Gravitational lensing beyond geometric optics

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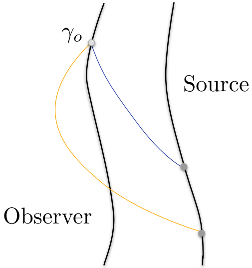
Traditional gravitational lensing

- Light, high-freq. GWs, etc. move on null geodesics,



- Intensity follows from cross-sectional areas of null geodesic bundles.
- Polarization is parallel-transported, . . .

First principles

Maxwell theory doesn't look anything like this:

$$dF = 0, \qquad d \star F = 4\pi J.$$

How does one go from here to the rules of geometric optics?

Deriving geometric optics

Consider a 1-parameter family of fields [Ehlers (1967), ...]

$$F(x;\omega) \sim e^{i\omega S(x)} \sum_{n=0}^{\infty} \frac{f_n(x)}{\omega^n}.$$

Substituting into Maxwell's equations while using $\omega \to \infty$ gives

- Momentum fluxes point along $k_a := \nabla_a S$, and this is null.
- Amplitude transported along null geodesics: $2k \cdot \nabla f_0 + f_0 \nabla \cdot k = 0$.
- Amplitudes evolve via cross-sectional areas: $\nabla_a(|f_0|^2 k^a) = 0$.

When is geometric optics sufficient?

If there's one lengthscale r, fractional corrections look like

$$1 + \frac{(\cdots)}{\omega r} + \frac{(\cdots)}{(\omega r)^2} + \ldots$$

So everything is fine if $\omega r \gg 1$ (i.e. $r \gg \lambda$).

Two lengthscales?

More dimensionless combinations are possible:

$$1 + \left[\frac{(\cdots)}{\omega r} + \frac{(\cdots)}{\omega \ell} + \frac{(\cdots) r}{\omega \ell^2} + \ldots \right] + \ldots$$

Could some effects grow with distance r?

Why generalize geometric optics?

1. Wave effects might accumulate over large distances.
2. Learn something using high precision measurements.
3. Caustics.

Generalizing geometric optics

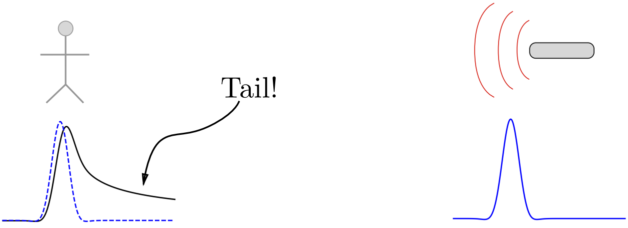
Straightforward to extend high-frequency ansatz to higher orders in ω^{-1} [Anile (1976), . . .]

But:

1. Is this ansatz appropriate? Too special?
2. Too general? Maybe equations allow too much.
3. How are field properties related to source properties?

Signaling experiments

Intervening spacetime distorts signals:



(output) $= \mathcal{T}[$(input)$]$ with $\mathcal{T}$ nonlocal and usually linear.

Find "transfer function" $\mathcal{T}[\cdot]$

Method: Green functions

For an EM field, there's some $G_{aa'}(x, x')$ such that

$$\nabla^b \nabla_b G_{aa'} - R_a{}^b G_{ba'} = -4\pi\delta(x, x').$$

Measured fields then look like

$$F_{ab}[J] = 2 \int \nabla_{[a} G_{b]b'} J^{b'} dV'$$

Lots known about Green functions. Can use well-developed bitensor results [Hadamard, Friedlander, DeWitt, Poisson, ...]

Point sources

Specialize for sources satisfying

$$J^a(x) = \int d\tau [\cancelto{\text{boring}}{q\dot{z}^a} + \mu^{ab}(\tau)\nabla_b + \ldots]\delta(\gamma_s(\tau), x)$$

- Radiation depends on dipole moment $\mu^{ab} = \mu^{[ab]}(\tau)$.
- Find $F_{ab} = F_{ab}[\mu^{cd}]$ as μ^{cd} varies rapidly...

Fourier transforms

$$F_{ab}(x) = (\cdots) - 2 \int \nabla_{a'} \nabla_{[a} G_{b]b'}(z(\tau), x) \mu^{a'b'}(\tau) d\tau$$

So if $\mu(\tau) \sim e^{i\omega\tau}$,

$$F_{ab}(x) \sim \mathcal{F}[\nabla \nabla' G](\omega)$$

Everything reduces to Fourier transforms of $G(z(\tau), x)$ wrt τ.

High-frequency limits

Geometric optics should be recovered as $\omega \to \infty$.

$\hookrightarrow$ Look at high-ω behavior of Fourier transforms.

"Almost-geometric" optics

[High-frequency signals] $\leftrightarrow$ [Most singular bits of $G(z(\cdot), x)$]

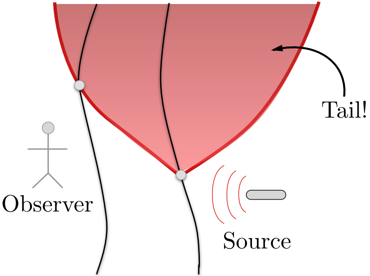
[Riemann-Lebesgue lemma, Paley-Wiener thms, etc.]

An immediate conclusion

Singularities of $G(z(\cdot), \gamma_o)$ concentrated on null geodesics from $\gamma_o \to \mathcal{S}$
$\hookrightarrow$ Geometric optics at leading order

All $O(\omega^{-n})$ effects come from neighborhood of observer's past light cone

What about tails?



Smooth tails away from null geodesics:

$\hookrightarrow$ $\mathcal{F}$[(smooth)](ω) decays faster than any polynomial, so

tails give nonperturbative corrections.

Detailed calculation gives

- (Polarization-dependent) phase shifts
- Frequency-dependent source directions
- Intensity shifts
- ...

All in terms of Hadamard-type transport equations along null geodesics.

Conclusions

1. Framework to describe gravitational lensing beyond geometric optics
2. Connects to bitensor constructions used in other contexts
3. Tail effects are "exponentially suppressed" away from null geodesics

Work in progress: Find interesting applications!