Second-order self-force: a progress report

Adam Pound
in collaboration with Wardell, Warburton, Miller, Barack; Moxon, Flanagan, Hinderer; Yamada, Tanaka, Isoyama

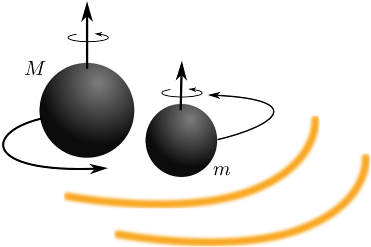
University of Southampton

14 July 2016

Outline

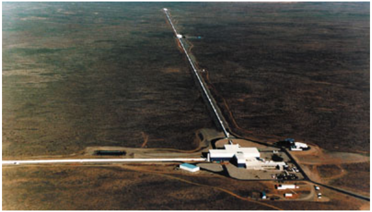
1. Why second-order self-force?
2. Self-force theory: replacing an extended object with a puncture
3. First application: quasicircular orbits in Schwarzschild

Gravitational waves and comparable-mass binaries

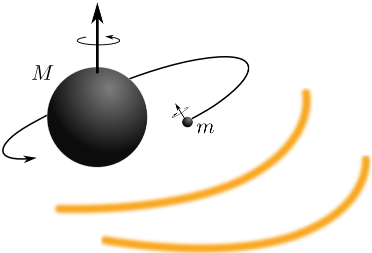


- first directly detected waves were generated by inspiral and merger of two comparable-mass black holes

- ground-based detectors will continue to observe such systems
- but they will not be able to observe binaries with very unequal masses

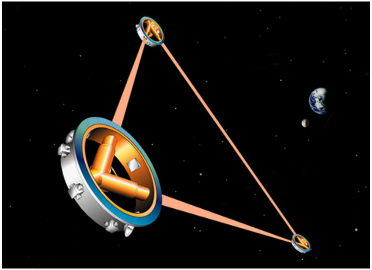


Extreme-mass-ratio inspirals (EMRIs)

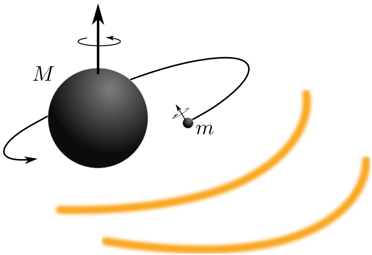


- space-based detector eLISA will observe extreme-mass ratio inspirals of stellar-mass BHs or neutron stars into massive BHs

- small object spends $\sim M/m \sim 10^5$ orbits near BH $\Rightarrow$ unparalleled probe of strong-field region around BH
- measure deviations from GR (with $\gtrsim 10$ times precision of other probes)



Modeling EMRIs: why second-order self-force?



- highly relativistic, strong fields
- disparate lengthscales
- long timescale: inspiral is slow, produces $\sim \frac{M}{m} \sim 10^5$ wave cycles

- treat m as source of perturbation of M's metric $g_{\mu\nu}$:

$$\mathsf{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \ldots$$

- represent motion of m via worldline z^μ satisfying

$$\frac{D^2 z^\mu}{d\tau^2} = \epsilon F_1^\mu + \epsilon^2 F_2^\mu + \ldots$$

- scaling arguments show we need F_2^μ for accurate model

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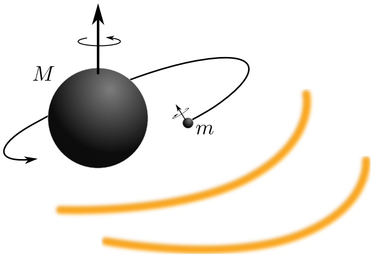

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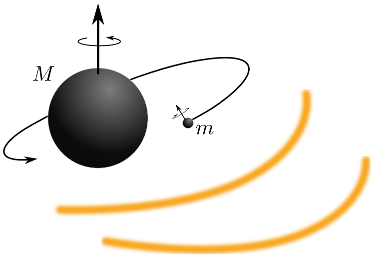

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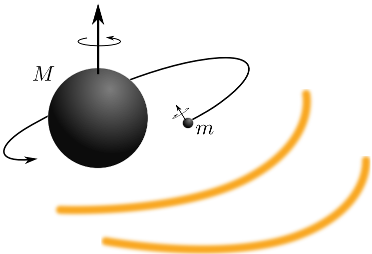

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- disparate lengthscales
  $\Rightarrow$ *can't use numerical relativity*
- long timescale: inspiral is slow, produces $\sim \frac{M}{m} \sim 10^5$ wave cycles
  $\Rightarrow$ *need a model that is accurate over those $\sim 10^5$ cycles*

