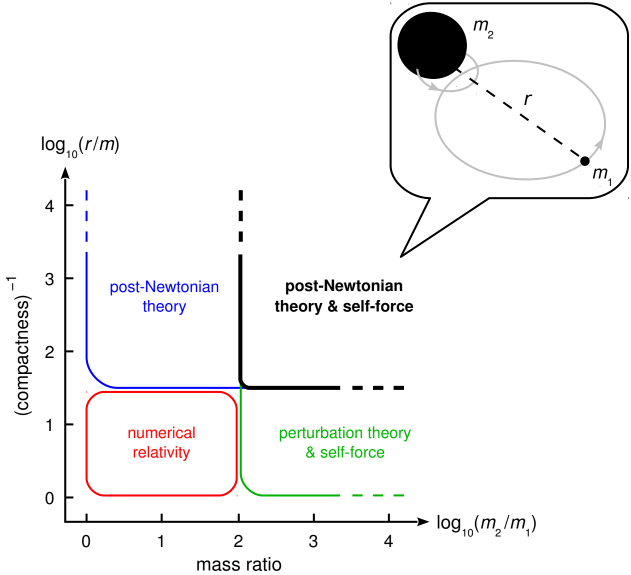
Phys. Rev. D **91** 124014 (2015), arXiv:1503.01374 [gr-qc]
Phys. Rev. D **92** 084021 (2015), arXiv:1506.05648 [gr-qc]



$\log_{10}(r/m)$
(compactness)$^{-1}$
4
3
2
1
0
post-Newtonian theory
numerical relativity
perturbation theory & self-force
0
1
2
3
4
$\log_{10}(m_2/m_1)$
mass ratio



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r
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Averaged redshift for eccentric orbits

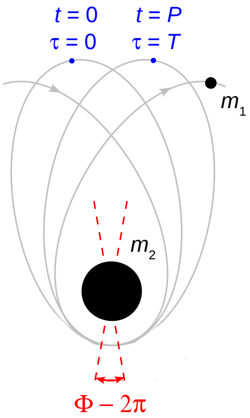
- Generic eccentric orbit parameterized by the two frequencies

$$n = \frac{2\pi}{P}\,, \quad \omega = \frac{\Phi}{P}$$

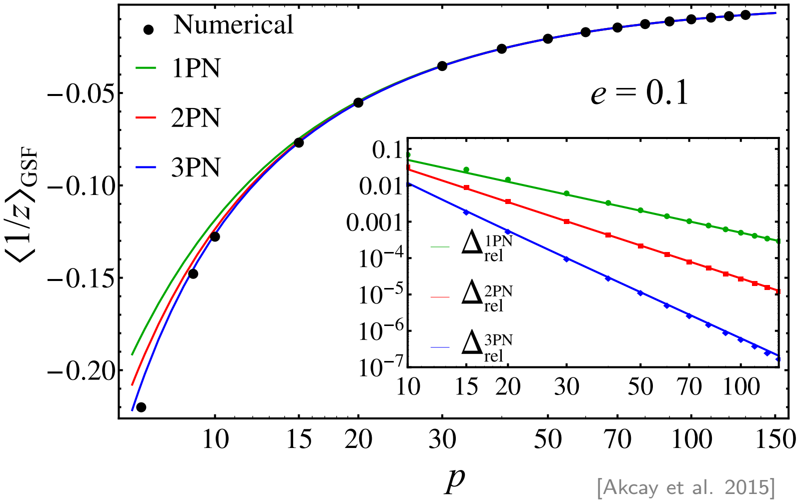
- Time average of redshift $z = \mathrm{d}\tau/\mathrm{d}t$ over one radial period

$$\langle z \rangle \equiv \frac{1}{P}\int_0^P z(t)\,\mathrm{d}t = \frac{T}{P}$$

- Coordinate-invariant relation $\langle z \rangle(n, \omega)$ is well defined in GSF and PN frameworks



Averaged redshift *vs* semi-latus rectum



[Akcay et al. 2015]

