The Cosmological Memory Effect

Alexander Tolish

With Robert Wald
The University of Chicago

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The Memory Effect

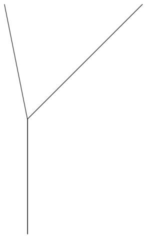
Memory—a permanent change in relative positions of test particles after passage of gravitational wave.

How do we isolate radiative gravitational effects (vs., *e.g.*, Hubble flow)? In an asymptotically flat spacetime, place the detector near $\mathscr{I}^+$; "peel off" dominant $1/r$ term of curvature.

Can we define memory in a spacetime that is not asymptotically flat?

The Memory Effect for Idealized Sources

AT and Wald (2014); AT, Bieri, Garfinkle, and Wald (2014):



What general features of the memory effect can we learn from studying particle interactions?

Retarded metric perturabation:

$$h_{ij} \sim \frac{\Theta(t-r)}{r}.$$

Electric components of the curvature:

$$R_{i00}{}^{j} \sim \frac{\delta'(t-r)}{r}.$$

Integrated geodesic deviation equation:

$$\Delta D^j = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\tau} d\tau' R_{i00}{}^{j}(\tau') D^i.$$

Memory effect:

$$\Delta D^i \sim \frac{\Theta(t-r)}{r}.$$

Takeaways

Memory occurs instantaneously when the detector intersects the decay event's future light cone.

Other effects ("velocity kick") take place over long times, carry higher orders of $1/r$.

Discrete, idealized particle sources $\rightarrow$ discrete, idealized memory effect.

Attributable to single, easily identifiable feature in curvature (δ')—*one which can be generalized unambiguously* to waves on backgrounds without notion of $\mathscr{I}^+$.

Memory Effect in General Spacetimes

Our model of memory—instantaneous change in relative separations of detector test particles, caused by δ' curvature discontinuities radiating away from point-particle interactions.

Only one event contributes to idealized memory and all spacetimes are locally comparable—*we can compare memory of "similar sources" in different spacetimes*.

We will concentrate on comparing memory of similar sources on Minkowski and FLRW backgrounds.

Gravitational Discontinuities on FLRW Backgrounds

Spatially flat FLRW Background:

$$ds^2 = a^2(\eta)\left(-d\eta^2 + dx^2 + dy^2 + dz^2\right)$$

How do we get δ' term in curvature? Durrer (2008):

$$\delta R_{i00}{}^{j} = -\frac{1}{2}\Bigg[\left(\partial_i\partial_k - \frac{1}{3}\delta_{ik}\nabla^2\right)\Phi + \left(\ddot{\Psi} + \frac{\dot{a}}{a}\left(\dot{\Psi} - \dot{\Phi}\right)\right)\delta_{ik} - \partial_{(i}\left(\dot{\Xi}_{k)} + \frac{\dot{a}}{a}\Xi_{k)}\right) + \left(\ddot{\hbar}_{ik} + \frac{\dot{a}}{a}\dot{\hbar}_{ik}\right)\Bigg]\delta^{jk}$$

Φ, Ψ, Ξ_i, $\hbar_{ij}$—gauge-invariant metric potential fields: Bardeen (1980), Durrer (1990).

Memory requires $\Theta(\eta - r)$-discontinuity in Φ, Ψ, Ξ_i, $\hbar_{ij}$.

Only $\hbar_{ij}$ can possess discontinuities away from particle sources.

Tensor-Mode Perturbations

For $g_{ab} = a^2\left(\eta_{ab} + h_{ab}\right)$ and T_{ab}, tensor-mode perturbations are found with a transverse-traceless projector TT:

$$\begin{aligned} \hslash_{ij} &= \mathrm{TT}[h_{\mu\nu}]\,, \\ \mathscr{T}_{ij} &= \mathrm{TT}[T_{\mu\nu}]\,. \end{aligned}$$

Tensor-mode perturbations satisfy the wave equation

$$-\ddot{\hslash}_{ij} - 2\frac{\dot{a}}{a}\dot{\hslash}_{ij} + \nabla^2 \hslash_{ij} = -16\pi\,\mathscr{T}_{ij}\,.$$

The Retarded Gravitational Field

Two spacetimes: $\left(M, \eta_{ab} + \tilde{h}_{ab}\right), \left(M, a^2(\eta_{ab} + h_{ab})\right)$ covered by $(\eta, \mathbf{r})$ with locally similar source events $\tilde{T}_{ab}$, T_{ab}.

$$-\ddot{\tilde{\hbar}}_{ij} + \nabla^2 \tilde{\hbar}_{ij} = -16\pi \tilde{\mathscr{T}}_{ij} \,,$$

$$\tilde{G}(x, x') = \frac{1}{4\pi} \delta\left(-(\eta - \eta')^2 + |\mathbf{r} - \mathbf{r}'|^2\right) \Theta(\eta - \eta')\,;$$

$$-\ddot{\hbar}_{ij} - 2\frac{\dot{a}}{a}\dot{\hbar}_{ij} + \nabla^2 \hbar_{ij} = -16\pi \mathscr{T}_{ij} \,,$$

$$G(x, x') = \frac{1}{4\pi}\frac{a(\eta')}{a(\eta)} \delta\left(-(\eta - \eta')^2 + |\mathbf{r} - \mathbf{r}'|^2\right) \Theta(\eta - \eta') + V(x, x')\Theta\left((\eta - \eta')^2 - |\mathbf{r} - \mathbf{r}'|\right) \Theta(\eta - \eta')\,.$$

“Direct terms” in the retarded fields—

$$\tilde{\hbar}_{ij}^{\text{dir}}(x) = 4\int d^4x'\delta\left(-(\eta-\eta')^2 + |\mathbf{r}-\mathbf{r}'|^2\right)\Theta(\eta-\eta')\tilde{\mathscr{T}}_{ij}(x')$$

$$\hbar_{ij}^{\text{dir}}(x) = 4\int d^4x'\frac{a(\eta')}{a(\eta)}\delta\left(-(\eta-\eta')^2 + |\mathbf{r}-\mathbf{r}'|^2\right)\Theta(\eta-\eta')\mathscr{T}_{ij}(x')$$

—are discontinuous away from source; they do cause memory.

$V(x,x')$ is model-dependent, but smooth—the “tail term”

$$\hbar_{ij}^{\text{tail}}(x) = 4\int d^4x' V(x,x')\Theta\left((\eta-\eta')^2 - |\mathbf{r}-\mathbf{r}'|\right)\Theta(\eta-\eta')\mathscr{T}_{ij}(x')$$

is continuous away from source; it doesn’t cause memory.

We are only concerned with $\hbar_{ij}^{\mathrm{dir}}$, but we drop the "dir" superscript.

Normalize $a = 1$ at the source event. Solving the field equations:

$$\hbar_{ij}(\eta, \mathbf{r}) = \frac{1}{a(\eta)} \tilde{\hbar}_{ij}(\eta, \mathbf{r}) \, .$$

To leading order:

$$\delta R_{i00}{}^{j}(\eta, \mathbf{r}) = \frac{1}{a(\eta)} \tilde{R}_{i00}{}^{j}(\eta, \mathbf{r}) \, .$$

Comparison of Memories

$$\Delta D^i(r) = \frac{1}{1+z}\Delta \tilde{D}^i(r)$$

(r is proper distance at emission)

$$\Delta D^i(d) = \Delta \tilde{D}^i(d)$$

(d is proper distance at detection)

$$\Delta D^i(d_L) = (1+z)\Delta \tilde{D}^i(d_L)$$

(d_L is luminosity distance)