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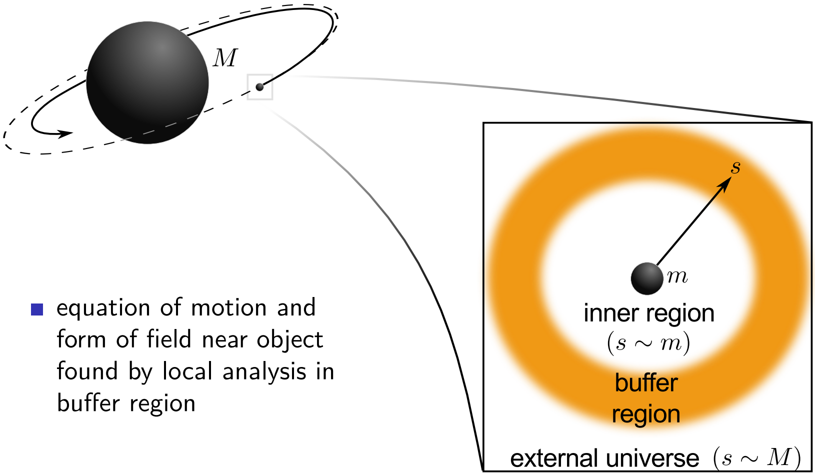
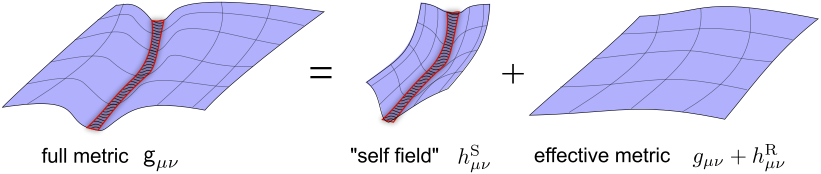
Matched asymptotic expansions



- equation of motion and form of field near object found by local analysis in buffer region

Punctures and EOM [Detweiler & Whiting; Barack et al; AP; Gralla; Harte]

- split field into “self-field” and “effective field”
- locally replace self-field with singular field



- replaces object with a *puncture*, a local singularity in the field, moving on z^μ, equipped with the object's multipole moments
- z^μ geodesic in $g_{\mu\nu} + h^{R}_{\mu\nu}$:

$$\frac{D^2 z^\mu}{d\tau^2} = -\frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(g_\nu{}^\rho - h^{R\rho}_{\nu}\right)\left(2h^{R}_{\rho\sigma;\lambda} - h^{R}_{\sigma\lambda;\rho}\right)u^\sigma u^\lambda + O(m^3)$$

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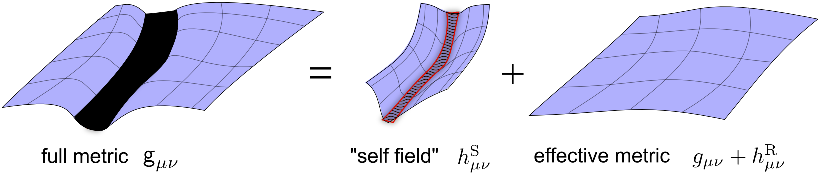

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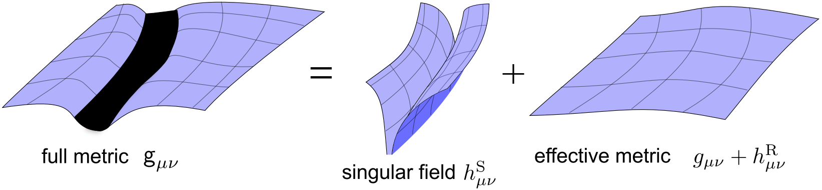

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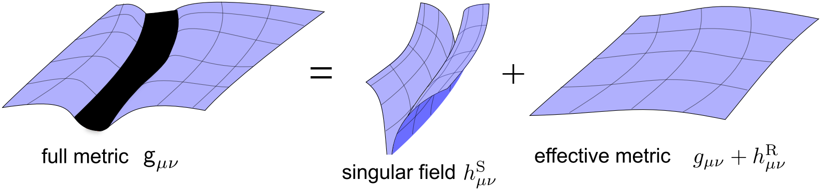