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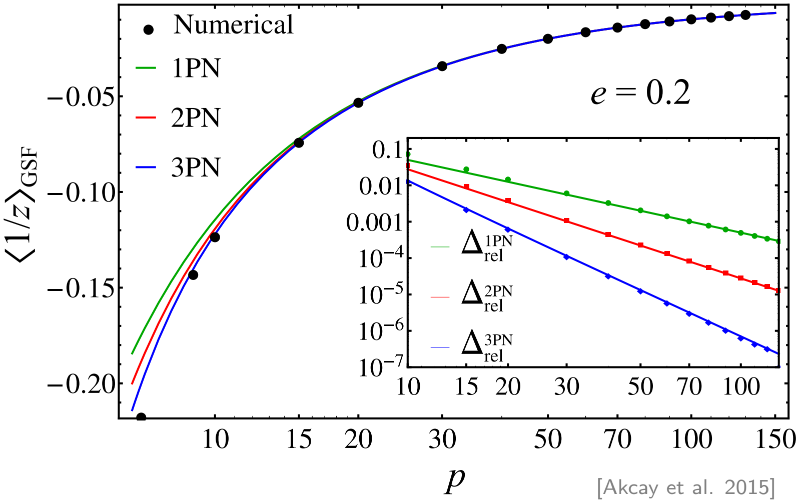

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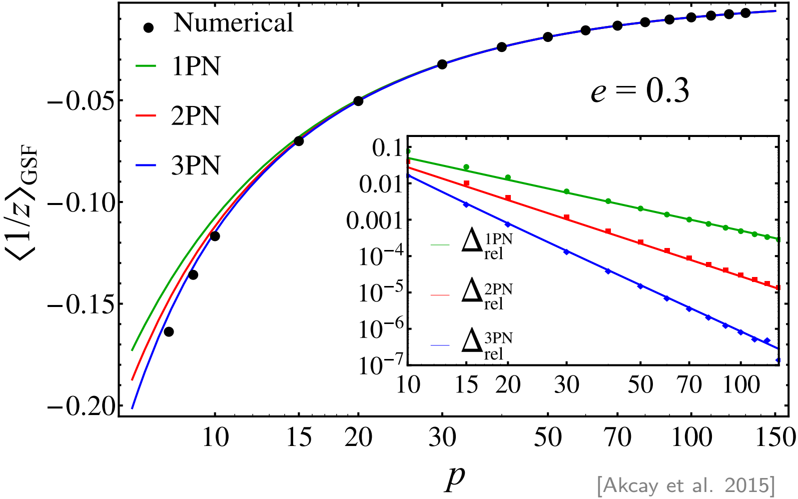

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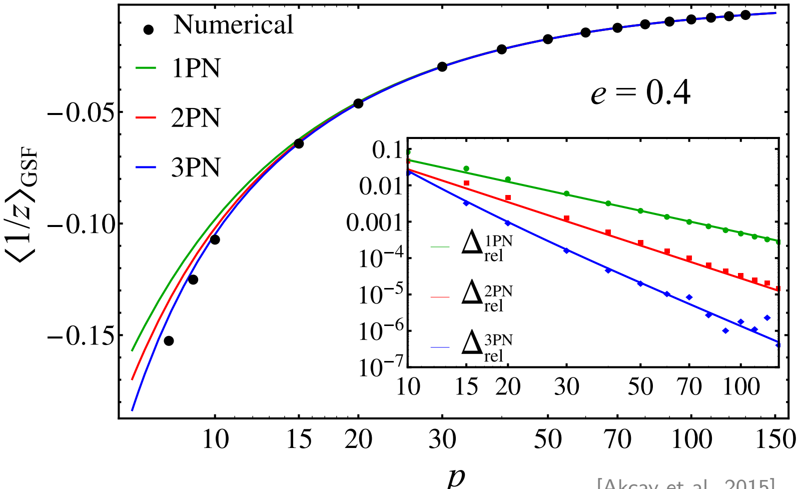

[Akcay et al. 2015]

Extracting post-Newtonian coefficients

	Coeff.	Exact value [Akcay et al. 2015]	Fitted value [Akcay et al. 2015]	Fitted value [Meent, Shah 2015]
1PN	e^2	4	4.0002(8)	$4 \pm 6 \times 10^{-12}$
	e^4	-2	$-2.00(1)$	$-2 \pm 4 \times 10^{-10}$
	e^6	0		$0 \pm 4 \times 10^{-9}$

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2PN	e^2	7	$7.02(2)$	$7 \pm 6 \times 10^{-9}$
	e^4	$1/4$		$1/4 \pm 4 \times 10^{-7}$
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3PN	e^2	$-14.312097\cdots$	$-14.5(4)$	$-14.3120980(5)$
	e^4	$83.382963\cdots$		$83.38298(7)$
	e^6	$-36.421975\cdots$		$-36.421(3)$

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New coefficients at 4PN and 5PN orders [van de Meent, Shah 2015]