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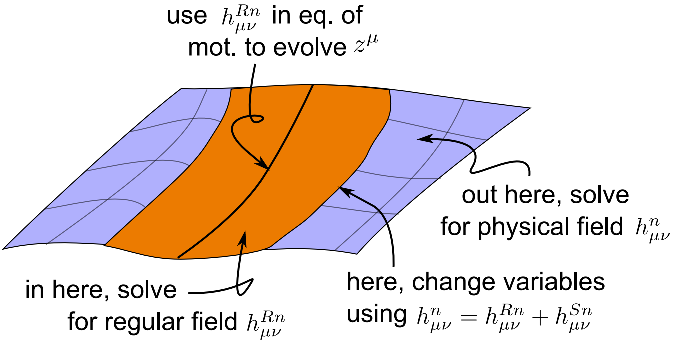
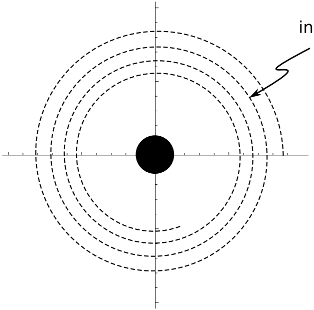
How you replace an object with a worldline

- in region near z^μ, move $h^S_{\mu\nu}$ to RHS of field equation, solve for $h^R_{\mu\nu}$



Quasicircular orbits [AP, Wardell, Warburton, Miller, Barack]



Introduce slow time $\tilde{t} \sim \epsilon t$

- multiscale expansion of the worldline:
  - radius $r_p = r_0(\tilde{t}) + \epsilon r_1(\tilde{t}) + \dots$
  - frequency $\Omega = \Omega_0(\tilde{t}) + \epsilon\Omega_1(\tilde{t}) + \dots$
  - orbital phase $\phi_p = \frac{1}{\epsilon}\int \Omega d\tilde{t}$
- multiscale expansion of the field:

$$h^n_{\mu\nu} = \sum_{ilm} h^n_{ilm}(\tilde{t}, r) e^{-im\phi_p(\tilde{t})} Y^{ilm}_{\mu\nu}$$

- solve numerically for h^n_{ilm} at fixed $\tilde{t}$ using standard frequency-domain methods from 1st order
- evolve $\tilde{t}$ dependence using equation of motion
- we take a "snapshot", doing our calculations at some $\tilde{t} = \tilde{t}_0$

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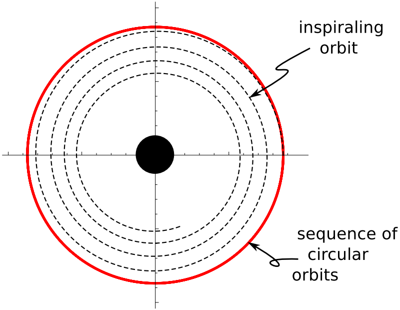

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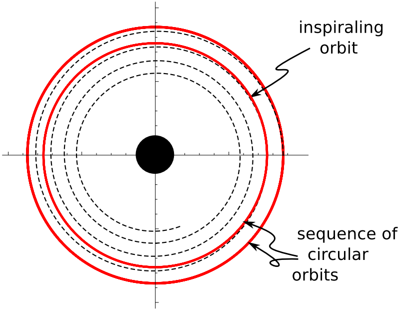

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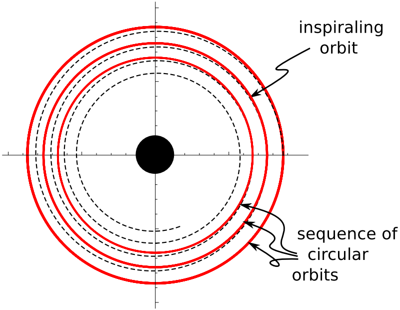

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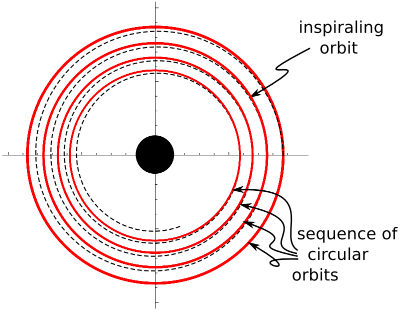

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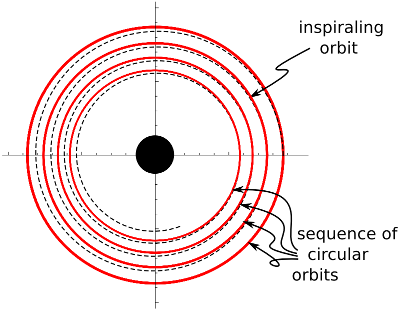

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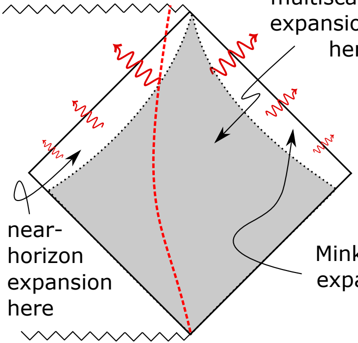