First law of binary mechanics

- Canonical ADM Hamiltonian H of two point masses m_a
- Variation δH + Hamilton's equation + orbital averaging:

$$\delta M = \omega\,\delta L + n\,\delta R + \sum_a \langle z_a \rangle\,\delta m_a$$

- First integral associated with the variational first law:

$$M = 2\,(\omega L + nR) + \sum_a \langle z_a \rangle m_a$$

- These relations are satisfied up to *at least* 3PN order

Applications of the first law

- Conservative dynamics beyond the geodesic approximation
- Shift of the Schwarzschild separatrix and singular curve
- Calibration of EOB potentials for generic bound orbits

$$\frac{\partial M}{\partial m_1} = \langle z \rangle - \omega \frac{\partial \langle z \rangle}{\partial \omega} - n \frac{\partial \langle z \rangle}{\partial n}$$

$$\frac{\partial L}{\partial m_1} = - \frac{\partial \langle z \rangle}{\partial \omega}$$

$$\frac{\partial R}{\partial m_1} = - \frac{\partial \langle z \rangle}{\partial n}$$

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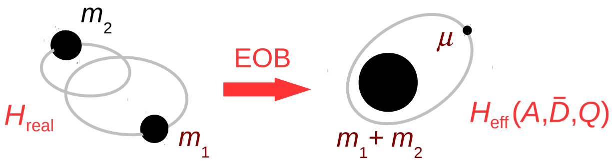
EOB dynamics beyond circular motion

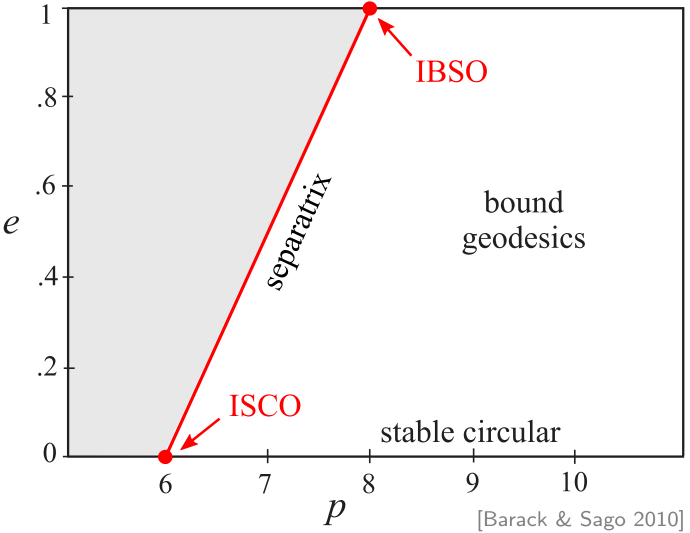


- Conservative EOB dynamics controlled by "potentials"

$$A = 1 - 2M/r + \nu\, a(r) + \mathcal{O}(\nu^2)$$
$$\bar{D} = 1 + \nu\, \bar{d}(r) + \mathcal{O}(\nu^2)$$
$$Q = \nu\, q(r)\, p_r^4 + \mathcal{O}(\nu^2)$$

- Functions $a(r)$, $\bar{d}(r)$ and $q(r)$ related to $\langle z \rangle_{\text{GSF}}(n, \omega)$
- Recently computed numerically [Akcay & van de Meent 2016]

Schwarzschild separatrix



[Barack & Sago 2010]

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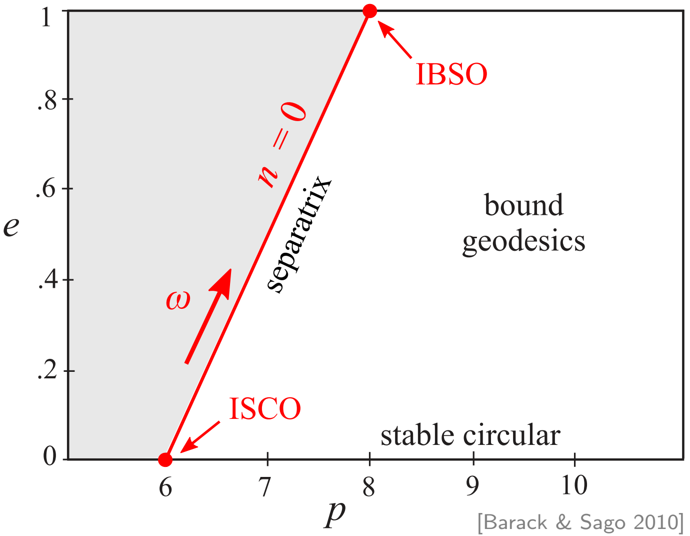