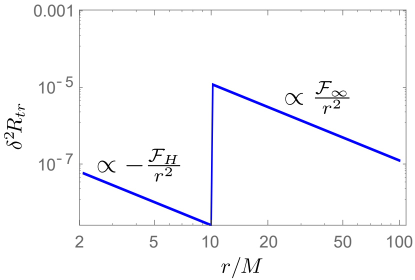
- multiscale expansion works on large scales, but not globally
- so use different expansions in different regions

- post-Minkowski and near-horizon expansions provide BCs $h^{\infty}_{\mu\nu}$ and $h^{\mathcal{H}}_{\mu\nu}$ for multiscale expansion
- we subtract the BCs from the field, numerically solve ODEs for residual $h^{\mathcal{R}}_{\mu\nu} = h_{\mu\nu} - h^{S}_{\mu\nu} - h^{\infty}_{\mu\nu} - h^{\mathcal{H}}_{\mu\nu}$,

$\ell = 0$, dissipative sector

- field equation:

$$\partial_r^2 h_{tr}^{\mathcal{R}2} \sim \delta^2 R_{tr}[h^1, h^1] - \partial_r^2 h_{tr}^{S2} - \partial_{\tilde{t}} h_{tt}^1$$



What comes out of the solution?

- balance law: $\dot{E}_0 + \delta \dot{M}_{BH} = \mathcal{F}_\infty$
- first major result/consistency check of numerical implementation

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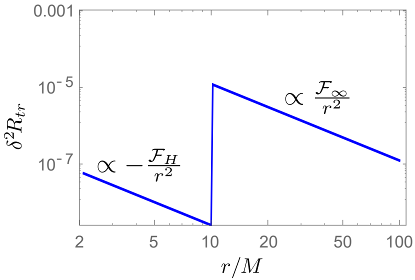

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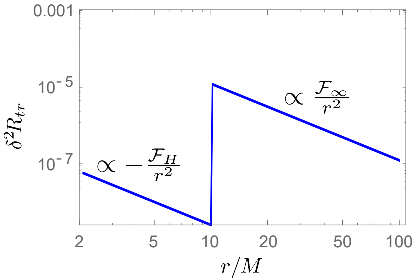

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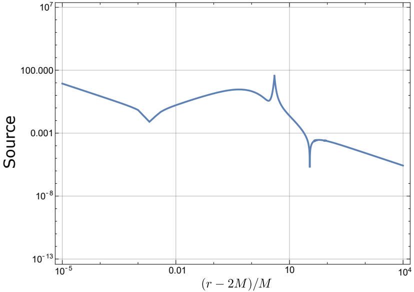

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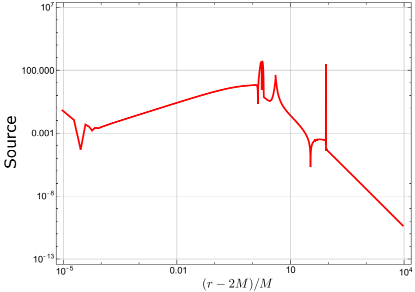

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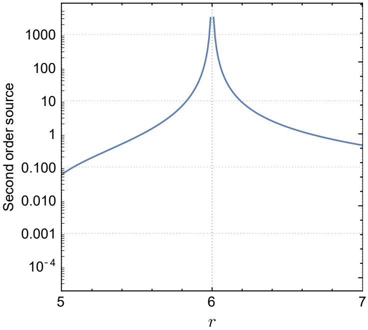

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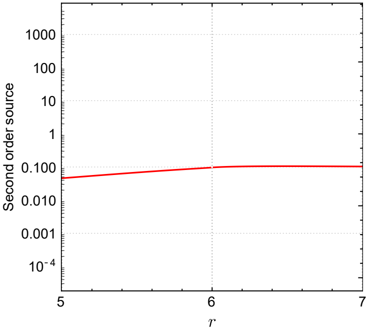

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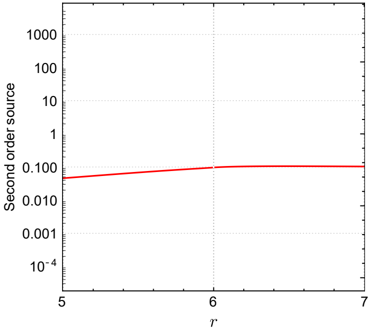

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Conclusion

Status of formalism

- basic formalism in place
- practical multiscale expansion under development

Status of implementations

- everything is working properly for quasicircular orbits in Schwarzschild—just more labour needed