[Barack & Sago 2010]

Shift of the Schwarzschild separatrix

- Separatrix $\omega = \omega_{\text{sep}}(e)$ characterized by the condition

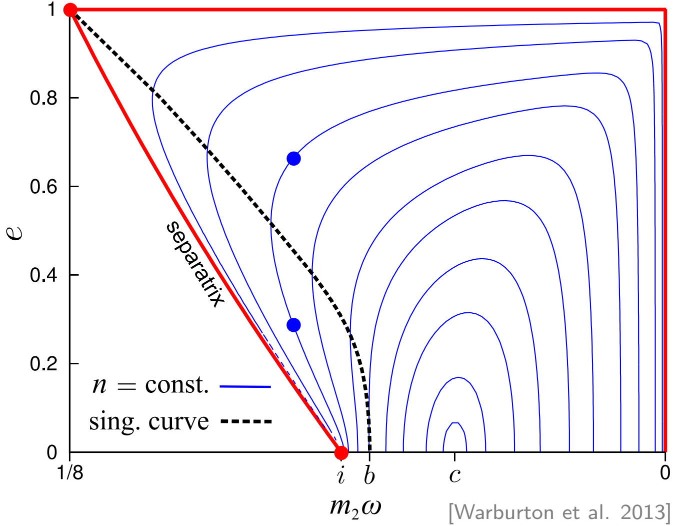
$$n = 0$$

- GSF-induced shift of Schwarzschild ISCO frequency
[Barack & Sago 2009; Le Tiec et al. 2012; Akcay et al. 2012]

$$\frac{\Delta\omega_{\text{isco}}}{\omega_{\text{isco}}} = 1.2101539(4)\, q$$

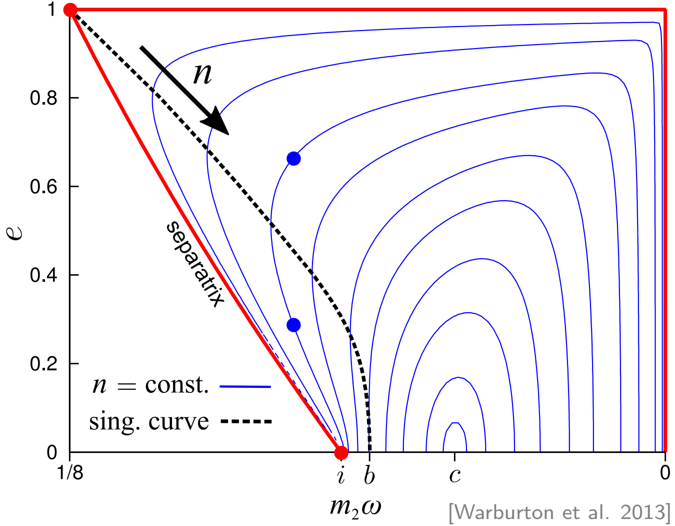
- GSF-induced shift of Schwarzschild IBSO frequency ?
- $\mathcal{O}(q)$ shift in $\omega = \omega_{\text{sep}}(e)$ controlled by $\langle z \rangle_{\text{GSF}}(n, \omega)$

Schwarzschild singular curve



[Warburton et al. 2013]

Schwarzschild singular curve



[Warburton et al. 2013]

Shift of the Schwarzschild singular curve

- Singular curve $\omega = \omega_{\mathrm{sing}}(n)$ characterized by condition

$$\left| \frac{\partial(n,\omega)}{\partial(M,L)} \right| = 0$$

- In the test-particle limit $q \to 0$ this is equivalent to

$$\left[\left(\partial^2_{n\omega} \langle z \rangle \right)^2 - \partial^2_n \langle z \rangle \, \partial^2_\omega \langle z \rangle \right]^{-1} = 0$$

- $\mathcal{O}(q)$ shift in $\omega = \omega_{\mathrm{sing}}(n)$ controlled by $\langle z \rangle_{\mathrm{GSF}}(n,\omega)$

Summary

- GSF/PN comparison for eccentric orbits relying on $\langle z \rangle(n, \omega)$
- First law of mechanics for eccentric-orbit compact binaries
- Numerous applications of the first law:
  - Conservative dynamics beyond the geodesic approximation
  - Shift of the Schwarzschild separatrix and singular curve
  - Calibration of EOB potentials for generic bound orbits
  - ...

Prospects

- GSF/PN comparison for eccentric orbits relying on $\langle \psi \rangle(n, \omega)$
- Extension of the first law to precessing spinning binaries